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# Image Denoising Using BM3D Combining Tetrolet Prefiltering

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**Abstract:** Image denoising is the hot spot of digital image processing and BM3D algorithm achieves an outstanding denoising performance, but BM3D has a sharp drop in denoising result when noise standard deviation reaches 40. In order to resolve this problem, an improved version of BM3D is proposed which combines BM3D with Tetrolet prefiltering. Tetrolet transform on strongly noisy image is done to remove part of the noise, then BM3D is executed. The experimental results show that the proposed modification delivers better performance than BM3D on the same level of running time, both in terms of objective criteria and visual quality.

**Key words:** Image denoising, BM3D, tetrolet prefiltering, PSNR, SSIM

## INTRODUCTION

The denoising of noisy images is an important task in image processing which has been got an increasing attention since the higher image quality requirement is needed in imaging analysis and follow-up applications. A wide variety of denoising methods have been proposed in the past decades, such as probability theory disciplines, statistics disciplines, partial differential equations, linear and nonlinear filtering, spectral and multiresolution analysis. Recently, a novel denoising algorithm, 3D transform domain collaborative filtering (BM3D), is introduced by Dabov *et al.* (2007) which is currently recognized as one of the best denoising algorithm. However, the denoising performance of BM3D drops sharply when noise standard deviation reaches 40 (Hou *et al.*, 2011). Dabov *et al.* (2008, 2009) improved the original algorithm. When noise level is low, it has a better denoising effect (Dabov *et al.*, 2008); but for relatively high noise level, the denoising performance is even worse than the original BM3D. To the method proposed by Dabov *et al.* (2009), it really promotes the quality of denoising image, but it causes a big problem of the computational burden as the time is nearly fifty times than before. In this paper, an improved version of BM3D is proposed which combines BM3D with Tetrolet prefiltering to achieve a better denoising performance when the additive noise is strong. That is, Tetrolet transform is applied to strongly noisy image to remove part of the noise before the execution of BM3D. Experimental results show that the proposed method outperforms BM3D as well as the methods combining BM3D with other wavelets frefiltering. On the other hand, although the denoising effect of our method is slightly worse than BM3D-SAPCA (Dabov *et al.*, 2009), our proposed method greatly reduces the execution time than it.

## TETROLET TRANSFORM

Tetrolets are adaptive Haar-type wavelet filter bank whose supports are the shapes called tetrominoes made by connecting four equal-sized squares (Krommweh, 2010). They are some geometric shapes in the famous computer game 'Tetris' (Demaine *et al.*, 2004). Disregarding rotations and reflections there are five basic free kinds of tetrominoes which are shown in Fig. 1.

Through the averaging sum or differences of every 4 pixels in a 2×2 square, low-pass or high-pass filters are obtained in the 2D classical Haar case. The matrix has four fixed 2×2 squares for disjoint covering of a 4×4 board (Wang *et al.*, 2013).

$$W := (\varepsilon[m,n])_{m,n=0}^{3} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \tag{1}$$

If the local image geometry is taken into account, Tetrolet transformation of the image is formed. The image $a = [a(i,j)]_{i,j=1}^{N}$ is divided into 4×4 blocks. Each block is covered by four tetrominoes which form the adaptive basis. These tetrominoes are presented by $\{I_0, I_1, I_2, I_3\}$ which are bijectively mapping into {0, 1, 2, 3} as their corresponding order. Then the discrete basis functions of Tetrolets are defined as follows:

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$$\phi_{I_v}[i,j] := \begin{cases} 1/2, (i,j) \in I_v \\ 0, \ \text{else} \end{cases} \quad (2)$$

$$\psi^l_{I_v}[i,j] := \begin{cases} \varepsilon[l, L(i,j)], & (i,j) \in I_v \\ 0, & \text{else} \end{cases} \quad (3)$$

where, $I_v$ is the reference block index, I = 1, 2, 3. Obviously, there are 117 kinds of tilings for disjoint covering of a 4×4 board with any four tetrominoes (Krommweh, 2010). The Tetrolet basis can adapt to local structures when we compute the 'optimal' partition of the block, because of their more partitions than the conventional Haar wavelets. Fig. 2a shows the example of the fixed squares of the 2D Haar wavelets. Fig. 2b shows one of the 117 kinds of tilings. Its corresponding local structure can be seen in Fig. 2c. The tiling in Fig. 2b is more efficient than the fixed squares in Fig. 2a. In fact, Fig. 2a, b show two of the 117 kinds of tilings. Thus, the original Haar wavelet is a special transform of the Tetrolet.

The flow chart of the algorithm of image Tetrolet transform is shown in Fig. 3. In order to obtain a processed image approximation, one can apply a suitable

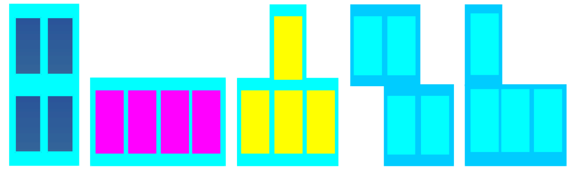

Fig. 1: The five free kinds of tetrominoes

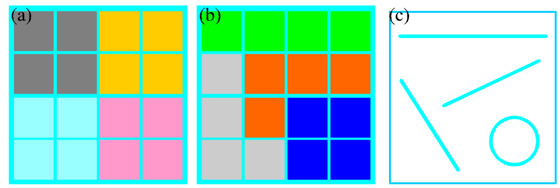


Fig. 2(a-c): Two examples of covering a 4×4 board and the local structure of the second (a) The fixed squares of the 2D Haar wavelets; (b) One of the 117 kinds of tilings for disjoint covering of a 4×4 board with tetrominoes; (c) Corresponding local structure of the second

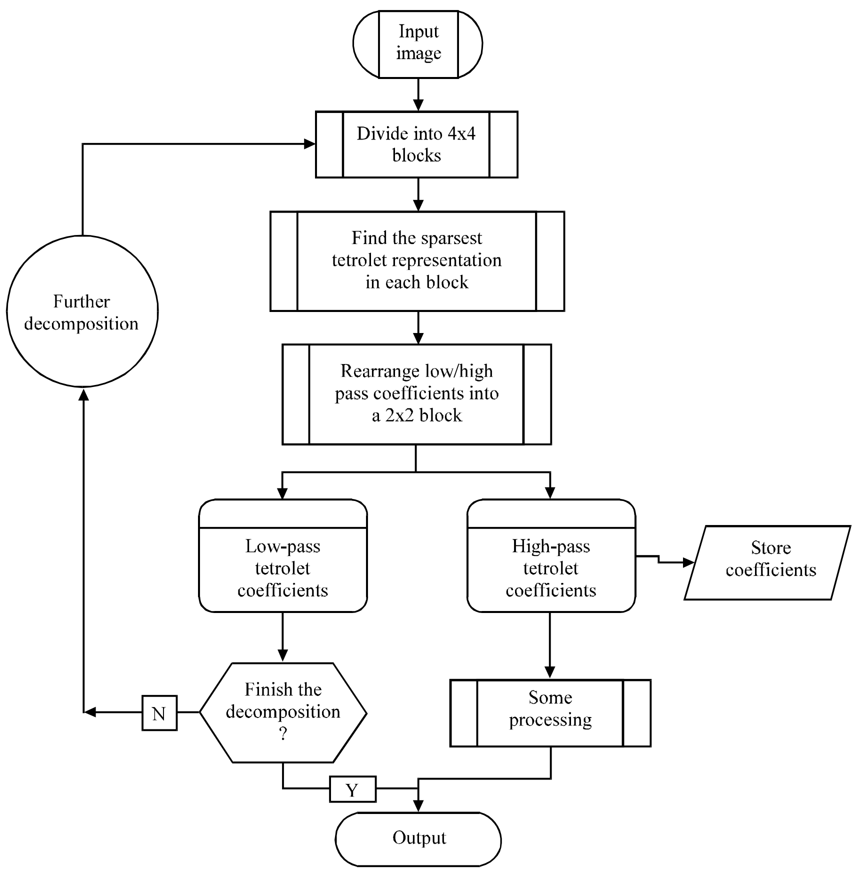


Fig. 3: Adaptive tetrolet transform algorithm

shrinkage procedure to the Tetrolet coefficients and reconstruct the image.

The geometric adaptive transform is particularly suited to the sparse representation of an image. The basic functions of tetromino support can adapt different directions in images. Because of its non-redundancy, the transform in denoising applications is not very effective (Krommweh, 2010). However, it can be combined with other denoising technologies such as BM3D which is described in the following paragraph.

## BM3D COMBINING TETROLET PREFILTERING

Suppose that an original image y has been corrupted by additive noise according to the following model:

$$z(x) = y(x)+\eta(x) \tag{4}$$

Our task is to get the original image y as soon as possible from the noisy image z. Here we recall the state-of-the-art image denoising algorithm, BM3D which takes the advantages of the transform domain denoising method and the local average method. It is based on an enhanced sparse representation in transform domain and shrinks the transform spectrum to attenuate the noise.

The algorithm contains the following two major steps Dabov *et al*. (2007):

- The first step estimates the denoised image

Firstly, find out blocks which are close to the currently processed one using the Eq. 5 and 6. Then store them together to form a 3D group (BM):

$$d(Z_{x_R}, Z_x) = \frac{\| \gamma'(T_{2D}^{ht}(Z_{x_R})) - \gamma'(T_{2D}^{ht}(Z_x)) \|_2^2}{(N_1^{ht})^2} \tag{5}$$

$$S_{x_R}^{ht} = \left\{ x \in X : d(Z_{x_R}, Z_x) \le \tau_{match}^{ht} \right\} \tag{6}$$

Secondly perform 3D transform to the group, denoise by hard-thresholding of the transform coefficients, then invert the 3D transform and return the estimates of the blocks to their original positions.

$$\hat{Y}_{S_{x_R}^{ht}}^{ht} = T_{3D}^{ht^{-1}}(\gamma(T_{3D}^{ht}(Z_{S_{x_R}^{ht}}))) \tag{7}$$

Finally use weighted average all of the obtained overlapping estimates to obtain the basic estimate of the image y:

$$\omega_{x_R}^{ht} = \begin{cases} \dfrac{1}{\sigma^2 N_{har}^{x_R}}, & \text{if } N_{har}^{x_R} \ge 1 \\ 1, & \text{otherwise} \end{cases} \tag{8}$$

$$\hat{y}^{basic}(x) = \frac{\sum_{x_R \in X} \sum_{x_m \in S_{x_R}^{ht}} \omega_{x_R}^{ht} \hat{Y}_{x_m}^{ht, x_R}(x)}{\sum_{x_R \in X} \sum_{x_m \in S_{x_R}^{ht}} \omega_{x_R}^{ht} \chi_{x_m}(x)}, \forall x \in X \tag{9}$$

- The second step is based both on the original noisy image and the basic estimate

Firstly, determine the locations of the blocks close to the reference one by block-matching in the basic estimate. With the obtained locations, form two 3D groups which are from the noisy image and the basic estimate.

$$S_{x_R}^{wie} = \left\{ x \in X : \frac{\| \hat{Y}_{x_R}^{basic} - \hat{Y}_x^{basic} \|_2^2}{(N_1^{wie})^2} \le \tau_{match}^{wie} \right\} \tag{10}$$

Secondly perform 3D transform to both groups. Perform Wiener filtering on the noisy one with the energy spectrum of the basic estimate. The next practices are similar to the corresponding practices in the first step:

$$W_{S_{x_R}^{wie}} = \frac{|T_{3D}^{wie}(\hat{Y}_{S_{x_R}^{wie}}^{basic})|^2}{|T_{3D}^{wie}(\hat{Y}_{S_{x_R}^{wie}}^{basic})|^2 + \sigma^2} \tag{11}$$

$$\hat{Y}_{S_{x_R}^{wie}}^{wie} = T_{3D}^{wie^{-1}}(W_{S_{x_R}^{wie}} T_{3D}^{wie}(Z_{S_{x_R}^{wie}})) \tag{12}$$

Finally use a weighted average of the estimates to get a final estimate image.

$$\omega_{x_R}^{wie} = \sigma^{-2} \| W_{S_{x_R}^{wie}} \|_2^{-2} \tag{13}$$

$$\hat{y}^{final}(x) = \frac{\sum_{x_R \in X} \sum_{x_m \in S_{x_R}^{wie}} \omega_{x_R}^{wie} \hat{Y}_{x_m}^{wie, x_R}(x)}{\sum_{x_R \in X} \sum_{x_m \in S_{x_R}^{wie}} \omega_{x_R}^{wie} \chi_{x_m}(x)}, \forall x \in X \tag{14}$$

As stated above, BM3D denoising method can achieve rather good results in both the objective evaluation and subjective visual. However, when the noise level is high, the probability densities of the different $d(Z_{XR}, Z_x)$ are likely to overlap heavily. Blocks with greater ideal distances than the threshold are grouped as similar, whereas blocks with smaller such distances are left out (Dabov *et al*., 2007).

In order to minimize the impact of the above problem, we propose to combine BM3D with Tetrolet prefiltering. For a strongly noisy image, Tetrolet prefiltering is performed to remove part of the noise. Due to its full-consideration to the local structure of an image, the details of the original image can be well preserved. At the same time, part of the noise is filtered out. Because local

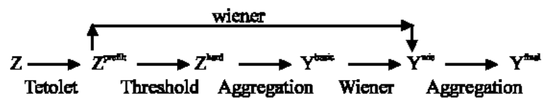


Fig. 4: General process of the proposed algorithm

structures are not damaged, BM3D denoising algorithm can be performed which makes full use of the internal similar details of the image. The proposed algorithm is summarized in Fig. 4.

## NUMERICAL EXPERIMENTS

In this section, the denoising results of the proposed BM3D method are presented by compassion with the original BM3D, BM3D combined with other wavelets prefiltering as well as BM3D-SAPCA. All experiments were conducted on the computer with a Core (TM) 2 2.00 GHz Duo CPU and 2 GB Rom. When performing the Tetrolet prefiltering of the noisy image, we apply the complete wavelet decomposition to image and use wavelet shrinkage with global hard-thresholding by choosing a certain number of largest wavelet coefficients. The determination of the number of retained coefficients is empirical. In general, the more seriously the image is noised, the smaller the number is. In order to reduce the execution time, a relaxed version ('Tetrolet 16 rel edge') is adopted (Krommweh, 2010). To our delight, the experimental results show that it does not reduce denoising effect at all.

The evaluation criterion is given in two commonly used ways, one is the Peak Signal to Noise Ratio (PSNR) and the other is the Structural Similarity Index (SSIM). PSNR is based on the mean squared error (MSE) which needs to be minimized.

$$\mathrm{MSE} = \frac{1}{M \times N} \| f - \hat{f} \|^2 = \frac{1}{M \times N} \sum_{x=1}^{M} \sum_{y=1}^{N} (f_{xy} - \hat{f}_{xy})^2 \tag{15}$$

$$\mathrm{PSNR} = 10 \log_{10}\left(\frac{255^2}{\mathrm{MSE}}\right) \tag{16}$$

A serious problem of PSNR is that it does not measure the resulting image quality directly and can attribute similar scores to images with large differences in psychovisual quality (Da Silva *et al*., 2012). To compare images which correlate more appropriately with the human perception, SSIM is proposed by Wang *et al*. (2004) where higher values are given to more similar pairs of two images. To facilitate the observation and comparison, every SSIM value is multiplied by 256, calculated as:

$$\mathrm{SSIM} = \frac{256(2\mu_A\mu_B + c_1)(2\sigma_{AB} + c_2)}{(\mu_A^2 + \mu_B^2 + c_1)(\sigma_A^2 + \sigma_B^2 + c_2)} \tag{17}$$

where, $\mu_A$, $\mu_B$, $\sigma^2_A$ and $\sigma^2_B$ are the averages and variances of image A and B respectively, $\sigma_{AB}$ is the covariance between A and B and $c_1$ and $c_2$ are predefined constants.

In the presence of weak additive noise, BM3D is stable and performs well. However it becomes unreliable with the increase of noise because the BM3D can't get an effective sparse representation. When the noise variance level is more than 40, the grouping performs badly that blocks matched are not really similar in the noise free image (Shen *et al*., 2012). Table 1 and 2 give the experimental results to the high noise level image ($\sigma$>40) in terms of PSNR and SSIM, respectively. Obviously, our algorithm performs stably and satisfactory which is superior to the original BM3D method as well as the methods combining BM3D with other wavelets frefiltering, especially in the case of SSIM. Tetrolet frefiltering removes noise by obtaining an efficient image representation that characterizes the significant image features in a compact form. Removing part of the noise makes subsequent block groups efficient, where there are more similar blocks to the reference one as it reduces interference from strong noise and improves the accuracy of matching. Conventionally, the 2D discrete wavelet transform (DWT) works on the rows and columns of an image independently. Therefore, the horizontal and vertical directions are preferred and the DWT fails to achieve optimal results with images that contain geometric structures in other directions (Krommweh, 2010). While Tetrolet transform, whose local basis is adapted to the image geometry, takes the image's significant features into consideration. The detail of an image can be well preserved.

Fig. 5 and 6 show that less artifacts are introduced such as the area around the stem of peppers. What's more, the details of the denoised image such as the edges of peppers and cap are better reserved by the proposed method than by the original one. The enlarged fragments in Fig. 6 help to demonstrate the good quality of the denoised images in terms of faithful detail preservation and very few disturbing artifacts.

In addition, compared with BM3D-SAPCA, the denoising effect of our method is slightly worse, but the execution time is significantly decreased indeed.

Table 1: PSNR of denoised image

| Peppers 256 | | +BM3D | | | | | |
|---|---|---|---|---|---|---|---|
| σ | BM3D | haar | bior1.5 | db4 | sym4 | coief2 | Proposed tetrolet+BM3D |
| 41 | 27.5517 | 27.5517 | 27.5517 | 27.5517 | 27.5517 | **27.6220** | 27.5271 |
| 45 | 26.8736 | 27.0888 | 27.0527 | 27.1031 | **27.1355** | 27.1131 | 27.0890 |
| 50 | 25.9787 | 26.4428 | 26.3764 | 26.4318 | 26.4451 | 26.4467 | **26.5721** |
| 60 | 24.7851 | 25.1585 | 25.0878 | 25.0927 | 25.1186 | 25.1259 | **25.6206** |
| 70 | 23.9468 | 24.1798 | 24.1403 | 24.1342 | 24.1294 | 24.1470 | **24.6676** |
| 80 | 23.1321 | 23.4657 | 23.3779 | 23.4194 | 23.4118 | 23.4366 | **23.7585** |
| 100 | 21.8571 | 22.2025 | 22.1594 | 22.1458 | 22.1929 | **22.2441** | 22.2198 |

Table 2: SSIM of denoised image

| Peppers 256 | | +BM3D | | | | | |
|---|---|---|---|---|---|---|---|
| σ | BM3D | haar | bior1.5 | db4 | sym4 | coief2 | Proposed tetrolet+BM3D |
| 41 | 206.245 | 206.245 | 206.245 | 206.245 | 206.245 | 204.697 | **206.654** |
| 45 | 200.568 | 200.529 | 201.835 | 199.140 | 200.551 | 200.595 | **203.364** |
| 50 | 192.462 | 195.091 | 194.680 | 193.155 | 196.057 | 194.827 | **199.212** |
| 60 | 179.208 | 184.402 | 184.551 | 183.208 | 184.213 | 185.086 | **193.618** |
| 70 | 170.731 | 176.707 | 176.093 | 175.125 | 175.063 | 176.357 | **185.546** |
| 80 | 161.677 | 170.775 | 168.510 | 168.166 | 167.766 | 168.884 | **179.296** |
| 100 | 148.234 | 157.596 | 155.652 | 155.045 | 154.738 | 156.144 | **167.462** |

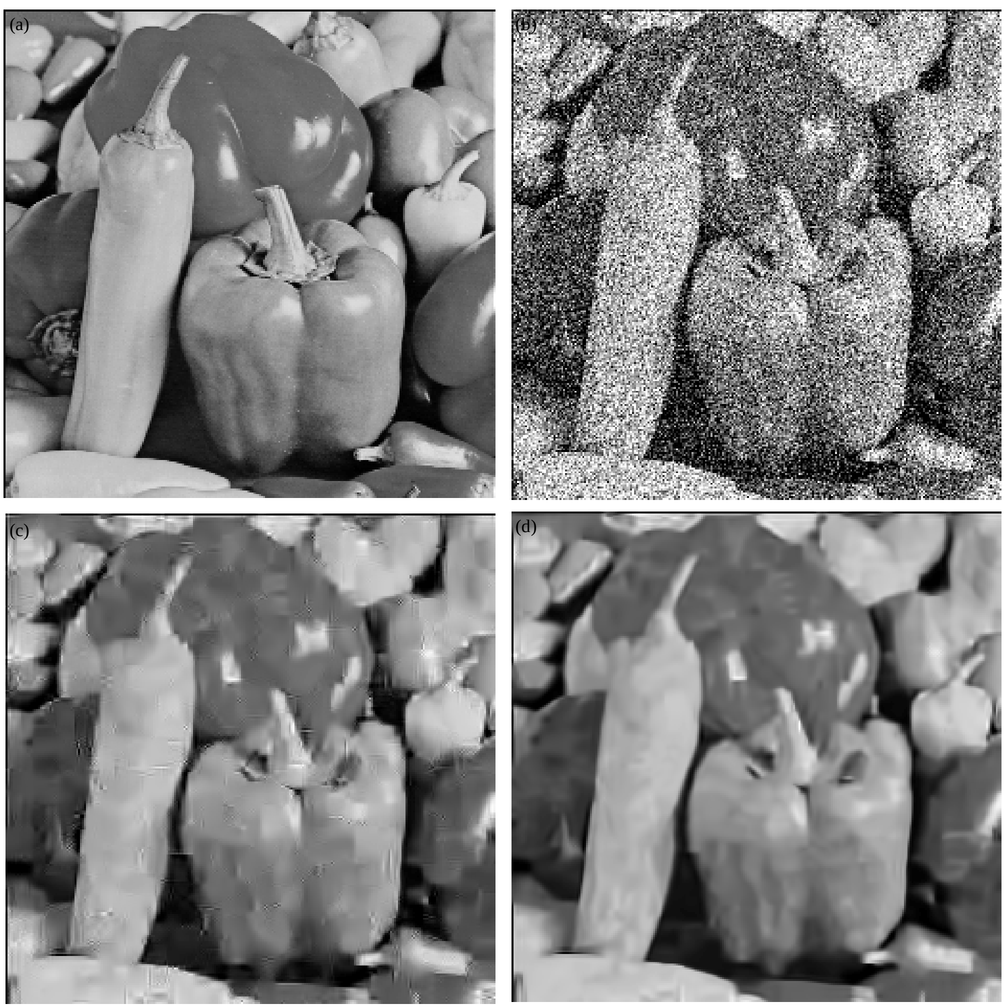


Fig. 5(a-d): Experimental results for image peppers 256 (a) Original image; (b) Nosiy image (σ = 60); (c) Result of BM3D, PSNR = 24.78 dB; (d) Result of our method, PSNR = 25.62 dB


Fig. 6(a-d): Experimental results for image Lena512 ($\sigma$ = 60) (a) Result of BM3D, PSNR = 26.69 dB, SSIM = 180.62; (b) Result of our method, PSNR = 27.76dB, SSIM = 193.46; (c) Fragment of image (a); (d) Fragment of image b

Table 3: PSNR, SSIM and time performance of denoised image

| peppers 256 ($\sigma$ = 60) | BM3D | BM3D-SAPCA | Tetrolet+BM3D |
|---|---|---|---|
| PSNR | 24.7851 | 26.0603 | 25.6206 |
| SSIM | 179.208 | 194.5272 | 193.618 |
| Time | 2.3 | 80.8841 | 5.0 |

Table 3 presents the comparison of PSNR, SSIM and corresponding running time.

Overall, the proposed method outperforms in terms of objective criteria and obtains good visual quality for relatively high levels of noise.

## CONCLUSION

In this study, an effective algorithm is proposed for image denoising by combining BM3D with Tetrolet prefiltering. This algorithm inherits the advantages of Tetrolet's well-preservation of image edge and other detail structure and BM3D's excellent denoising performance. Improving PSNR greatly at the same time, it significantly up-grades SSIM and also gets a good visual effect.

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