

<http://ansinet.com/itj>

ITJ

ISSN 1812-5638

# INFORMATION TECHNOLOGY JOURNAL

**ANSI***net*

Asian Network for Scientific Information  
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

## Simulation and Analysis of Random Switching Frequency Space Vector Pulse Width Modulation

<sup>1</sup>Guoqiang Chen, <sup>2</sup>Zhihong Wu, <sup>2</sup>Yuan Zhu and <sup>1</sup>Junwei Zhao

<sup>1</sup>School of Mechanical and Power Engineering, Henan Polytechnic University, China

<sup>2</sup>Chinese-German School for Postgraduate Studies, Tongji University, China

---

**Abstract:** In order to solve the severe problem of mechanical vibration, audible noise and electromagnetic interference resulting from the prominent amplitude harmonic, the random space vector pulse width modulation has important application prospects and has been widely discussed. The random switching frequency PWM technique has the most excellent performance among several types of random strategies. Aiming at the spectrum spreading performance, the study simulates and discusses the random switching frequency pulse width modulation of the two-level three-phase inverter. The two factors-switching frequency range width and probability distribution law-that affect the spreading effectiveness are analyzed in theory based on the theoretical spectra expression. The switching frequency range width factor in theory is consistent with the experiment result. The simulation verifies the theoretical analysis. The simulation shows that it is feasible to find an optimal probability distribution law that can improve the spectrum spreading effectiveness.

**Key words:** Three-phase inverter, random switching frequency, probability distribution law, spectrum, random PWM

---

### INTRODUCTION

The two-level three-phase inverter has been widely utilized in all kinds of fields because it can generate the alternate electronic source with the variable frequency and amplitude (Hava, 1998). Pulse Width Modulation (PWM) technique lays the foundation for the inverter. Based on the volt-second balance principle, the output voltage is a series of voltage pulses through the power electron device (Hava, 1998; Chen, 2007; Holmes and Lipo, 2003). Because of the switching characteristic, the undesirable harmonic waves are inevitable besides the desired fundamental wave (Holmes and Lipo, 2003). The harmonic waves around the switching frequency and its multiples exhibit prominent amplitudes (Holmes and Lipo, 2003; Holmes, 1995). The clustered harmonics bring out many severe problems, such as mechanical vibration, audible noise and electromagnetic interference (Yin *et al.*, 2010; Ma *et al.*, 2008; Zhu *et al.*, 2008; Wu *et al.*, 2009). So, the methods to spread the spectra have been proposed and discussed (Carlosena *et al.*, 2007; Chen *et al.*, 2012a, b, 2013). The random switching frequency PWM technique has the most excellent performance among several types of random strategies. The theoretical formulas of the output voltage spectra are always complex and sometimes the unified formulas cannot be found (Yin *et al.*, 2010; Ma *et al.*, 2008; Zhu *et al.*, 2008; Wu *et al.*, 2009).

The PWM technique can be implemented through the triangle intersection method and the direct digital method (Hava, 1998; Holmes and Lipo, 2003). The triangle intersection and the digital implementation are equivalent to some extent. With the performance improvement of the digital chipsets such as TI TMSF2812, Freescale MC56F8XXX and Infineon TC1767 and TC1797 (Chen *et al.*, 2012a), the direct digital implementation has gotten wide applications. The Space Vector PWM (SVPWM) is most widely utilized in the motor control that is the typical application of PWM.

The computer simulation is a powerful tool for analyzing the PWM technique, especially for the random PWM. Almost all the PWM strategies can be analyzed through the computer simulation without complex formulas. Research on the random switching frequency PWM through the computer simulation has been reported. Ma *et al.* (2008) analyzed the spectra of the random frequency PWM for SPWM (Sinusoidal PWM) based on the simulation in MATLAB/Simulink and the experiment. Zhu *et al.* (2008) simulated the random frequency PWM in PSIM.

Based on the theory in the public literature, the paper simulates the spectra of RSFSVPWM (Random Switching Frequency SVPWM) using the simulation model built in MATLAB/ Simulink. The study is structured as follows. The principle of RSFSVPWM is introduced firstly.

Secondly, the study analyzes the theoretical spectra of the fixed and random frequency SVPWM based on the theoretical formulas for the fixed frequency SVPWM reported in the published literature. Thirdly, the effects of the switching frequency range and the probability distribution law on the spectra are simulated and analyzed. Some conclusions are drawn finally.

**PRINCIPLE OF RSFSVPWM**

To the classical two-level three-phase voltage source inverters, there are only eight switching states and the corresponding space voltage vectors and the notation are shown in Fig. 1. An arbitrary reference voltage vector with the amplitude  $U_s$  and the phase angle  $\theta$  can be generated by the two adjacent active vectors and the inactive vectors based on volt-second balance principle in a PWM period  $T_s$  (Chen, 2007; Holmes and Lipo, 2003; Bowes and Lai, 1997). The duration time of the two active vectors and the zero vectors can be gotten by:

$$\begin{cases} T_1 = \sqrt{3}T_s U_s \sin(\pi/3 - \theta) / U_{DC} \\ T_2 = \sqrt{3}T_s U_s \sin \theta / U_{DC} \\ T_0 = T_s - T_1 - T_2 \end{cases} \quad (1)$$

where,  $U_{DC}$  is the DC voltage.

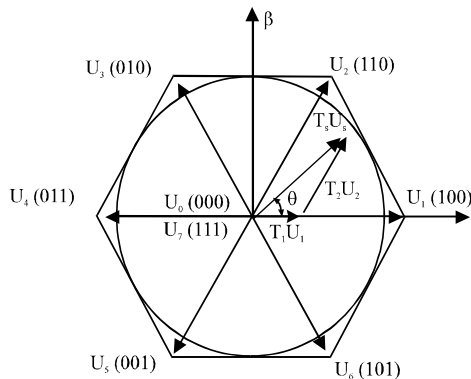


Fig. 1: Basic vectors and vector summation

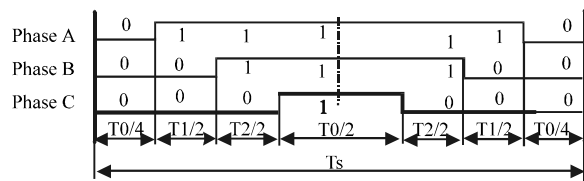


Fig. 2: SVPWM duration assignment strategy in the first sector

In the first sector, the sequence of the working basic vectors shown in Fig. 2 has been widely used because of the excellent performance.

In the deterministic SVPWM strategies, the PWM period  $T_s$  is always constant in a system. But the PWM period varies randomly in a range in RSFSVPWM (Kim *et al.*, 2009).

**THEORETICAL SPECTRUM ANALYSIS**

**Spectra of deterministic SVPWM:** The theoretical spectrum expressions for SVPWM strategies are always complex and the computation is tedious. Based on Double Fourier Integral, the phase voltage theoretical expression for SVPWM can be expressed as follows and the spectra are shown in Fig. 3 (Holmes and Lipo, 2003).

$$V_{Ao} = \frac{A_{m0}}{2} + \sum_{n=1}^{\infty} [A_{mn} \cos n(\omega_0 t + \theta_0) + B_{mn}] + \sum_{m=1}^{\infty} A_{m0} \cos(\omega_c t + \theta_c) + B \sin m(\omega_c t + \theta_c)$$

DC Offset      Fundamental component and baseband harmonics      Carrier harmonics

$$\sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \{A_{mn} \cos[m(\omega_0 t + \theta_0)] + B_{mn} \sin[(\omega_c t + \theta_c) + (\omega_0 t + \theta_0)]\}$$

Sideband harmonics

(2)

where,  $\omega_c$  is the carrier angular frequency,  $\theta_c$  is the switching offset angle for carrier,  $\omega_0$  is the modulation wave angular frequency,  $\theta_0$  is the phase offset angle for the modulation wave and  $A_{00}$ ,  $B_{00}$ ,  $A_{mn}$  and  $B_{mn}$  are the coefficients determined by the SVPWM strategy.

The spectra show pronounced maxima at the switching frequency and its multiples (Holmes, 1995; Quang and Dittrich, 2008). The variable switching frequency is a straightforward solution because it can destroy the clustering characteristic. The variable switching frequency PWM can be realized through many techniques. For example, the hysteresis current control can be regarded as a variable switching frequency

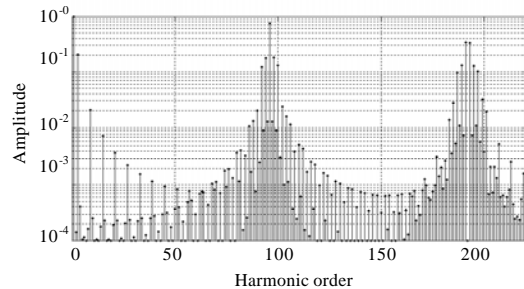


Fig. 3: Theoretical spectrum of the phase voltage ( $M = 0.9$ ,  $\omega_c/\omega_0 = 96$ )

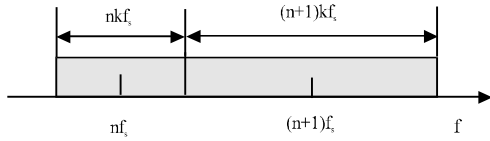


Fig. 4: Diagram of the switching frequency range ( $f_s = 1/T$ )

strategy, so a more uniform spectrum distribution can be gotten. In the digital system, the random switching frequency SVPWM is more feasible.

**Range of random switching frequency:** Kirlin *et al.* (2002) discussed the required range width of the switching frequency through a defined HSF (Harmonic Spread Factor) and the experiment showed that the width of the switching frequency range should not be less than half the average switching frequency (Kirlin *et al.*, 2002; Wang, 2004).

The theoretical analysis of the switching frequency range can be explained in Fig. 4. The aim of the randomization is to spread the clustered spectra. From Fig. 3, we can find that the clustering characteristic results from the fixed switching frequency. If the switching frequency and its multiples can uniformly cover the entire spectrum axis as shown in Fig. 4, the spectra can be spread uniformly.

In order to cover the entire spectrum axis, the smallest range should satisfy the relationship:

$$nf_s + \frac{nkf_s}{2} = (n+1)f_s - \frac{(n+1)kf_s}{2} \quad (3)$$

where,  $k$  is the range factor and  $n = 1, 2, 3, \dots$

So, we can get:

$$k = \frac{2}{2n+1} \quad (4)$$

The range factor is not constant and highly depends on the harmonic order, as shown in Table 1 and Fig. 5. In the last row of Table 1, the average factor of the former  $n$  factors is given. The conclusion-not less than half the average switching frequency-given by Kirlin *et al.* (2002) is consistent with the average factor of the former two or three factors: 0.53 or 0.45 which is no coincidence. The optimum range factor can be regarded as the weighted mean of all the factors in the first row in Table 1. The maximum harmonic amplitudes are concentrated around three specific frequencies (1st, 2nd and 3rd order) (Quang and Dittrich, 2008), so the optimum factor is almost determined by former two or three factors.

**Probability distribution law of switching frequency:** The maximum frequency, minimum frequency and average frequency are determined by the frequency factor and the

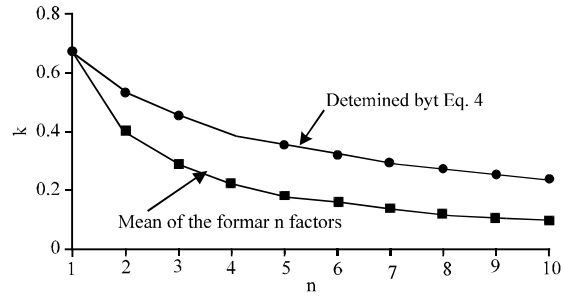


Fig. 5: Relationship between the range factor and the average factor

Table 1: Switching frequency range factor

n	1	2	3	4	5
k	2/3	2/5	2/7	2/9	2/11
$\bar{k}$	0.67	0.53	0.45	0.39	0.35

hardware characteristic of the switching devices. In theory, there are infinite numbers of probability distribution laws that satisfy the above constraints. From Fig. 4, we can find that the spectrum spreading effectiveness depends not only on the switching frequency range, but also on the probability distribution law. Due to ease of implementation, the symmetrical distributions have been widely discussed and utilized, especially the uniform distribution (Drissi *et al.*, 2003). Figure 6 presents four symmetrical probability laws. If the frequency factor is approximate 0.5 and HSF does not saturate, the uniform distribution can make the switching devices work at any frequency over the frequency interval with equal chance. So, the uniform distribution law maybe can get more spreading effectiveness than the normal and the triangle distribution laws on this occasion. If the factor is less than 0.5, the distribution in Fig. 6d may get more uniform spreading effectiveness than the other three laws.

## SIMULATION MODEL AND RESULTS

**Simulation model:** The simulation model developed by Chen and Kang (2011) is updated and used to simulate the random switching frequency SVPWM. The generated random number using the MATLAB functions must be mapped into the frequency interval and then the corresponding switching period is computed. Other distributions can be derived from the uniform distribution (Chen *et al.*, 2008; Gentle, 2003). For example, the symmetrical triangle distribution can be gotten by adding two independent random variables that conform to the same uniform distribution. Because a random variable that conforms to the normal distribution is unbounded, so the values that exceed three standard deviations are omitted in the simulation. Figure 7 shows the program flowcharts

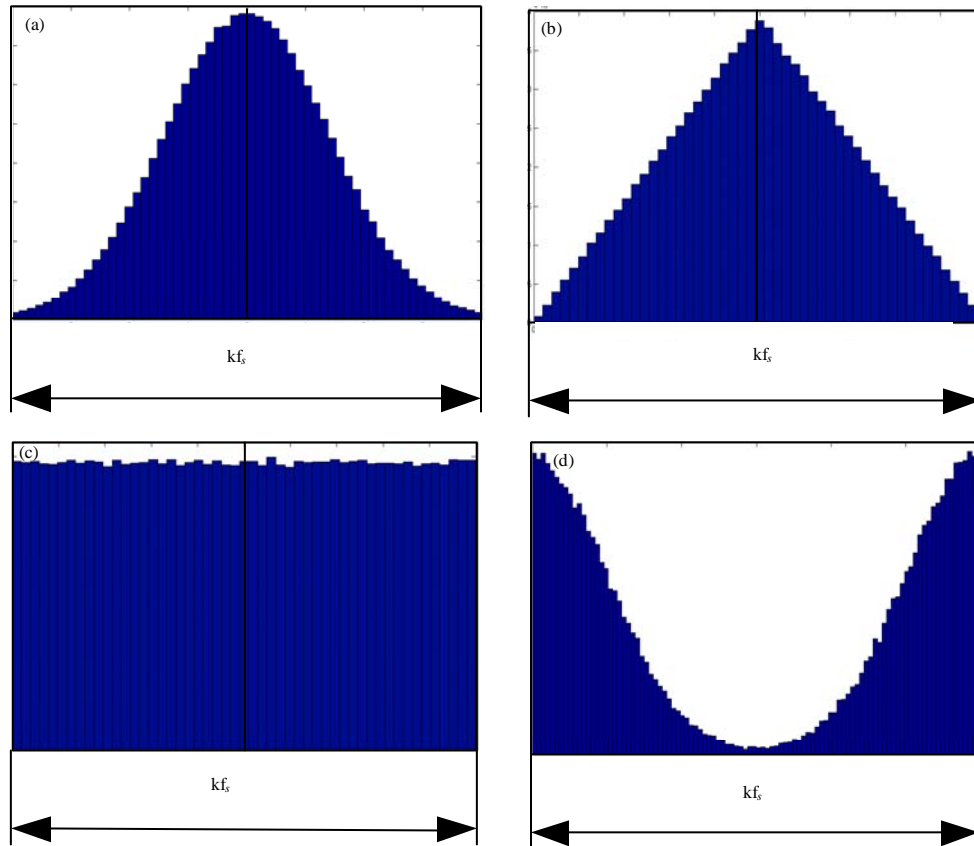


Fig. 6(a-d): Four types of symmetrical probability distribution laws, (a) Normal, (b) Triangle, (c) Uniform and (d) Defined in the paper

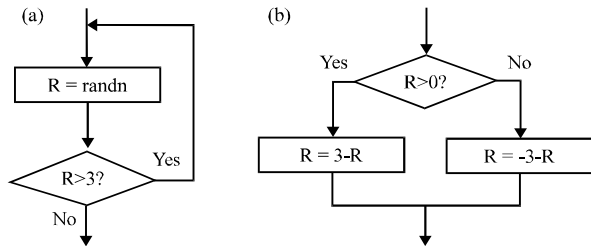


Fig. 7(a-b): Program flowchart for generating random number

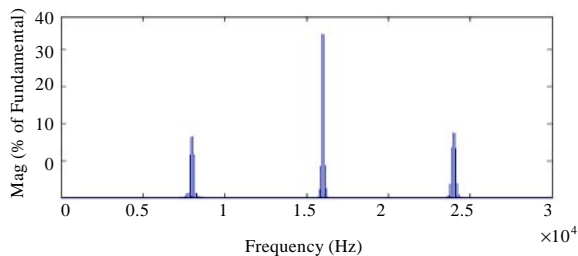


Fig. 8: Fixed switching frequency SVPWM (8000 Hz)

for generating the random variables that conform to the specific probability distribution laws (Fig. 7a for Fig. 6a); both Fig. 7a and b for Fig. 6d).

**Simulation results:** The simulation results are shown in Fig. 8-18. The fundamental frequency is 30 Hz. The FFT is employed to analyze the line-to-line voltage spectra. From the figures, some phenomena can be found as follows:

- RSFSVPWM can produce excellent spectra spreading effectiveness. The maximum harmonic amplitude is largely reduced by about ten times
- With the standard deviation becoming bigger, the spectrum spreading range around the average switching frequency and its multiples becomes wider. The standard deviation increases from a-d in the probability laws in Fig. 6 and the spread effectiveness difference can be intuitively found from Fig. 9-12

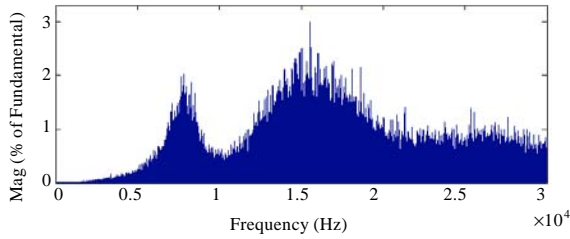


Fig. 9: RSFSVPWM (uniform distribution, [6000,10000]Hz)

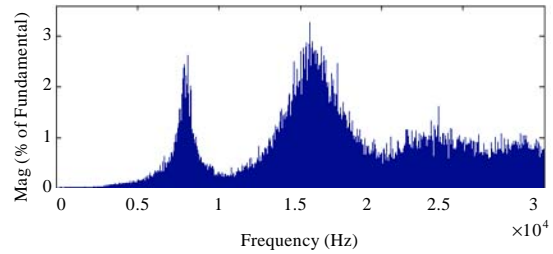


Fig. 13: RSFSVPWM (distribution shown in Fig. 6d), [7000, 9000]Hz)

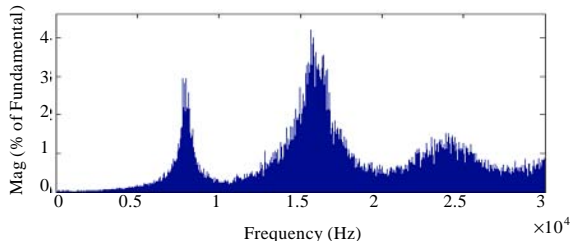


Fig. 10: RSFSVPWM (normal distribution, [6000,10000]Hz)

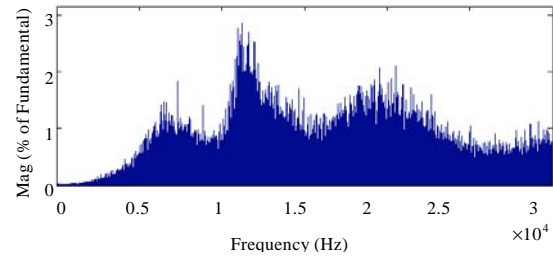


Fig. 14: RSFSVPWM (distribution shown in Fig. 6d), [5333, 10667]Hz)

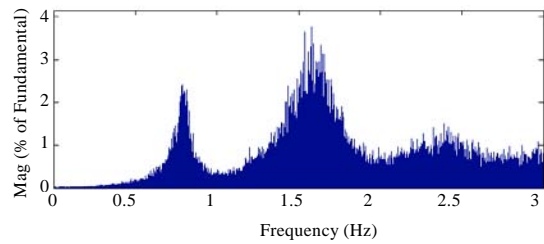


Fig. 11: RSFSVPWM (triangle distribution, [6000, 10000]Hz)

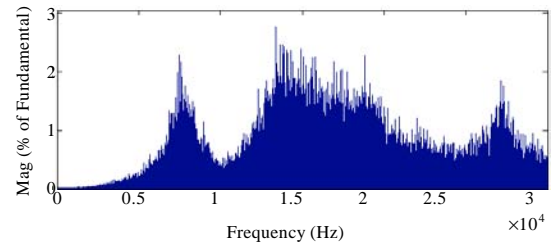


Fig. 15: RSFSVPWM (distribution shown in Fig. 6d), [6400, 9600]Hz)

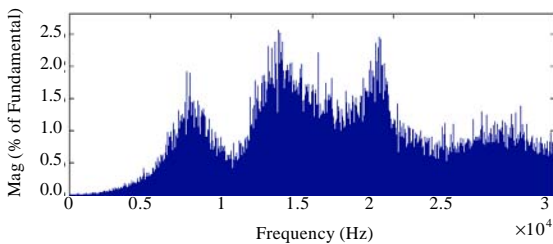


Fig. 12: RSFSVPWM (distribution shown in Fig. 6d), [6000, 10000]Hz)

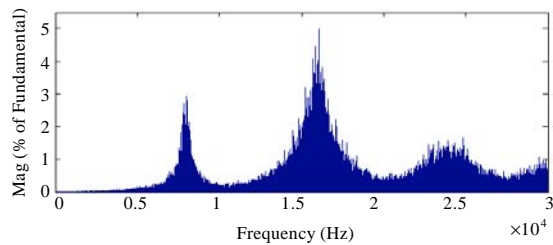


Fig. 16: RSFSVPWM (uniform distribution, [7000,9000]Hz)

- From the perspective of the spectra spreading effectiveness around the average switching frequency, the difference between the uniform distribution and the distribution defined in the paper is subtle. But from the spectra amplitudes

around the double average switching frequency, there is large difference. So, it is possible to find an optimal probability distribution law that can make the spectra spread around the double average switching frequency more uniformly

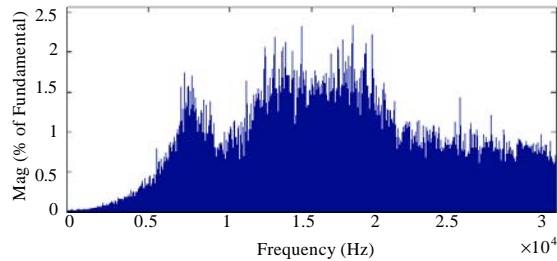


Fig. 17: RSFSVPWM (uniform distribution, [5333, 10667]Hz)

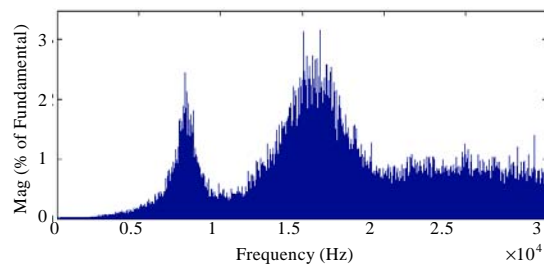


Fig. 18: RSFSVPWM (uniform distribution, [6400, 9600]Hz)

The spectra shown in Fig. 8-18 are only the spectra of the stochastic voltage signal samples. The more complete evaluation should be done using the probability density function and the probability density function for DC-DC was discussed by Carlosena *et al.* (2007) and Drissi *et al.* (2003). So, the randomization parameters for RSFSVPWM can be accurately computed using the theory of the stochastic process to get the required harmonic spectra, but the computing model and procedure may be complicated and this is the future work of the author.

The switching period optimization SVPWM can not only reduce the harmonic distortion, but also reduce the harmonic amplitude peaks (Chen *et al.*, 2013), so the more excellent performance can be gotten if RSFSVPWM is combined with the switching period optimization SVPWM, the random zero vector distribution PWM and the random pulse position PWM (Chen *et al.*, 2012a, b, 2013).

### CONCLUSION

The two factors-the switch frequency range width and the probability distribution law- that affect the spectrum spreading performance are discussed. The range width factor is almost determined by the former two or

three factors. The probability distribution law has heavy effects on the spreading effectiveness, especially on the double average switching frequency. The shape of the spectra of RSFSVPWM is complex, so the computer simulation is a powerful tool for the selection and assessment of the frequency range width and the probability distribution law.

### ACKNOWLEDGMENTS

This study is supported by Henan Polytechnic University doctoral science foundation with grant No. 648495 and the science and technology key project of Education Department of Henan Province of China with grant No. 13B460027.

### REFERENCES

- Bowes, S.R. and Y.S. Lai, 1997. The relationship between space-vector modulation and regular-sampled PWM. *IEEE Trans. Ind. Electron.*, 44: 670-679.
- Carlosena, A., W.Y. Chu, B. Bakkaloglu and S. Kiaei, 2007. Randomized carrier PWM with exponential frequency mapping. *IEEE Trans. Power Electron.*, 22: 960-966.
- Chen, G., 2007. PWM Inverse Technology and Application. China Electronic Press, Beijing, China.
- Chen, G. and J. Kang, 2011. Simulation platform development for random space vector PWM. *Applied Mech. Mater.*, 44-47: 3433-3437.
- Chen, G., M. Zhang and J. Zhao, 2012a. Harmonic distortion factor of a hybrid space vector PWM based on random zero-vector distribution and random pulse position. *Adv. Inform. Sci. Serv. Sci.*, 4: 242-250.
- Chen, G., Z. Wu and Y. Zhu, 2012b. Realization of random space vector pulse width modulation based on infineon tricore TC1767/TC1797. *Int. J. Digital Content Technol. Appl.*, 6: 624-632.
- Chen, G., J. Zhao and J. Kang, 2013. Switching period optimization method to random space vector PWM. *Int. J. Advancements Comput. Technol.*, 5: 181-189.
- Chen, G., X. Shang and J. Zhao, 2008. Principle and realization for generating data sequence with errors. *J. Henan Polytechnic Univ. (Nat. Sci.)*, 27: 560-564.
- Drissi, K.E.K., P.C.K. Luk, B. Wang and J. Fontaine, 2003. Effects of symmetric distribution laws on spectral power density in randomized PWM. *IEEE Power Electron. Lett.*, 1: 41-44.
- Gentle, J.E., 2003. Random Number Generation and Monte Carlo Methods. 2nd Edn., Springer, USA., ISBN-13: 9780387001784, Pages: 381.

- Hava, A.M., 1998. Carrier based PWM-VSI drives in the overmodulation region. Ph.D. Thesis, University of Wisconsin-Madison, Madison, WI., USA.
- Holmes, D.G. and T.A. Lipo, 2003. Pulse Width Modulation for Power Converters: Principles and Practice. John Wiley and Sons, USA., ISBN-13: 9780471208143, Pages: 744.
- Holmes, D.G., 1995. The significance of zero space vector placement for carrier based PWM schemes. Conference Record of the IEEE Industry Applications Conference, 30th IAS Annual Meeting, Volume 3, October 8-12, 1995, Orlando, FL., USA., pp: 2451-2458.
- Kim, Ki.S., Y.G. Jung and Y.C. Lim, 2009. A new hybrid random PWM scheme. IEEE Trans. Power Electron., 24: 192-200.
- Kirilin, R.L., M.M. Bech and A.M. Trzynadlowski, 2002. Analysis of power and power spectral density in PWM inverters with randomized switching frequency. IEEE Trans. Ind. Electron., 49: 486-499.
- Ma, F., Z. Wu and Y. Li, 2008. Analysis and design of the random frequency PWM inverters. Proc. CSEE, 28: 67-71.
- Quang, N.P. and J.A. Dittich, 2008. Vector Control of Three-Phase AC Machines: System Development in the Practice. Springer, Berlin, Germany, ISBN-13: 9783540790297, Pages: 340.
- Wang, Z., 2004. Comparative investigation of PWM techniques for a new drive system for electric vehicles. M.S. Thesis, University of Nevada, Reno, NV., USA.
- Wu, S., J. Zhang and L. Wang, 2009. The space vector PWM technique base on the random switching frequency. Mar. Electr. Electron. Technol., 29: 25-28.
- Yin, Z., Y. Zhong, J. Liu and Q. Zhu, 2010. Random PWM control technology of frequency control system based on Markov chain. Electr. Mach. Control, 14: 41-46.
- Zhu, M., F. Kong and Z. Xu, 2008. Simulation study of a Random Frequency PWM (RFPWM) inverter. Tech. Autom. Appl., 27: 50-53.