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Two-dimensional DOA Estimation with L-shaped Array Based on a Jointly Sparse Representation

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Abstract: Sparse representation is a recently introduced method for Direction-of-Arrival (DOA) estimation. For Quasi-stationary signals in many practical applications, such as speech and audio signals, a jointly sparse representation algorithm based on Khatri-Rao product is proposed to estimate the two-dimensional (2-D) DOA with an L-shaped array in this paper. First, the elevation and azimuth angles are independently estimated through two Multiple-Measurement Vectors (MMV) models. Second, corresponding pair-matching of the elevation and azimuth angles is obtained by extracting the sparse coefficients from the sparse recovery. The uniqueness condition of the sparse recovery problem is given, which indicates that the proposed approach is capable of handling the multi-source data and increasing the degrees of freedom of arrays. The simulation results demonstrate the effectiveness of the proposed approach, with the smallest Root Mean Square Error (RMSE) compared with KR-MUSIC and KR-CAPON.

Key words: Direction-of-arrival, estimation, L-shaped array, quasi-stationary signal, khatri-rao, product, multiple-measurement vectors

INTRODUCTION

Two-dimensional (2-D) DOA (i.e., elevation and azimuth angles) estimation with various array geometries plays an important role in many practical applications such as communications, radar, sonar, microphone array system and radio astronomy (Kikuchi *et al.*, 2006; Wang *et al.*, 2011). The L-shaped array consists of two orthogonal Uniform Linear Arrays (ULAs). It has some advantages in the geometric structure and coverage area with the smallest Cramer-Rao lower bound values compared with circular array and rectangular array. The main interest in this paper is a 2-D DOA estimation problem for the L-shaped array.

In the traditional methods for the L-shaped array, the elevation and azimuth angles are independently obtained from each ULA by using MUSIC or ESPRIT methods and they are matched into pairs according to the cross-correlation matrix (Kikuchi *et al.*, 2006; Wang *et al.*, 2011). The performances of these methods become poor in low Signal-to-Noise Ratio (SNR) environments. In recent years, a kind of novel direction finding methods based on the sparse representation have been proposed, with some salient characteristics, such as high resolution and improved robustness to noise. Reference (Li *et al.*, 2011) proposed a sparse 2-D DOA estimation method but it only provides us with the capability of processing cases with more sensors of sub-array than sources. In most practical

cases, speech and audio signals are quasi-stationary (short-time stationary) signals and their statistical distribution keeps piecewise stationarity (Ma *et al.*, 2010).

In this paper, we propose a new jointly sparse decomposition method based on KR product for 2-D DOA estimation by making full use of the statistic characteristics of quasi-stationary signals. This method can handle the multi-source data situation (i.e., $M < K \leq 2M-2$, K is the number of signal sources, M is the number of sensors for each ULA). Reference (Palamisamy and Kishore, 2011) also realized the 2-D DOA estimation by using a 2-D spatial spectrum peak search method with 2-D KR subspace, such as KR-MUSIC method. Compared with the method by Palamisamy and Kishore (2011), the 2-D DOA estimation problem in this paper can be cast into two 1-D DOA estimation problems according to the structure of L-shaped array, in the sequel, a jointly sparse Multiple-Measurement Vectors (MMV) model based on KR product is proposed to independently estimate the elevation and azimuth angles. Pairs of the corresponding elevation and azimuth angles can be obtained from the source information contained by the sparse resolutions.

For an L-shaped array, 2-D DOA estimation can be effectively achieved by the jointly sparse MMV models with KR product and will be demonstrated by the experiment results in this paper. Although, the jointly sparse optimization method leads to a higher

computational complexity, we observe that our approach has some other advantages, including the increased resolution, no prior knowledge of the signal source number, with respect to the existing method (Ma *et al.*, 2010; Palanisamy and Kishore, 2011).

DATA MODEL

Consider an L-shaped array (Kikuchi *et al.*, 2006) consisting of two ULAs employing M-element sensors for each with spacing d and suppose that K narrowband signals $\{S_k(t)\}_{k=1}^K$ impinge on the array from distinct directions at the elevation angles $\{\theta_k\}_{k=1}^K$ and azimuth angles $\{\varphi_k\}_{k=1}^K$, as shown in Fig. 1. The elevation angle θ_k and the azimuth angle φ_k are measured clockwise relatively to the z or x axis, while the projected azimuth angle $\bar{\varphi}_k$ is measured counterclockwise relatively to the x axis in the x-y plane. Then the received signals at the ULAs along the z and x axes are given by:

$$\begin{cases} z(t) = A_z(\theta)s(t) + n_z(t) \\ x(t) = A_x(\bar{\varphi}, \theta)s(t) + n_x(t) \end{cases} \quad (1)$$

where, $t = 0, 1, \dots, T-1, T$, is the total number of snapshots, $z(t) = [z_1(t), \dots, z_M(t)]^T$ and $x(t) = [x_1(t), \dots, x_M(t)]^T$, $s(t) = [s_1(t), \dots, s_K(t)]^T$, represents the source signals, $n_z(t) = [n_{z1}(t), \dots, n_{zM}(t)]^T$ and $n_x(t) = [n_{x1}(t), \dots, n_{xM}(t)]^T$ represent the spatial noise. The steering matrices are given by:

$$\begin{cases} A_z(\theta) = [a_z(\theta_1), a_z(\theta_2), \dots, a_z(\theta_K)] \\ A_x(\bar{\varphi}, \theta) = [a_x(\bar{\varphi}_1, \theta_1), a_x(\bar{\varphi}_2, \theta_2), \dots, a_x(\bar{\varphi}_K, \theta_K)] \end{cases} \quad (2)$$

where, $a_z(\theta_k) = [1, e^{-j2\pi d \cos \theta_k/\lambda}, \dots, e^{-j2\pi (M-1)d \cos \theta_k/\lambda}]^T$ and $a_x(\bar{\varphi}_k, \theta_k) = [1, e^{-j2\pi d \cos \varphi_k \sin \theta_k/\lambda}, \dots, e^{-j2\pi (M-1)d \cos \varphi_k \sin \theta_k/\lambda}]^T$. λ is the signal carrier wavelength and $d \leq \lambda/2$.

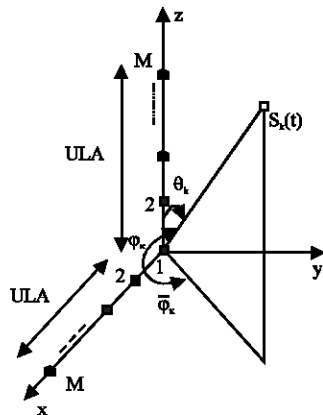


Fig. 1: L-shaped array configuration for 2-D DOA estimation

In order to formulate the DOA estimation problem in (1) as a sparse representation problem, we must introduce a two-dimensional over-complete dictionary D in terms of all possible source locations, that is:

$$D = [a_x(\bar{\varphi}_1, \theta_1), a_x(\bar{\varphi}_2, \theta_2), \dots, a_x(\bar{\varphi}_{N1}, \theta_{L1})]$$

where, N1 and L1 are the number of the sampling grid of all the azimuth angle and elevation angle locations of interest. N1 and L1 are typically much greater than the number of sources K or the number of sensors M. The computational complexity will attain $O((N1 \times L1)^3 \log(N1 \times L1))$ if we acquire the sparse solution by using the Basis Pursuit (BP) method. Because the computational amount will grow rapidly with the improvement of the spatial resolution, it is rare to estimate the elevation angle and the azimuth angle directly by using two-dimensional sparse representation for (1). In this paper, according to the relationship equation $\cos(\varphi_k) = \cos(\bar{\varphi}_k) \sin(\theta_k)$ between the elevation angle θ_k , the azimuth angle φ_k and the projected azimuth angle $\bar{\varphi}_k$, we simplify a 2-D DOA estimation problem into two 1-D DOA estimation problems and the computational complexity of sparse representation will reduce to:

$$2O(N^3 \log(N))$$

where, N is the number of the sampling grid of all the azimuth angle or elevation angle locations of interest. The received signal x(t) in (1) is represented as Eq. 3:

$$x(t) = A_x(\varphi)s(t) + n_x(t) \quad (3)$$

where, the steering matrices $A_x(\varphi)$ is given by:

$$A_x(\varphi) = [a_x(\varphi_1), a_x(\varphi_2), \dots, a_x(\varphi_K)] \quad (4)$$

With $a_x(\varphi_k) = [1, e^{-j2\pi d \cos \varphi_k/\lambda}, \dots, e^{-j2\pi (M-1)d \cos \varphi_k/\lambda}]^T$, $1 \leq k \leq K$. Some common assumptions are made as follows:

- **A1:** The noise signals are the zeros-mean wide-sense stationary with unknown covariance matrices C_z and C_x and are statistically independent of the source signals
- **A2:** Each source signal is mutually uncorrelated zero-mean wide-sense quasi-stationary with the frame length L and the second order statistics are time-varying but remain static over a shot period of time

The second order statistics are defined as:

$$E\{s_k(t)l^2\} = \sigma_{kn}, \forall t \in [(n-1)L, nL-1], n=1, 2, \dots, N \quad (5)$$

where, N is the number of frames and T = NL.

JOINTLY SPARSE REPRESENTATION BASED ON KR PRODUCT

The covariance matrices of two sub-arrays are defined as:

$$\begin{cases} R_z = E[z(t)z^H(t)] = A_z(\theta)R_s A_z^H(\theta) + C_z \\ R_x = E[x(t)x^H(t)] = A_x(\phi)R_s A_x^H(\phi) + C_x \end{cases} \quad (6)$$

where, $R_s = \text{diag}(\sigma_s)$, $\sigma_s = [\sigma_{1n}, \sigma_{2n}, \dots, \sigma_{kn}]^T$. Let us apply Property 1 from (Ma *et al.*, 2010) to the covariance models in (6) to obtain:

$$\begin{cases} Z = \text{vec}(R_z) = (A_z^*(\theta) \odot A_z(\theta))\sigma_s + \text{vec}(C_z) \\ X = \text{vec}(R_x) = (A_x^*(\phi) \odot A_x(\phi))\sigma_s + \text{vec}(C_x) \end{cases} \quad (7)$$

where, $\text{vec}(\cdot)$ denotes the column vector stacking operation, $A_z^* \odot A_z(\theta)$ and $A_x^* \odot A_x(\phi)$ are KR product matrices (Ma *et al.*, 2010), formulated by:

$$\begin{cases} A_z^*(\theta) \odot A_z(\theta) = [a_z^*(\theta_1) \otimes a_z(\theta_1), \dots, a_z^*(\theta_k) \otimes a_z(\theta_k)] \\ A_x^*(\phi) \odot A_x(\phi) = [a_x^*(\phi_1) \otimes a_x(\phi_1), \dots, a_x^*(\phi_k) \otimes a_x(\phi_k)] \end{cases} \quad (8)$$

where, \otimes denotes the Kronecker product. The vectors Z and X of model in (7) can be viewed as the new array output signals. Thus, σ_s is the source signal vector, $A_z^*(\theta) \odot A_z(\theta)$ and $A_x^*(\phi) \odot A_x(\phi)$ are the virtual array steering matrices. The virtual array dimension given by M^2 is greater than the physical array dimension M, which enhances the equivalent aperture of sensor array and provides us with the capability of processing cases where there are less sensors of sub-array than sources (i.e., $M < K$).

The new array output signals from all frames are stacked into two new matrices Ψ_z and Ψ_x , we can write:

$$\begin{cases} \Psi_z = [Z_1, \dots, Z_N] = (A_z^*(\theta) \odot A_z(\theta))P_s + \text{vec}(C_z)1_N^T \\ \Psi_x = [X_1, \dots, X_N] = (A_x^*(\phi) \odot A_x(\phi))P_s + \text{vec}(C_x)1_N^T \end{cases} \quad (9)$$

where, $1_N = [1, \dots, 1]^T$ and $P_s = [\sigma_{s1}, \dots, \sigma_{sN}]$, σ_{sk} denotes the power vector of k-th frame. The source power distributions over the time frame are different and non-stationary, so that the matrix P_s can maintain a full row rank condition. The assumption is made as follow:

- **A3:** The real matrix $[P_s^T 1_N]$ is of full column rank.

This assumption implies that any linear combination of sources cannot result in a wide-sense-stationary source. Given the over complete bases $\{a_z(\tilde{\theta}_q)\}_{q=1}^Q$ and $\{a_x(\tilde{\phi}_q)\}_{q=1}^Q$, $Q \gg M$, where $\{\tilde{\theta}_q\}_{q=1}^Q$ and $\{\tilde{\phi}_q\}_{q=1}^Q$ are specified angle directions sampled in 0-180°, we can rewrite (9) into:

$$\begin{cases} \Psi_z = (A_z^*(\tilde{\theta}) \odot A_z(\tilde{\theta}))\tilde{P}_s + \text{vec}(C_z)1_N^T \\ \Psi_x = (A_x^*(\tilde{\phi}) \odot A_x(\tilde{\phi}))\tilde{P}_s + \text{vec}(C_x)1_N^T \end{cases} \quad (10)$$

where, the matrix \tilde{P}_s is the sparse representation coefficient matrix, each column of which shares the identical sparse structure, i.e., the K nonzero elements of each column should appear in the same rows of \tilde{P}_s , therefore the sparse representations of Eq. 10 are regarded as the formulation of K jointly sparse MMV model (Davies and Eldar, 2012) which can enhance the number of recognizable signal sources. From (10), we independently obtain the elevation and azimuth angles estimates from the peaks of \tilde{P}_s and match the elevation and azimuth angles by the source information contained by \tilde{P}_s . In order to remove the redundancy of array manifold matrices, we introduce the column orthogonal matrix G (Ma *et al.*, 2010), then the KR product matrices can be relatively represented by:

$$A_z^*(\tilde{\theta}) \odot A_z(\tilde{\theta}) = GB(\tilde{\theta})$$

and:

$$A_x^*(\tilde{\phi}) \odot A_x(\tilde{\phi}) = GB(\tilde{\phi})$$

with:

$$G = [\text{vec}(J_{M-1}), \dots, \text{vec}(J_1), \text{vec}(J_0), \text{vec}(J_1^T), \dots, \text{vec}(J_{M-1}^T)]_{M^2 \times (2M-1)}$$

$$J_m = \begin{bmatrix} 0_{M-m,m} & I_{M-m} \\ 0_{m,m} & 0_{m,M-m} \end{bmatrix}$$

$m = 0, 1, \dots, M-1$:

$$B(\tilde{\theta}) = [b(\theta_1), \dots, b(\theta_Q)]_{(2M-1) \times Q}$$

$b(\theta_k) = [e^{j2\pi(M-1)d \cos\theta_k/\lambda}, \dots, e^{j2\pi d \cos\theta_k/\lambda}, \dots, 1, e^{-j2\pi d \cos\theta_k/\lambda}, \dots, e^{-j2\pi(M-1)d \cos\theta_k/\lambda}]^T$, $k = 1, 2, \dots, Q$. The same definition as shown above holds for the term $B(\tilde{\phi})$ also.

The noise compositions of Eq. 10 are eliminated by post-multiplied by an orthogonal complement projection matrix $P^\perp = I_N - 1_N 1_N^T$ of 1_N , then the jointly sparse model is re-expressed as:

$$\begin{cases} \Psi_z P^1 = GB(\tilde{\theta})(\tilde{P}_s P^1) = GB(\tilde{\theta})\Phi \\ \Psi_x P^1 = GB(\tilde{\phi})(\tilde{P}_s P^1) = GB(\tilde{\phi})\Phi \end{cases} \quad (11)$$

This operation does not damage the sparse structure of (\tilde{P}_s) , only affects the numerical values of nonzero elements of \tilde{P}_s , i.e., $|\text{supp}(\Phi)| = |\text{supp}(\tilde{P}_s)| = K$. From the analysis above, the source location estimating problems are equivalent to finding the solutions of the Φ in (11). Once Φ is obtained, the DOA estimates are determined from its sparse structure by plotting it on the grid of direction samples.

MMV RECOVERY

Because system (11) are the underdetermined equations, it is necessary to examine the condition for a single sparse solution. According to the sufficient and necessary condition for the measurement $Y = AX$, with $|\text{supp}(X)| = K$, to uniquely determine the jointly sparse matrix X is $K < [\text{spark}(A) - 1 + \text{rank}(Y)]/2$ (Davies and Eldar, 2012), we can draw a conclusion as follow:

Theorem 1: The condition of the unique solution of the K jointly sparse matrix Φ of (11) is $K \leq 2M - 2$.

Proof: Take the equation $\Psi_z P^1 = GB(\tilde{\theta})\Phi$ as the example. From the sufficient and necessary condition, we must find the least number of linear dependent columns of $GB(\tilde{\theta})$ (i.e., $\text{spark } GB(\tilde{\theta})$) and the rank of $\Psi_z P^1$. Since the matrix G is of full column rank, we can get $\text{rank}(GB(\tilde{\theta})) = \text{rank}(B(\tilde{\theta}))$. The matrix $B(\tilde{\theta})$ has Vander-monde structure, then, we have the result: $\text{rank}(B(\tilde{\theta})) = \min(2M - 1, Q) = 2M - 1$. This implies that the any $2M$ columns are linear dependent, as a consequence, we can get $\text{spark}(GB(\tilde{\theta})) = 2M$.

From the Assumption A3, suppose that there exists the matrix $\Gamma \in \mathbb{R}^{N \times (N-K-1)}$ which makes the matrix $[P_s^T, 1_N \Gamma]^T$ be of full rank, then we have $\text{rank}(P[P_s^T, 1_N \Gamma]) = \text{rank}(P^1) = N - 1$. It is easy to see that $P[P_s^T, 1_N \Gamma] = [P^1 P_s^T, 0 P^1 \Gamma]$. $P^1 P_s^T$ and $P^1 \Gamma$ and are also of full column matrices, thus $\text{rank}(P^1 P_s^T) = K$ and $\text{rank}(\Psi_z P^1) = \text{rank}(GB(\tilde{\theta}))$. $(P^1 P_s^T)^T = \text{rank}(GB(\tilde{\theta})) = \text{rank}(B(\tilde{\theta})) = \min(2M - 1, K)$. As a result, we will show that $K < [\text{spark}(GB(\tilde{\theta})) - 1 + \text{rank}(\Psi_z P^1)]/2 \Rightarrow K < 2M - 1$.

The same results as shown above holds for the equation $\Psi_x P^1 = GB(\tilde{\phi})\Phi$ also. This condition further shows that the MMV model in (11) can increase the number of recognizable signal sources most to $2M - 2$ but the existing SMV model (Li *et al.*, 2011) can process the situation where K is not more than $M - 1$.

Note that Φ is jointly sparse, we can recover Φ by using the following $l_{2,1}$ -norm minimization formulation:

$$\begin{cases} \min_{\Phi} \|\bar{\Psi}_z - GB(\tilde{\theta})\Phi\|_F^2 + \lambda_z \|\Phi\|_{2,1} \\ \min_{\Phi} \|\bar{\Psi}_x - GB(\tilde{\phi})\Phi\|_F^2 + \lambda_x \|\Phi\|_{2,1} \end{cases} \quad (12)$$

where, $\|\cdot\|_F$ denotes the Frobenius norm, is the $l_{2,1}$ -norm of Φ , λ_z and λ_x are the $\|\Phi\|_{2,1} = \sum_{i=1}^Q \|\Phi(i,:)\|_2$ regularization parameters, $\bar{\Psi}_z = \Psi_z P^1$ and $\bar{\Psi}_x = \Psi_x P^1$. The elevation and azimuth angles $\{\hat{\theta}_k\}_{k=1}^K$ and $\{\hat{\phi}_k\}_{k=1}^K$ will be independently estimated from the position indices of $B(\tilde{\theta})$ and $B(\tilde{\phi})$ corresponding to the nonzero values of the two solutions $\hat{\Phi}_z$ and $\hat{\Phi}_x$ in (12). Pairs of the corresponding elevation and azimuth angles can be made by a computationally efficient manner from the source information contained by $\hat{\Phi}_z$ and $\hat{\Phi}_x$. The method is summarized as follows: (1) Select the nonzero-value row vector sets $\{\hat{\Phi}_z^k\}_{k=1}^K$ and $\{\hat{\Phi}_x^k\}_{k=1}^K$. (2) Calculate the l_2 -norm of the first vector $\hat{\Phi}^1$. Between all K row vectors $\{\hat{\Phi}_z^k\}_{k=1}^K$, the smallest value is corresponding to $\hat{\phi}_1$. Thus obtain the first combination $(\hat{\theta}_1, \hat{\phi}_1)$ and eliminate $\hat{\Phi}_z^1$ from the set $\{\hat{\Phi}_z^k\}_{k=1}^K$ at the same time. (3) The rest $K - 1$ angle combinations may be deduced by the same way. This pair-matching method takes advantage of the source information contained by the sparse coefficients, which can reduce the computational load.

EXPERIMENTAL RESULTS

In this section, we carry out the simulations to analyze the performance of the proposed method in 2-D DOA estimation of quasi-stationary signals and compare it with the previous methods, including KR-CAPON, KR-MUSIC (Ma *et al.*, 2010). The elements of each ULAs with $M = 5$ sensors were separated by a half-wavelength, $d = 1\text{m}$ and the number of signals $K = 7$ was given in advance and $M < K \leq 2M - 1$. The elevation azimuth angles defined according to Fig. 1 were set as:

$$\{(\theta_k, \phi_k)\}_{k=1}^7 = \{(30^\circ, 50^\circ), (50^\circ, 90^\circ), (70^\circ, 150^\circ), (90^\circ, 70^\circ), 110^\circ, 130^\circ, (130^\circ, 30^\circ), (150^\circ, 110^\circ)\}$$

The total number of snapshots was $T = 10000$ and the number of frames was $N = 50$, the quasi-stationary source signals were locally stationary zero-mean complex Gaussian process with the variances randomly varying from one frame to another shown in Fig. 2. All the results were analyzed in 500 independent trails in total. Both elevation and azimuth angles were calculated by CVX(convex) toolbox associated with Matlab in (12). The SNR is defined as:

$$\text{SNR} = (1/T) \sum_{t=0}^{T-1} E \left\{ \|\mathbf{A}(\theta) \mathbf{s}(t)\|_2^2 \right\} / E \left\{ \|\mathbf{n}(t)\|_2^2 \right\}$$

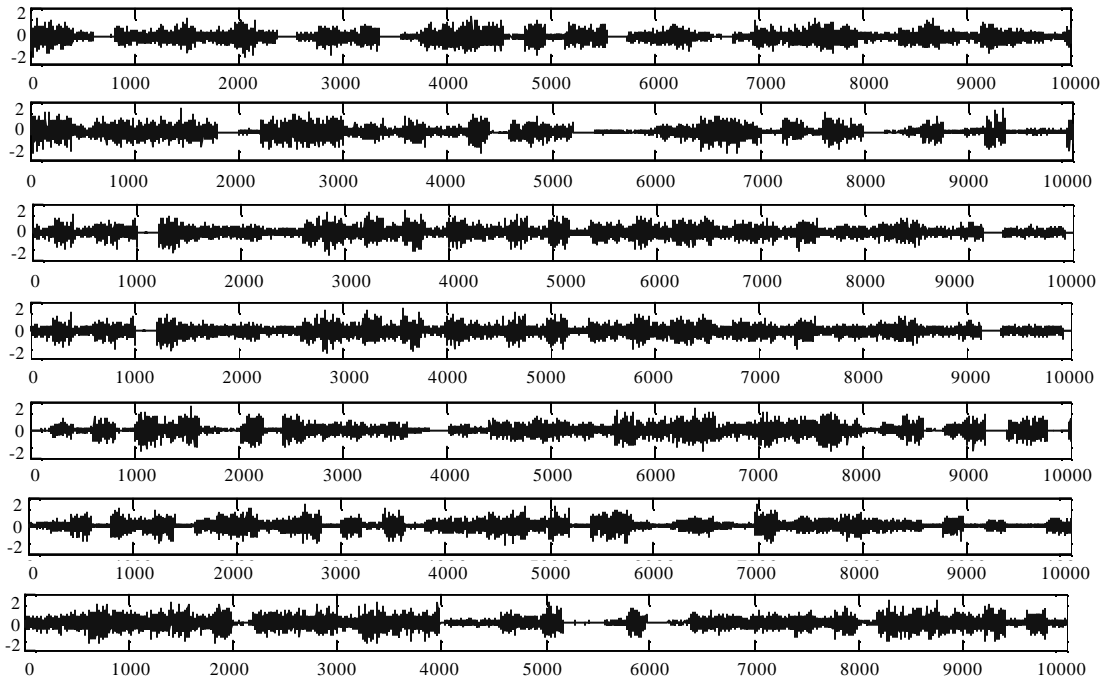


Fig. 2: The real parts of 7 signal sources

The estimation precision of DOA is restricted by the resolution of the grid set and regularization parameter (Stoica and Babu, 2012). Coarse grid results in quantization error, but fine grid will increase the computational complexity and inter-column correlation of array manifold matrices, which does not satisfy the Restricted Isometry Property (RIP) condition. A large λ_x (or λ_z) emphasizes the role of the l_{21} -norm term, which may cause wrong DOA estimation. A small λ_x (or λ_z) emphasizes the role of the Frobenius norm term, which may produce many spurious peaks in spatial spectrum. The grid and regularization parameter in this paper were chosen by experiments, with 1° spacing grid and $\lambda_z = \lambda_x = 0.34$, where the peaks of the spectra are in good agreement with the true DOAs whether high SNR or low SNR. Figure 3 demonstrates that spatial spectra are obvious without false peaks when λ_x is between 0.30 and 0.36 in the different SNR environments.

The pair-matching probability is the ratio of the number of success trails to the total number of trails, which is evaluated to measure the performance of the proposed algorithm. From Fig. 4 we can observe that success ratio is 100% from -3dB. The Root Mean Square Error (RMSE) is used as another performance measured index and defined as:

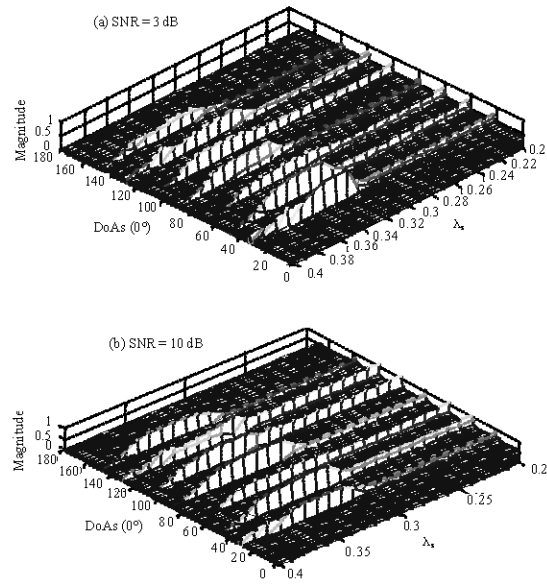


Fig. 3(a-b): The spatial spectra versus the regularization parameters

$$RMSE = \left(\sum_{k=1}^7 \sum_{m=1}^{500} [(\hat{\theta}_k^m - \theta_k)^2 + (\hat{\phi}_k^m - \phi_k)^2] / 3500 \right)^{1/2}$$

where, (θ_k, ϕ_k) and $(\hat{\theta}_k^m, \hat{\phi}_k^m)$ denote the true and estimated DOA angles, respectively. Figure 5 describes the RMSE

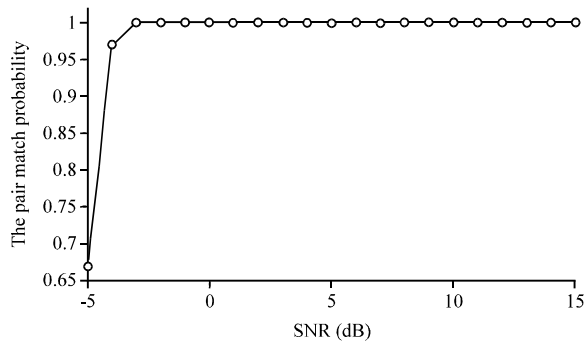


Fig. 4: The pair matching probability versus input SNR

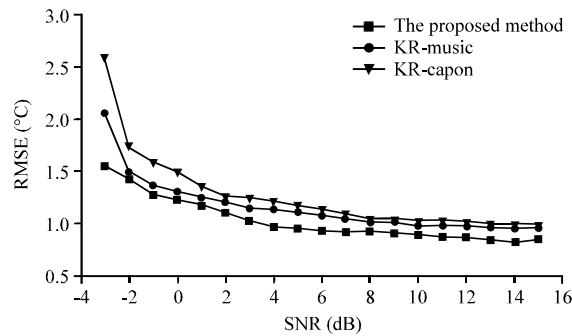


Fig. 5: RMSE of the DOA estimates versus input SNR

of the conventional and proposed methods when the SNR changed. The proposed method has the smallest RMSE compared with KR-MUSIC and KR-CAPON.

CONCLUSION

We have described a new method for 2-D DOA estimation, which can enlarge the number of recognized sources. Experimental results showed that the proposed algorithm outperformed the spatial spectrum estimation. We will also concern about the problem of wideband 2-D DOA estimation.

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