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Projective Synchronization for a Class of Unknown Chaotic Systems Based on Adaptive Fuzzy Control

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Abstract: A practical projective synchronization problem for master and slave chaotic systems with uncertainties and external disturbances was addressed in this study. Practical projective synchronization between the master chaotic system and slave systems was achieved by an adaptive fuzzy controller. Then the underlying parameter adaptation and the stability analysis were carried out based on Lyapunov based technique. In the design of the proposed projective synchronization method, the knowledge of the uncertainties and the external disturbances were not required. Finally, simulation results were given and one could see the effectiveness of the proposed practical projective synchronization method.

Key words: Chaotic systems, adaptive fuzzy control, synchronization, projective synchronization

INTRODUCTION

Chaotic systems were widely used in system engineering due to their attractive features, such as a broadband spectrum, a noise-like wave form, an extreme sensitivity to initial conditions as well as parameter variations and an unpredictable behavior (Boulkroune and M'saad, 2011). Up to now, much attention had been paid to the problem for chaos synchronization and one can see that many kinds of synchronization methods had been introduced, including complete synchronization, phase synchronization, anti-synchronization, intermittent lag synchronization, lag synchronization, intermittent generalized synchronization, generalized synchronization, projective synchronization and so forth (Carroll *et al*., 1996; Zhang and Sun, 2004; Rosenblum *et al*., 1997; Mainieri and Rehacek, 1999). Projective synchronization meant that the master and the slave systems could be synchronized proportionally according to a scaling factor. As a result the problem of projective synchronization had been further studied (Niu *et al*., 2012).

But in real world applications, for example in secure communication, the receiver plants often suffered from various uncertainties such as parameter perturbation and external disturbance, which will influenced the accuracy of the communication. In recent years, adaptive fuzzy control had been found to be an effectively tolls in the control of uncertain nonlinear systems. The fuzzy controller had strong ability to handle system uncertainties and external disturbances (Yu *et al*., 2011).

To the best of the author's knowledge, there were few studies on the robust adaptive fuzzy projective synchronization between chaotic systems. Then, it was advisable to do the study on this area. In this study, adaptive modified function projective synchronization based on adaptive fuzzy control between chaotic systems with systems uncertainties and external disturbance was proposed.

PROBLEM DESCRIPTION AND PRELIMINARIES

Problem description: Consider the master chaotic system of the form:

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad i = 1, 2, \cdots n-1, \\ \dot{x}_n &= f(x) \end{aligned} \tag{1}$$

where, $x = [x_1, x_2,..., x_n]^T$ is the state vector and the system output variable, respectively. f(x) is nonlinear but continuous function. Eq. 1 can be rewritten as the following form:

$$\dot{x} = Ax + Bf(x) \tag{2}$$

Where:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

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Then the slave chaotic system can be given as:

$$\dot{y} = Ay+Bf(y)+Bu+Bd(t) \tag{3}$$

with u∈R is the control input, d (t) is unknown external disturbance. The objective of this study is to design an adaptive fuzzy controller such that the master system (1) and the slave system (2) can achieve projective synchronization. By modified function projective synchronization, we mean:

$$\lim_{t\to\infty}\|x(t)-\gamma y(t)\| = 0 \tag{4}$$

where, $\gamma \neq 0$ is a constant.

Remark 1: The constant λ can be called as scaling factor. Then complete synchronization is the special case of projective synchronization when $\lambda = 1$.

Description of the fuzzy logic system: The fuzzy logic systems that employ singleton fuzzification, sum-product inference and center-off-sets defuzzification can be modeled by:

$$\alpha(x) = \frac{\sum_{j=1}^{N}\theta_j \Pi_{i=1}^{n}\mu_{F_i^j}(x_i)}{\sum_{j=1}^{N}[\Pi_{i=1}^{n}\mu_{F_i^j}(x_i)]} \tag{5}$$

where, α (x) is the output of the fuzzy system, x is the input vector, $\mu_{F_i^j}(x_i)$ is x'$_i$ membership of jth rule and θ_j is the centroid of the jth consequent set. Equation 5 can be rewritten as following equation:

$$\alpha(x) = \theta^T \Psi(x) \tag{6}$$

with $\theta = [\theta_1,..., \theta_N]^T$, $\Psi(x) = [p_1(x), p_2(x),..., p_N(x)]^T$ and the fuzzy basis function can be expressed as:

$$p_j(x) = \frac{\Pi_{i=1}^{n}\mu_{F_i^j}(x_i)}{\sum_{j=1}^{N}[\Pi_{i=1}^{n}\mu_{F_i^j}(x_i)]}$$

Lemma 1 (Wang, 1994; Yu *et al.*, 2011): Let f (x) be a continuous function defined on a compact set Ω. Then for any $\varepsilon>0$ scalar, there exists a fuzzy system as the form Eq. 6 and satisfy:

$$\sup_{x\in\Omega}|f(x)-\alpha(x)| \le \varepsilon \tag{7}$$

MAIN RESULTS

Let define the synchronization error as:

$$e = x(t)-\gamma y(t) = [e_1, e_2,... e_n]^T \tag{8}$$

and the filtered synchronization error as:

$$s = \left[\frac{d}{dt}+\lambda\right]^{n-1} e_1 = \lambda^{n-1}e_1 + (n-1)\lambda^{n-2}e_2 + \cdots + (n-1)\lambda e_{n-1} + e_n \tag{9}$$

where, $\lambda>0$ then the polynomial H (s) = λ^{n-1}+ (n-1) λ^{n-2} s+...+s^{n-1} is Hurwitz. Equation 9 can be rewritten as the following form:

$$s = S_\lambda^t e \tag{10}$$

with $C\lambda = [\lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots (n-1)\lambda]^T$.

The dynamic of can be described by:

$$\dot{s} = C^T e + w_1^{(n)} \tag{11}$$

where, $C = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots (n-1)\lambda]^T$.

From the master system (1) and the slave system (3) and (11) can be rewritten as:

$$\dot{s} = C^T e + \alpha(x,y) - \gamma u \tag{12}$$

where, $\alpha(x,y) = f(x)-\gamma f(y)-\gamma d(t)$ represents the unknown nonlinear function. Since α (x,y) is unknown, then it can be approximated, through the fuzzy logic system (7), by:

$$\alpha(x,y) = \theta^T \Psi(x,y) + \delta \tag{13}$$

where, δ is the approximation error. According to Lemma 1, there exits a constant $\varepsilon>0$ satisfies $|\delta\le\varepsilon|$. Then after some straightforward manipulation, yields:

$$\begin{aligned} s\alpha(x,y) &= s\theta^T\psi(x,y) + s\delta \\ &\le s\frac{\|\theta\|\psi^T(x,y)\psi(x,y)}{1\|\theta\|}1 + s\delta \\ &\le \frac{1}{21^2}s^2\|\theta\|^2\psi^T(x,y)\psi(x,y) + \frac{1}{2}1^2 + \frac{1}{2}s^2 + \frac{1}{2}\varepsilon^2 \end{aligned} \tag{14}$$

where, $1>0$ is a positive constant.

From above discusses, the controller can be designed as:

$$u = \frac{1}{\gamma}C^T e + \frac{1}{21^2}s\hat{\tau}\psi^T\psi + ks \tag{15}$$

where, k>0 is design parameter and its value will be define later. is the estimation of the unknown fuzzy system parameter $\tau = \|\theta\|^2$.

The adaptive law can be chosen as:

$$\dot{\hat{\tau}} = \frac{r}{2l^2} s^2 \psi^T \psi - m\hat{\tau} \tag{16}$$

with is constant. Then we are ready to give the following results.

Theorem 1: Consider the master chaotic system (2) and s lave chaotic system (3). Then, the proposed fuzzy controller, defined as (15B16), can guarantee:

- All signals in the closed-loop system are bounded
- The synchronization error of the closed-loop error system converges to an adjustable region of the origin
- A projective synchronization between the master systems (1) and the slave system (3) can be practically achieved, i.e., $\|x(t)-\gamma y(t)\| \le c$, where is the synchronization error bound and it can be made sufficiently small if the design parameters are chosen appropriately

Proof: From Eq. 12-14, gives:

$$\begin{aligned} s\dot{s} &= sC^T e + s\alpha(x,y) - \gamma su \\ &\le sC^T e + \frac{1}{2l^2} s^2 \tau \psi^T(x,y)\psi(x,y) + \frac{1}{2} l^2 + \frac{1}{2} s^2 + \frac{1}{2}\varepsilon^2 - \gamma su \end{aligned} \tag{17}$$

Then substitute Eq. 15 into 17, yields:

$$s\dot{s} \le \left(\frac{1}{2} - k\right) s^2 - \frac{1}{2l^2} s^2 \tilde{\tau} \psi^T(x,y)\psi(x,y) + \frac{1}{2} l^2 + \frac{1}{2}\varepsilon^2 \tag{18}$$

where, $\tilde{\tau} = \hat{\tau} - \tau$ is the estimation error of the fuzzy parameter.

Let define the following Lyapunov function:

$$V = \frac{1}{2} s^2 + \frac{1}{2r} \tilde{\tau}^2 \tag{19}$$

The time derivative of Eq. 19 is:

$$\dot{V} = s\dot{s} + \frac{1}{r} \tilde{\tau} \dot{\hat{\tau}} \tag{20}$$

From Eq. 16 and 18, we have:

$$\dot{V} \le (-k + \tfrac{1}{2}) s^2 - \frac{m}{2r} \tilde{\tau}^2 + \frac{1}{2} l^2 + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\tau^2 \tag{21}$$

where the inequality:

$$-\hat{\tau}\tilde{\tau} \le -\tilde{\tau}(\tilde{\tau} + \tau) \le \frac{1}{2}\tilde{\tau}^2 + \frac{1}{2}\tau^2$$

is used. If we choose $k > 1/2$ and let:

$$a_0 = \min\{2(k - \tfrac{1}{2}), m\},\ b_0 = \frac{1}{2} l^2 + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\tau^2$$

then (21) can be rewritten as:

$$\dot{V} \le a_0 V + b_0 \tag{22}$$

Further, we have:

$$V(t) \le \left(V(t_0) - \frac{b_0}{a_0} \right) e^{-a_0(t-t_0)} + \frac{b_0}{a_0}, \forall t \ge t_0 \tag{23}$$

As a result, all signals in the closed-loop system are bounded and the synchronization error of the closed-loop error system converges to an adjustable region of the origin. From Eq. 23, we have:

$$\lim_{t\to\infty} s^2 \le \frac{2b_0}{a_0} \tag{24}$$

It is clear that we can make the projective synchronization error small if we choose large k, m and small l. This ends the proof.

SIMULATION STUDIES

In this section two simulation studies are carried out to indicate the effectiveness of the proposed projective synchronization method. The examples are selected as Duffing oscillator and Genesio chaotic system.

Example 1: The Duffing chaotic system can be de described as:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -p_1 x_2 - p_2 x_1 - p_3 x_1^3 + q\cos(\omega t) \end{aligned} \tag{25}$$

A typical chaotic behavior of duffing system can be achieved with, $p_1 = 0.4$, $p_2 = -1.1$, $p_3 = 1$, $q = 2.1$, $\omega = 1.8$. The unforced Duffing chaotic system exhibits chaotic behavior with the initial values $x = [1.6, 0]^T$ is shown in Fig. 1.

The slave system is constructed as:

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= -p_1 y_2 - p_2 y_1 - p_3 y_1^3 + q\cos(\omega t) + u + d(t) \end{aligned} \tag{26}$$

with d (t) is the external disturbance.

In the simulation, the fuzzy systems have x_1, x_2, y_1, y_2 as the inputs. And for each input, this study define 11 Gaussian membership functions distributed on [-5, 5]. The initial values of the system are chosen as y = $[-3.3]^T$, $\hat{\tau}$ (0) = 0. The design parameter are selected as γ = 2, r = 20, m = 0.05, k = 5, l = 0.2.

The simulation results are shown in Fig. 2-3. From Fig. 2 one can see a quick convergence of the synchronization error and the synchronize performance is good. Figure 3 shows the smoothness and boundedness of the control input.

Example 2: The Genesio chaotic system can be described as:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -6x_1 - 2.92x_2 - 1.2x_3 + x_1^2 \end{aligned} \tag{27}$$

The slave system is constructed as:

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3, \\ \dot{y}_3 &= -6y_1 - 2.92y_2 - 1.2y_3 + y_1^2 + u(t) + 3\sin t \end{aligned} \tag{28}$$

In the simulation, the fuzzy systems have x_1, x_2, x_3, y_1, y_2, y_3 as the inputs. And for each input, this study define 11 Gaussian membership functions distributed on [-10,10]. The initial values of the system are chosen as x = $[-2, 2, 4]^T$, y = $[2, -2, -4]^T$, $\hat{\tau}$ (0) = 0.

The design parameter are selected as γ = 2, r = 20, m = 0.05, k = 5, l = 0.2.

The simulation results are shown in Fig. 4-5. From Fig. 4 one can see a quick convergence of the synchronization error and the synchronize performance is good. Figure 5 shows the smoothness and boundedness of the control input.

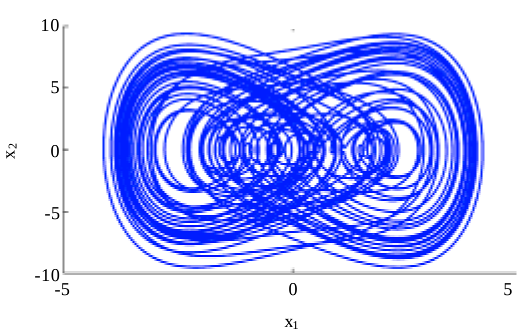

Fig. 1: The chaotic attractor of Duffing system

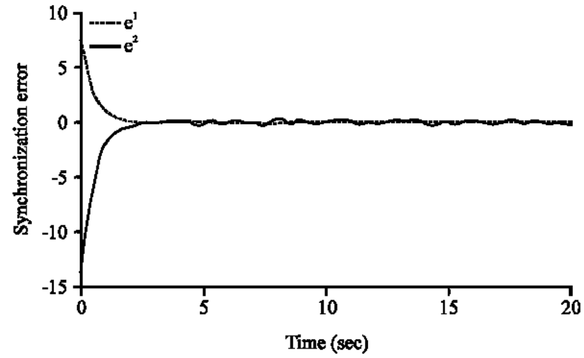

Fig. 2: The responses of synchronization errors

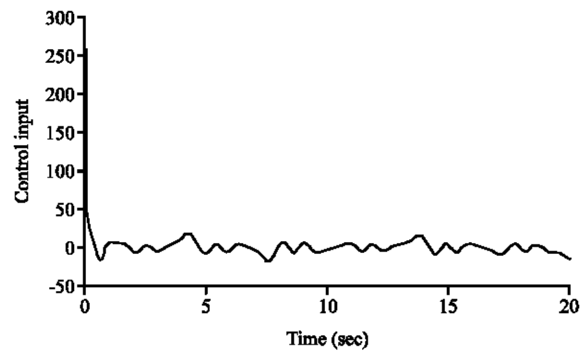

Fig. 3: The control input

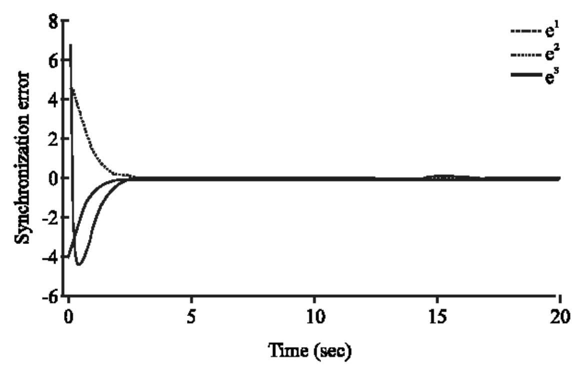

Fig. 4: The responses of synchronization errors

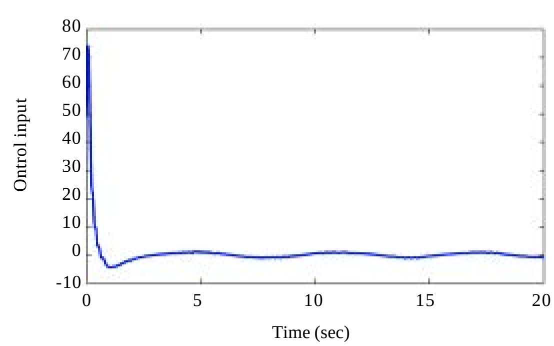

Fig. 5: The control input

CONCLUSION

An adaptive fuzzy approach has been presented in this study to handle the practical projective synchronization problem for a class of uncertain chaotic systems. The projective synchronization is obtained via an adaptive fuzzy controller. A suitable Lyapunov based analysis had been carried out to guarantee the stability and the synchronization error convergence of the proposed projective synchronization system. Simulation results were given to indicate the effectiveness of the proposed scheme.

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