http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (1): 234-238, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.234.238

The Influences of Multi-exponential Inversion on Relaxation Time Spectrum of Rocks

[1]Wang Qingyi, [2]Yin Quan, [2]Liu Yang and [2]Luo Hui
[1]Faculty of Mechincal and Electronical Information, China University of Geosciences, Wuhan, 430074, China
[2]Department of Control Science and Engineering, Huazhong University of Science and Technology, 430074, China

Abstract: Theoretical and practical research results both indicate that the induced polarization decaying curve of rocks has a close relationship with the pore structure and permeability of stratum in oil well logging. For better evaluating the reservoir characteristic from the induced polarization decaying data, the influence of parameters on inversion results should be analyzed in relaxation time spectrum. In this paper, the least square fitting method with penalty item is adopted to implement the multi-exponential inversion using the secondary field data of rocks, the continuous and smooth relaxation time spectrum is therefore obtained. The influence of the factors such as numbers of distribution points, relaxation time distribution, smooth factors, signal and noise ratio (SNR) on the spectrum are analyzed in detail. The results show that the penalty inversion may achieve good effect while the SNR is larger than 20 and the distribution points are between 32 and 64.

Key words: Induced polarization, relaxation time spectrum, smooth factor, multi-exponential fitting

INTRODUCTION

In oil well logging, the induced polarization based and Nuclear Magnetic Resonance (NMR) based technologies are two commonly used methods to evaluate the underground oil reservoir. Based on the theoretical and practical research proposed by Sturrock *et al.* (1999) and Titov *et al.* (2002), it is revealed that the permeability can be obtained from induced polarization decaying spectrum in time domain which can be used to further analyze the Reservoir information. Most researchers focus on the induced polarization decaying spectrum inversion in frequency domain but the induced polarization experiment of rocks shows little attention except (Tong *et al.*, 2004), in which how to get the relaxation time spectrum from the measured secondary field potential data is the highlight. Essentially speaking, the resolution of relaxation time spectrum is a multi-exponential inversion problem, in which the multi-exponential model with a series of different weight factors and exponential factors were used to fit the secondary field decaying curve. In fact, it is the problem to solve the first kind Fredholm integration equation proposed by Munn and Smith (1987). There have already been many inversion algorithms as followed Kokot and Zembaty (2009) and He and Geng-Ying (2005) proposed Non-negative Least Square (NNLS) algorithm also called Levenberg-Marquardt local search. Borgia *et al.* (1998) proposed Union-penalty Functions (UP) algorithm and Ghosh *et al.* (2008) revised it. Dunn *et al.* (1994) and Wang *et al.* (2001) proposed the Singular Value Decomposition (SVD) algorithm, Wang *et al.* (2003) proposed the Simultaneous Algebraic Reconstruction Technique (SIRT), Ke-Jia *et al.* (2008) proposed the Different Evolution (DE) algorithm and Zhang *et al.* (2009) proposed the Chaotic Immune Particle Swarm Optimization (CIPSO).

The NNLS algorithm based solution is possibly unstable and causes severe relaxation time spectrum oscillation which does not conform to the continuous inversion as well as continuous pore size distribution of the core. Based on the NNLS, the UP algorithm takes a smoothing factor as penalty item to smooth the relaxation time spectrum but the selection of the smoothing factor probably needs the man's intervention. The SVD uses the reduced order way to prevent the negative solution, nevertheless the inversion spectrum will be discontinuous which is somewhat computational intensive and time consuming. Because of the strong rely on initial value, the methods above may only give the local optimization results. In order to overcome this problem, the regularization optimization was used by Ghosh *et al.* (2008) to adjust the penalty item expecting to get the smooth spectrum. The regularization parameters are solved by DE algorithm which applies to data inversion

Corresponding Author: Wang Qingyi, Faulty of Mechincal and Electronical Information, China University of Geosciences, Wuhan, 430074, China

with less points and low SNR, however, the optimization parameter choice relies on empirical rules. In the CIPSO algorithm, the operators of clone, crossover, mutation and receptor editing are embedded into particle swarm optimization in order to make good convergence accuracy and computation time.

All the optimization algorithms described above are to solve the nonlinear questions with non-negative constraint. The inversion effect has been influenced by measurement error distribution, number of distribution point, smooth factors, relaxation time distribution and etc. All these parameters are relevant to physical features such as core solution concentration, power-on time, power supply current and etc. In this study, on the basis of modeling the induced polarization decaying features of rock in time domain, the secondary field decaying curve is obtained from an automatic control apparatus with high precision. After the inversion of relaxation time spectrum using damped least square method, the effects of the parameters variation on the inversion results are analyzed in detail from the experimental data.

MULTI-EXPONENTIAL INVERSION THEORY OF INDUCED POLARIZATION OF ROCKS

Theoretical analysis shows that relaxation time of induced polarization of rock has a close relationship with the pore groups. The attenuation of induced polarization in single pore follows the law of single-exponential decay, decay time constant T satisfies the condition in Eq. 1:

$$T = \frac{R^2}{D} \tag{1}$$

where, D stands for diffusion coefficient, R represents aperture. Actually, the internal aperture of rock varies in size, distributes in a very wide range. Therefore, y(t) which stands for secondary field potential of induced polarization, is the superposition of polarization potential of a series of single aperture. And then:

$$y(t) = \int_{T_{min}}^{T_{max}} x(T)e^{-\frac{t}{T}}dT \tag{2}$$

where, x(T) stands for non-negative weighting coefficient, T_{max} and T_{min} stand for the maximum and minimum constant relaxation time respectively which corresponds to the maximum and minimum aperture.

Discretizing the equation above, the result is given as follows:

$$y(t) = \sum_{i=1}^{m} x(T_i)e^{-\frac{t}{T_i}} \tag{3}$$

where, T_i stands for the relaxation time of type i. Assuming that the observed data number is m, the relaxation time component number is n, the following matrix will be obtained:

$$Y = AX \tag{4}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_m \end{bmatrix},\quad A = \begin{bmatrix} e^{-\frac{t_1}{T_1}} & e^{-\frac{t_1}{T_2}} & \dots & e^{-\frac{t_1}{T_n}} \\ e^{-\frac{t_2}{T_1}} & e^{-\frac{t_2}{T_2}} & \dots & e^{-\frac{t_2}{T_n}} \\ \dots & \dots & \dots & \dots \\ e^{-\frac{t_m}{T_1}} & e^{-\frac{t_m}{T_2}} & \dots & e^{-\frac{t_m}{T_n}} \end{bmatrix},\quad X = \begin{bmatrix} x(T_1) \\ x(T_2) \\ \dots \\ x(T_n) \end{bmatrix} \tag{5}$$

There will be noise in the process of actual measurements, therefore solutions can be only obtained by using least squares:

$$\min: e = \|AX\text{-}Y^2\| \tag{6}$$

In order to eliminate the instability when using the non-negative least squares algorithm, the solutions can be optimized by adding a penalty item in (7):

$$\min: e = \|AX\text{-}Y^2\|+\alpha\|X\|^2 \tag{7}$$

where, α is smooth factor, $\alpha\|X\|^2$ is penalty item. By solving the optimization problem, optimal solution will be obtained as follow:

$$X = (A^TA+\alpha I)^{-1} A^TY \tag{8}$$

For the non-negative constraint of X in physical sense, when the solution is negative, non-negative solution can be obtained by increasing α. Taking the measurement error of the actual system into account, the actual measurement value $\tilde{y}$ should be:

$$\tilde{y} = y(t)+\varepsilon(t) \tag{9}$$

where, $\varepsilon(t)$ is the measurement error.

Using limited bandwidth of Gaussian random noise to simulate the measurement error and the measurement error will be expressed as:

$$\varepsilon(t) = \delta\cdot\lambda(t)\cdot y(t) \tag{10}$$

where, $\lambda(t)$ stands for Gaussian random number with zero mean and unit standard deviation, δ represents the applied noise level.

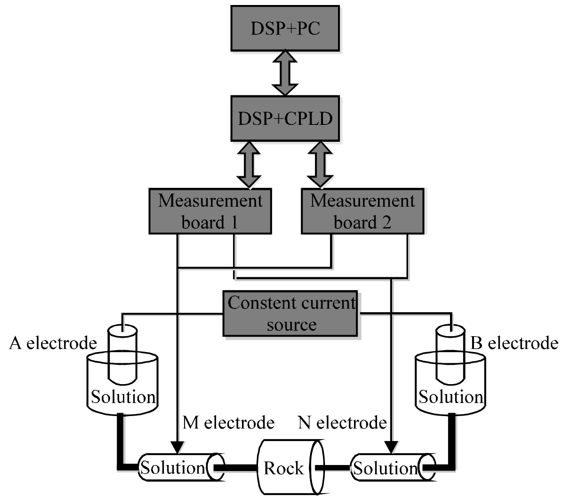

Fig. 1: The construction of measure apparatus

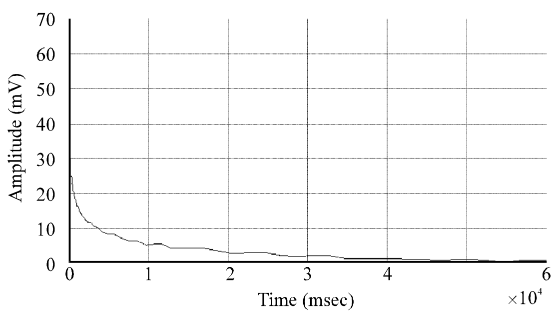

Fig. 2: Sampling of original data

SETUP OF MEASURE APPARATUS AND EXPERIMENTAL CONDITION

To acquire the measured data of rocks based on time domain induced polarization, the automatic control apparatus is designed. The whole construction of the apparatus is shown in Fig. 1. The function of measurement board 1 and 2 are to achieve the real-time values of the first and the secondary field potential individually.

Taking a group of actual measured secondary field potential signal of rocks as original data for sampling, the data graph is shown in Fig. 2 as followed. The experimental conditions are followed. The length of rock is 0.049 meter, diameter is 0.038 meter, the resistivity of saturated solution is 1.455 Ω·m, the value of power supply current is 5 mA, the measurement time of second field potential is 60 sec.

ANALYSIS OF INFLUENCING FACTORS ON MULTI-EXPONENTIAL INVERSION

Different inversion parameters and experimental conditions have a great impact on the inversion results in different multi-exponential inversion methods. In this study, the influences of the number of distribution point, range of relaxation time, smooth factor, SNR and other factors on the results of multi-exponential inversion are analyzed using penalty function method. The relationship between different factors is also studied by comparing to other algorithms and experimental conclusion.

THE INFLUENCE OF THE NUMBER OF DISTRIBUTION POINTS

The number of distribution points, affecting the inversion speed, also affects the shape of the relaxation time spectrum. Due to the complexity of the actual core porosity structure and widely aperture distribution, the use of few number of distribution points should cause less information about aperture structure contained in the relaxation time spectrum. As a consequence, the relaxation time spectrum can not accurately represent the actual nature of rocks. However, inversion consumed time will gradually increase with the increase in the number of relaxation time distribution. Setting the number of distribution points as 8, 16, 32, 64, 128, through uniform logarithmic distribution, the smooth factor is set to be 0.06 and range of the relaxation time distribution is set between 0.1~600000 sec. The inversion results are given in Fig. 3 as follows:

It can be concluded from the Fig. 3 that, when the number of relaxation time distribution is greater than 32, the difference between the inversions of the relaxation time spectrum is smaller. The more number of distribution points are, the higher resolution and more smooth curve of inversion will be obtained, peak time positions in the graphic are also consistent. When the number of distribution points set to be 8 and 16, the spectral resolution will be significantly worse, losing the characteristics of multi-peak in relaxation time spectrum. It means that the relaxation time spectrum is incomplete which is consistent with the previous analysis. Experimental results show that the number of distribution points is rational in the range from 32 to 64 which are basically the same as the conclusion from Tong *et al*. (2004).

THE INFLUENCE OF RELAXATION TIME DISTRIBUTION RANGE

The relaxation time distribution range corresponds to the pore diameter of the rock which influences on the truth and completeness of the inversion for relaxation time spectrum. Therefore, relaxation time distribution range plays an important part in the inversion. Choosing

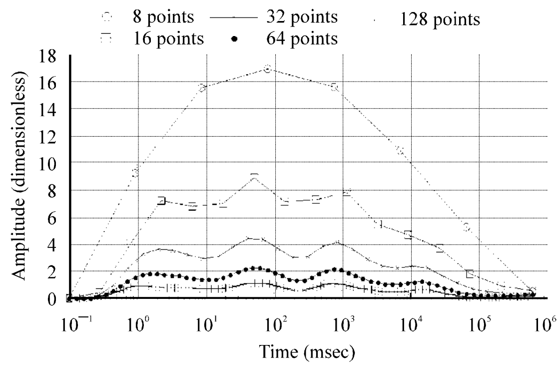

Fig. 3: Graphic of inversion in different numbers of distribution

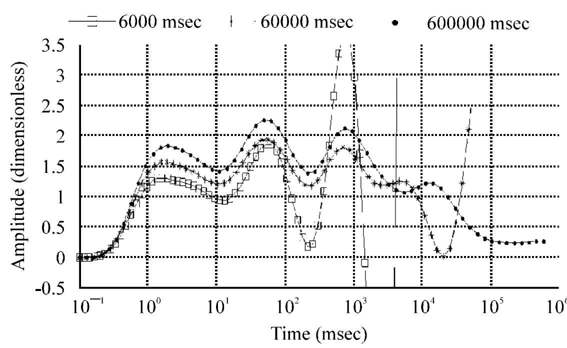

Fig. 4: Sample data inversion in different relaxation time distribution

0.1~6000, 0.1~ 60000 and 0.1~600000 msec as the relaxation time distribution range separately, picking 64 values from two sets of the original data in logarithmic uniform distribution way, smooth factor is 0.08 for each one, the multi-exponential inversion results are shown in Fig. 4.

When the relaxation time distribution range is too small, the completeness of the relaxation time spectrum can not be truly presented and the relaxation time spectrum turns out to be not convergent. In the case of the wide range, the inversion result does not show an obvious difference, the spectrum is convergent, only the termination of the spectrum is extended. So the relaxation time distribution range should be chosen according to the practical situation of the rock, for example the frequency range of argillaceous sandstone is from several Hz to a thousand Hz. The relaxation time range should span at least 3 orders of magnitude to guarantee the completeness of inversion for relaxation time spectrum and the minimum of the relaxation time distribution range should be less than or equal to 0.1 msec.

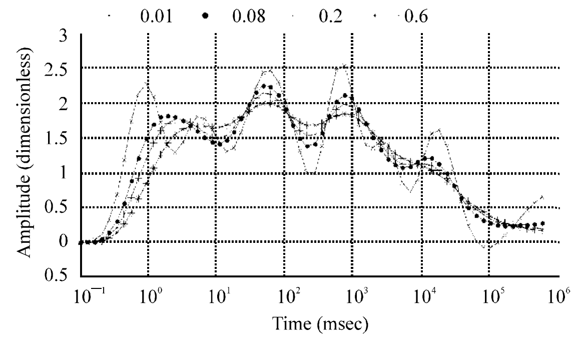

Fig. 5: Data inversion in different smooth factor

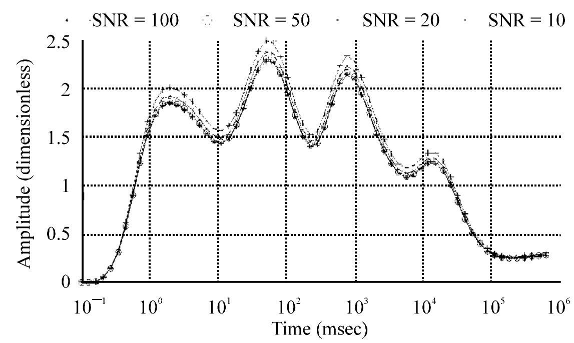

Fig. 6: Sample data inversion of different SNR

THE INFLUENCE OF THE SMOOTH FACTOR

Compared with the NNLS algorithm, when the inversion result is negative, the penalty function algorithm can be used to eliminate this negative result by adjusting the smooth factor. Choosing 0.01, 0.08, 0.2 and 0.6 as the smooth factors separately, the relaxation time distribution range of these data is 0.1-600000 msec, picking 64 values in logarithmic uniform distribution way, the inversion results are shown in Fig. 5.

We can conclude easily from Fig. 5 that, as the smooth factor increase, the peak values of relaxation time spectrum decreases while the valley value increases which make the inversion result flatter and may eliminate the negative values which may appear in inversion result. Besides, as the smooth factor grows, relaxation time spectrum converges faster. When the smooth factor is too small, the tail of the spectrum may be negative. The principle to choose the appropriate smooth factors is that it should be chosen as low as possible to make sure the inversion result is not negative.

THE INFLUENCE OF SNR

SNR decides the quality of the signal from sampled data and to some extent influences the inversion result of relaxation time spectrum of rocks. By adding noise signal into the original sampled data gradually, four experimental data sets are set up with SNR 100, 50, 20 and 10, respectively, the relaxation time distribution range is 0.1-600000 msec, picking 64 values in logarithmic uniform distribution way, smooth factor is 0.08, multi-exponential inversion results are shown in Fig. 6.

When the original data's SNR is 50 and 100, these two inversion results are similar with the former inversion results without noise but when the SNR is 10, even though the structure of the spectrum is much the same which just reflect the three-peak characteristic of the rock roughly, it's still different from the original inversion result. In order to guarantee the accuracy of inversion result, the SNR of the original sampled data should be larger than 20 which are basically similar with the conclusion from Ke-Jia *et al*. (2008).

CONCLUSION

The key to solve this problem is the choices of relevant parameters in the condition that the solution is non-negative. This paper investigates some factors' influence on the inversion result using penalty function algorithm, including the numbers of distribution point in multi-exponential fitting, relaxation time distribution range, smooth factor and SNR. It is concluded that numbers of distribution point affects the resolution ratio of relaxation time spectrum. To take rocks as an example, 32 to 64 is appropriate. If relaxation time distribution is too small, the inversion result will be not convergent and when if it's too large, there will be no obvious difference. The relaxation time distribution range should span 3 orders of magnitude at least and the minimum of the range should be less than or equal to 0.1 msec. Smooth factor can be used to eliminate the negative values in the inversion result.

ACKNOWLEDGMENT

We gratefully acknowledge many useful discussions with Mrs. Zeng Huaxiu, senior engineer serving on the Technology Center of China Petroleum Logging Co. Ltd.

REFERENCES

Borgia, G.C., R.J. Brown and P. Fantazzini, 1998. Uniform-penalty inversion of multiexponential Decay data. J. Magn. Reson., 132: 65-77.

Dunn, K.J., G.A. Latorraca and J.L. Warner, 1994. On the calculation and interpretation of NMR relaxation time distribution. Proceedings of the 69th Annual SPE Technical Conference and Exhibition, October, 1994, New Orleans, pp: 45-54.

Ghosh, S., K.M. Kevin and Y. Pan, 2008. A simulation based method to assess inversion algorithms for transverse relaxation data. J. Magn. Reson., 191: 226-230.

He, W. and L. Geng-Ying, 2005. Combination of inversion and fitting as an effective method for the analysis of NMR relaxation data. Acta Phy. Sin., 54: 1431-1436.

Ke-Jia, P., T. Yong-Ji and C. Hua, 2008. Multi-exponential inversion of T_2 Spectrum in NMR based on differential evolution algorithm. Acta Phys. Sin., 57: 5961-5961.

Kokot, S. and Z. Zembaty, 2009. Damage reconstruction of 3D frames using genetic algorithms with Levenberg-Marquardt local search. Soil Dyn. Earthquake Eng., 29: 311-323.

Munn, K. and D.M. Smith, 1987. A NMR technique for the analysis of pore structure: Numerical inversion of relaxation measurements. J. Colloid Interface Sci., 19: 117-126.

Sturrock, J.T., D. Lesmes and F.D. Morgan, 1999. Permeability estimation using spectral induced polarization measurements. Proceedings of the 12th EEGS Symposium on the Application of Geophysics to Engineering and Environmental Problems, 14 March, 1999, USA., pp: 409-415.

Titov, K., V. Komarov, V. Tarasov and A. Levitskic, 2002. Theoretical and experimental study of time domain-induced polarization in water-saturated sands. J. Applied Geophys., 50: 417-433.

Tong, M., W. Wang, L. Li, J. Yizhonga and S. Deqina, 2004. Estimation of permeability of shaly sand reservoir from induced polarization relaxation time spectra. J. Pet. Sci. Eng,, 45: 31-40.

Wang, W., P. Li and C. Ye, 2001. Multi-exponential inversion of nuclear magnetic relaxation signal. Sci. Chin. A Math., 44: 1477-1484.

Wang, Z.D., L.Z. Xiao and T.Y. Liu, 2003. New inversion method with multiexponent and its application. Sci. Chin. G, 4: 323-332.

Zhang, Y.D., L.N. Wu and G. Wei, 2009. Chaotic immune particle swarm optimization for multi-exponential fitting. J. Southeast Univ., 4: 678-683.