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Dynamical Properties Analysis of Rotor Systems with Uncertain-but-bounded Parameters

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Abstract: In this study, interval analysis method is discussed for estimating the dynamical properties of the rotor system with uncertain parameters. The error of dynamical parameters of an aeroengine rotor system is unavoidable in the course of manufacture and installation. Parameters of the rotor will vary due to friction during working. Critical speed and dynamical response are hard to be obtained by the traditional dynamical theory. Based on the variation of the rotor system parameters, variation of the natural frequencies of the rotor is obtained via interval analysis method and the variation range of critical speed is determined. The influence of parameter perturbations on natural frequencies is investigated. The illustrative numerical examples are provided to demonstrate the validity of interval analysis method. Compared with the Monte Carlo method, the calculated results show that the proposed method in this study is effective in evaluating bounds of the frequencies of uncertain rotor systems.

Key words: Interval analysis, rotor dynamics, eigenvalue, uncertain parameters

INTRODUCTION

The concept of uncertainty plays an important role in the investigation of various engineering problems. In particular, all engineering analysis and design problems involve uncertainty to varying degrees. The inaccuracy of measurements, for example, is often called 'uncertainty' which differs from the concept of error. An uncertainty is a possible value that error might take over a range and the dynamical response of structures with the uncertainty varies over a certain range.

Probabilistic model is the most widely used methods of solving uncertain problems in most engineering researches. In this case, all information about the structural parameters is provided by the probability density function of the structural parameters. However, probabilistic model is not the only way that one could describe the uncertainty and uncertainty does not equal randomness. In many practical problems, the sources of the uncertainty are too complex to allow analytical determination of the probability density function and meanwhile it's hard to obtain enough data to determine the probability density function. So, it can be seen that interval analysis can be used most conveniently.

Since, the mid-sixties, a new method called interval analysis has been developed. Moore (1979) have carried out the pioneering work. Deif (1991) obtained the solution theorem for interval matrix. Qiu *et al.* (1995) presented the interval perturbation method, semi-definite solution theorem and the inclusion theorem. Dimarogonas (1995) discussed the natural and forced vibration problems for interval rotor-bearing systems and solutions were developed using interval calculus. Hu and Qiu (2010) investigated the dynamical response of structures with uncertain-but-bounded parameters was studied via convex models and interval analysis methods. Yang *et al.* (2012) obtained the lower and upper bounds of dynamic response of structures with uncertainty by Laplace transform.

A substantial number of engineering vibration problems are indeed interval problems. All rotor dynamical analysis is performed with single-valued bearing properties (Li *et al*., 2012; Zhang *et al*., 2011). It is well known, however that the bearing properties change substantially with temperature which is controlled in machinery within relatively broad intervals. Manufacturing tolerances introduce another interval parameter, the bearing clearance which widens the bearing parameters further.

In this study, a new method is presented for computing the natural frequencies of a rotor system with uncertain parameters which could not be treated as being random, since no information is available on their probabilistic characteristics. The set of possible states of the system is described by interval matrices. By solving the generalized interval eigenvalue problem, the bounds on the critical speed of the rotor system with interval parameters are evaluated.

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INTERVAL AND INTERVAL OPERATIONS

In the following, the field of real numbers is denoted by R and its members are denoted by lower case letters. An interval is a subset of R with the following form (Sim *et al*., 2007; Chen *et al*., 2003):

$$X^I = [\underline{x}, \overline{x}] = \{t \mid \underline{x} \le t \le \overline{x}; \underline{x}, \overline{x} \in R\} \tag{1}$$

where, $\underline{x}$ is the lower bound and $\overline{x}$ is the upper bound. The set of closed real intervals will be denoted by I(R) and its members by upper case letters. Assume that I(R), I(R^n) and I($R^{n\times n}$) denote the sets of all closed real interval numbers, n-dimensional real interval vectors and n×n real interval matrices, respectively. $X^I = [\underline{x}, \overline{x}]$ is a member of I(R) and X^I be usually written in the following form:

$$X^I = [X^c - \Delta X, X^c + \Delta X] \tag{2}$$

where, X^c and ΔX denote the mean (or midpoint) value of X^I and the uncertainty (or the maximum width) in X^I, respectively. It follows that:

$$X^c = \frac{\underline{x} + \overline{x}}{2} \tag{3}$$

$$\Delta X = \frac{\overline{x} - \underline{x}}{2} \tag{4}$$

In terms of the interval addition, Eq. 2 can be put into the more useful form:

$$X^I = X^C + \Delta X^I \tag{5}$$

where, $\Delta X^I = [-\Delta X, \Delta X]$.

An n-dimensional real interval vector $X^I \in I(R^n)$ can be written as:

$$X^I = (X_1^I, X_2^I, \cdots, X_n^I)^T \tag{6}$$

The mean value and uncertainty of x^I are:

$$X^c = (X_1^c, X_2^c, \cdots, X_n^c)^T \tag{7}$$

$$\Delta X = (\Delta X_1, \Delta X_2, \cdots, \Delta X_n)^T \tag{8}$$

Similar expressions exist for an n×n interval matrix $A^I = [\underline{A}, \overline{A}] \in I(R^{n\times n})$.

$$A^I = A^C + \Delta A^I \tag{9}$$

where, $\Delta A^I = [-\Delta A, \Delta A]$ A^C and ΔA denote the mean matrix of A^I and the uncertain (or the maximum width) matrix of A^I, respectively. It follows that:

$$A^c = \frac{(\overline{A} + \underline{A})}{2} \text{ or } a_{ij}^c = \frac{(\overline{a}_{ij} + \underline{a}_{ij})}{2} \tag{10}$$

$$\Delta A = \frac{(\overline{A} - \underline{A})}{2} \text{ or } \Delta a_{ij} = \frac{(\overline{a}_{ij} - \underline{a}_{ij})}{2} \tag{11}$$

where, $A^c = (a^c_{ij})$ and $\Delta A = (\Delta a_{ij})$.

Let $X^I, Y^I \in I(R), X^I = [\underline{x}, \overline{x}], Y^I = [\underline{y}, \overline{y}]$. Two sets $X^I = [\underline{x}, \overline{x}]$ and $Y^I = [\underline{y}, \overline{y}]$ are equal if $\underline{x} = \underline{y}$ and $\overline{x} = \overline{y}$. An interval of zero width [x, x] will be called as the point interval and it is a regular real number. The operations for X^I+Y^I, X^I-Y^I, X^I-Y^I, $X^I\times Y^I$ and X^I/Y^I are:

$$X^I + Y^I = [\underline{x}, \overline{x}] + [\underline{y}, \overline{y}] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}] \tag{12}$$

$$X^I - Y^I = [\underline{x}, \overline{x}] - [\underline{y}, \overline{y}] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}] \tag{13}$$

$$X^I \times Y^I = [\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}] = [\min(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}), \max(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y})] \tag{14}$$

$$\frac{X^I}{Y^I} = \frac{\underline{x}, \overline{x}}{\underline{y}, \overline{y}} = [\underline{x}, \overline{x}] \times \frac{1}{\overline{y}}, \frac{1}{\underline{y}} \tag{15}$$

It is apparent that division is not defined if $0 \in [\underline{x}, \overline{x}]$ and that $[\underline{x}, \overline{x}]c = [c\underline{x}, c\overline{x}]$ for c>0 and $[\underline{x}, \overline{x}]c = [c\overline{x}, c\underline{x}]$ for c>0.

While commutativity and associativity are preserved in interval algebra, distributivity is not preserved that is:

$$X^I(Y^I + Z^I) \ne X^I Y^I + X^I Z^I \tag{16}$$

However, the following subdistributivity, or inclusion rule is true:

$$X^I(Y^I + Z^I) \subseteq X^I Y^I + X^I Z^I \tag{17}$$

The subdistributivity law, or inclusion theorem, is of fundamental importance to the interval calculus, because it proves that interval arithmetic operations always yield an upper bound for the interval of the function. Thus they provide a conservative estimate for it.

BOUNDS OF INTERVAL EIGENVALUES

The generalized eigenvalue problem is expressed as follows:

$$Ku = \lambda Mu \tag{18}$$

where, $K = (k_{ij})$ is stiffness matrix, $M = (m_{ij})$ is the mass matrix, u is the eigenvector and λ is the square of the frequency of free vibration.

Generalized interval eigenvalue problem: In a variety of applications it is often desirable to obtain solutions to the eigenvalue problem in which K and M are affected by uncertainties as subjected to:

$$\underline{K} \le K \le \overline{K} \text{ or } \underline{k}_{ij} \le k_{ij} \le \overline{k}_{ij},\ i, j = 1, 2, \cdots, n \tag{19}$$

$$\underline{M} \le M \le \overline{M} \text{ or } \underline{m}_{ij} \le m_{ij} \le \overline{m}_{ij},\ i, j = 1, 2, \cdots, n \tag{20}$$

$\underline{K} = (\underline{k}_{ij})$ and $\overline{K} = (\overline{k}_{ij})$ are the minimum and maximum allowable stiffness matrices of system $\underline{M} = (\underline{m}_{ij})$ and $\overline{M} = (\overline{m}_{ij})$ are the minimum and maximum allowed mass matrices of system. With the use of the interval matrix notation, the above equations can be rewritten as:

$$K \in K^I,\ K^I = [\underline{K}, \overline{K}] = [K^c - \Delta K, K^c + \Delta K] \tag{21}$$

$$M \in M^I, M^I = [\underline{M}, \overline{M}] = [M^c - \Delta M, M^c + \Delta M] \tag{22}$$

where, $K^I = [\underline{K}, \overline{K}]$ is a positive-semidefinite symmetric interval matrix and $M^I = [\underline{M}, \overline{M}]$ is a positive-definite symmetric interval matrix. Then K^C and M^C are interval stiffness and mass and ΔK and ΔM, respectively radius interval stiffness and mass which are given by:

$$K^c = \frac{\underline{K} + \overline{K}}{2},\ M^c = \frac{\underline{M} + \overline{M}}{2}, \Delta K = \frac{\overline{K} - \underline{K}}{2},\ \Delta M = \frac{\overline{M} - \underline{M}}{2}$$

For the sake of simplicity, Eq. 18 can be expressed by Eq. 10:

$$K^I u = \lambda M^I u \tag{23}$$

The above equation is called a generalized interval eigenvalue problem. Because K^I and M^I are defined as interval matrices, the associated eigenvalue of K^I and M^I similarly constitute interval variables $\lambda^I = [\underline{\lambda}, \overline{\lambda}] = (\lambda_i^I)$. The objective is to find all the possible eigenvalues $\lambda \in R^n$ satisfying the equation $Ku = \lambda Mu$, where, K and M are assuming all possible values inside K^I and M^I. This infinite number of solutions constitutes a region inside R^n which we will call Γ and is expressed as follows:

$$\Gamma = \{\lambda: \lambda \in R^n, Ku = \lambda Mu, u \ne 0, K \in K^I, M \in M^I\} \tag{24}$$

Solving the generalized interval eigenvalue problem 23 is synonymous to finding a multi-dimensional rectangle containing all eigenvalues Eq. 24 set for interval matrix sets (21) and (22). In other words, we seek the lower and upper bounds, or interval eigenvalues, on the eigenvalue set (24), i.e.:

$$\lambda^I = [\underline{\lambda}, \overline{\lambda}] = (\lambda_i^I),\ \lambda_i^I = [\underline{\lambda}_i, \overline{\lambda}_i],\ i = 1, 2, \cdots, n \tag{25}$$

Where:

$$\underline{\lambda}_i = \min_{K \in K^I, M \in M^I} \lambda_i(<K, M>) \tag{26}$$

$$\overline{\lambda}_i = \max_{K \in K^I, M \in M^I} \lambda_i(<K, M>) \tag{27}$$

in which:

$$\lambda_i(<K, M>) = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \ne 0}} \frac{u^T K u}{u^T M u} \tag{28}$$

where, $\Phi^i \subset R^n$ is an arbitrary i-dimensional subspace of n-dimensional real space.

Interval analysis method: Here, we will calculate the generalized interval eigenvalue problem making use of the interval mathematics. Clearly, the eigenvalue λ_i is considered a function of the element k_{ij} and m_{ij}. Then, by means of the natural interval extension, from Eq. 28 we obtain:

$$\lambda_i^I = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \ne 0}} \frac{u^T K^I u}{u^T M^I u} \tag{29}$$

In terms of the interval operations (Moore, 1979; Qiu *et al*., 1995), Eq. 29 can be rewritten as:

$$\lambda_i^I = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \ne 0}} \frac{[u^T \underline{K} u, u^T \overline{K} u]}{[u^T \underline{M} u, u^T \overline{M} u]} \tag{30}$$

By the interval division, we obtain:

$$\lambda_i^I = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \ne 0}} [\frac{u^T \underline{K} u}{u^T \overline{M} u}, \frac{u^T \overline{K} u}{u^T \underline{M} u}] \tag{31}$$

In terms of Eq. 21-22 and 31 can be rewritten as follow:

$$\lambda_i^I = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \ne 0}} [\frac{u^T (K^c - \Delta K) u}{u^T (M^c + \Delta M) u}, \frac{u^T (K^c + \Delta K) u}{u^T (M^c - \Delta M) u}] \tag{32}$$

Then, to obtain the lower and upper bounds on a particular λ_i, we can introduce Dief's assumption, i.e., signs of components of the associated eigenvector u^i remain unchanged, when matrices K and M range over the interval $K^I = [\underline{K}, \overline{K}]$ and $M^I = [\underline{M}, \overline{M}]$. Then we define:

$$S^i = \text{diag}(\text{sgn}(u_1^i), \text{sgn}(u_2^i), \cdots, \text{sgn}(u_n^i)) \quad u_j^i \neq 0 \quad i, j = 1, 2, \cdots, n \tag{33}$$

where, S^i is a diagonal sign matrix expressed by the sign of row elements $(u_1^i, u_2^i, ..., u_n^i)^T$ of the ith centered eigenvector u^i. In the sign matrix S^i of the eigenvectors within the interval of the eigenvalues, the bounds found for the eigenvalues are exact. Then:

$$S^i u^i = |u^i| > 0 \tag{34}$$

Substitution of Eq. 34 into 32, we have:

$$\lambda_i^I = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \neq 0}} \left[\frac{u^T(K^c - S^i \Delta K S^i)u}{u^T(M^c + S^i \Delta M S^i)u}, \frac{u^T(K^c + S^i \Delta K S^i)u}{u^T(M^c - S^i \Delta M S^i)u} \right] \tag{35}$$

Thus, from Eq. 35, we have:

$$\lambda_i^I = [\underline{\lambda}_i, \overline{\lambda}_i] = \left[\min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \neq 0}} \frac{u^T(K^c - S^i \Delta K S^i)u}{u^T(M^c + S^i \Delta M S^i)u}, \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \neq 0}} \frac{u^T(K^c + S^i \Delta K S^i)u}{u^T(M^c - S^i \Delta M S^i)u} \right] \tag{36}$$

According to the necessary and sufficient conditions of equality of interval variables, we obtain:

$$\underline{\lambda}_i = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \neq 0}} \frac{u^T(K^c - S^i \Delta K S^i)u}{u^T(M^c + S^i \Delta M S^i)u} \tag{37}$$

$$\overline{\lambda}_i = \min_{\Phi_i \subset R^n} \max_{\substack{u \in \Phi_i \\ u \neq 0}} \frac{u^T(K^c + S^i \Delta K S^i)u}{u^T(M^c - S^i \Delta M S^i)u} \tag{38}$$

The stationarity condition of the Rayleigh quotient is equivalent to the algebraic eigenvalue problem. Thus, the eigenvalue problem corresponding to the lower bound of Eq. 37 is:

$$(K^c - S^i \Delta K S^i)\underline{u}_i = \underline{\lambda}_i (M^c + S^i \Delta M S^i)\underline{u}_i \tag{39}$$

where, $\underline{u}_i$ is the eigenvector associated with $\underline{\lambda}_i$. Similarly, the eigenvalue problem corresponding to the upper bound of Eq. 38 is:

$$(K^c + S^i \Delta K S^i)\overline{u}_i = \overline{\lambda}_i (M^c - S^i \Delta M S^i)\overline{u}_i \tag{40}$$

where, $\underline{u}_i$ is the eigenvector associated with $\overline{\lambda}_i$.

Thus, we arrive at the following theorem.

If $K^I = [\underline{K}, \overline{K}] = [K^c - \Delta K, K^c + \Delta K]$ is a positive-semidefinite interval matrix and if $M^I = [\underline{M}, \overline{M}] = [M^c - \Delta M, M^c + \Delta M]$ is a positive-definite interval matrix, ΔK and ΔM are also positive-semidefinite real matrices. Signs of components of the associated eigenvect u^i, i.e., S^i = diag $(\text{sgn}(u_1^i), \text{sgn}(u_2^i), ..., \text{sgn}(u_n^i))$ $u_j^i \neq 0$ i, j = 1, 2,..., n, remain unchanged, when matrices K and M range over the interval $K^I = [\underline{K}, \overline{K}]$ and $M^I = [\underline{M}, \overline{M}]$, then the eigenvalue λ_i, i = 1, 2,..., n, of $K \in K^I$ and $M \in M^I$ range over the interval, i.e.:

$$\lambda_i^I = [\underline{\lambda}_i, \overline{\lambda}_i], \ i = 1, 2, \cdots, n \tag{41}$$

where, the lower bounds $\underline{\lambda}_i$ satisfy:

$$(K^c - S^i \Delta K S^i)\underline{u}_i = \underline{\lambda}_i (M^c + S^i \Delta M S^i)\underline{u}_i \tag{42}$$

and the upper bounds $\underline{\lambda}_i$ satisfy:

$$(K^c + S^i \Delta K S^i)\overline{u}_i = \overline{\lambda}_i (M^c - S^i \Delta M S^i)\overline{u}_i \tag{43}$$

NUMERICAL EXAMPLES

The error of dynamical parameters of an aeroengine rotor system is unavoidable in the course of manufacture and installation. Parameters of the rotor will vary due to friction during working. Critical speed and dynamical response are hard to be obtained by the traditional dynamical theory. Interval analysis method can deal with the uncertainty problems effectively.

Example: An application is presented for the dynamic response of a rotor with interval bearing properties. The dynamical model of the aeroengines rotor can be simplified by a Jeffcott rotor shown in Fig. 1 a rotor system consists of a massless shaft, a disk in the middle of the shaft and two self-contained bearings at the ends of the shaft. When the stiffness matrix, K and the mass matrix, M, of the structure are influenced by uncertain parameters, it is significant to solve the eigenvalue problem, $Ku = \lambda Mu$. The physical parameters of this

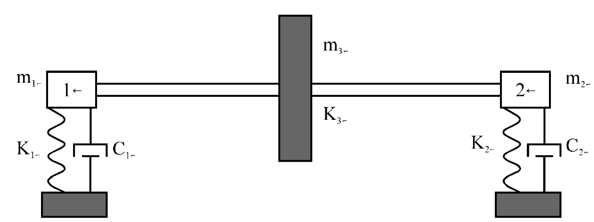

Fig. 1: Dynamical model of the aeroengines rotor simplified by a Jeffcott rotor

Table 1: Eigenvalues and eigenvectors for the nominal parameters K^C and M^C

	λ_{1c}	λ_{2c}	λ_{3c}
λ_{ic}	10211	18538	52084
u_{ic}	0.5755	-0.6509	0.4950
	0.7277	0.1314	-0.6732
	0.3732	0.7477	0.5493

Table 2: Comparison between interval analysis and monte carlo method on the natural frequency of the rotor

	Interval analysis method			Monte carlo method		
	$\underline{\lambda}_i$	$\overline{\lambda}_i$	$\Delta\lambda_i$	$\underline{\lambda}_i$	$\overline{\lambda}_i$	$\Delta\lambda_i$
λ_1	0.8543×10^4	1.1729×10^4	3186	0.8772×10^4	1.1672×10^4	2950
λ^2	1.5715×10^4	2.1911×10^4	6736	1.5933×10^4	2.1817×10^4	5884
λ^3	5.0282×10^4	5.4027×10^4	3745	5.0646×10^4	5.3972×10^4	3326

$\underline{\lambda}_i$: Lower eigenvalue bounds using the present method, $\overline{\lambda}_i$: Upper eigenvalue bounds using the present method, $\underline{\mu}_i$: Lower eigenvalue bounds using monte carlo method, $\overline{\mu}_i$: Upper eigenvalue bounds using monte carlo method and $\Delta\lambda_i$, $\Delta\mu_i$: Uncertainty

system are as follows $m_1 = m^2 = m_3 = 300$ kg, $K_3 = 8.8\times10^6$ N/m, K_1 and K_2 are interval values, $K^1{}_1 = [4\times10^6, 6\times10^6]$ N/m, $K^1{}_2 = [5\times10^6, 7.1\times10^6]$ N/m. For simplicity, we assume that the horizontal and vertical vibrations are not coupled.

We can obtain the dynamical equation of the system easily, as follows:

$$M\ddot{X} + KX = 0$$

The system mass matrix is:

$$M = \begin{pmatrix} [300,300] & 0 & 0 \\ 0 & [300,300] & 0 \\ 0 & 0 & [300,300] \end{pmatrix}$$

The interval stiffness matrix is:

$$K^I = \begin{pmatrix} \frac{K_3}{4} + K_1^I & -\frac{K_3}{2} & \frac{K_3}{4} \\ -\frac{K_3}{2} & K_3 & -\frac{K_3}{2} \\ \frac{K_3}{4} & -\frac{K_3}{2} & \frac{K_3}{4} + K_2^I \end{pmatrix} = \begin{pmatrix} [62,82] & [-44,-44] & [22,22] \\ [-44,-44] & [88,88] & [-44,-44] \\ [22,22] & [-44,-44] & [72,93] \end{pmatrix} \times 10^6$$

The eigenvalues and eigenvectors for the nominal rotor system K^C and M^C are summarized in Table 1. The upper and lower bounds on the eigenvalues are listed in Table 2. To facilitate comparison, the upper and lower bounds obtained by Monte Carlo method are also listed. Monte Carlo solutions approach to the exact results.

It can be seen from Table 2 that very good agreement between the interval evaluation and the Monte Carlo solution is obtained. But the interval of eigenvalues obtained by the Monte Carlo method is contained by that yielded by the interval analysis. That is to say, the lower bounds yielded by the interval analysis are smaller than those predicted by the Monte Carlo method. Likewise, the upper bounds furnished by the interval analysis are larger than those yielded by the Monte Carlo method.

CONCLUSION

A rational method for the solution of the generalized interval eigenvalue problem was presented with an application to rotor dynamics. Interval analysis method doesn't need the information of probability information of the uncertain parameters but only need the top and bottom limitation of the uncertain parameters and the frequency set relevant to the boundary of the uncertain parameters can be obtained. A Monte Carlo method was used to provide an exhaustive alternative to test the proximity of the evaluated interval solution to the true one. The numerical example shows that interval analysis can predict the range of the eigenvalues with sufficient accuracy and the interval of eigenvalues interval obtained by the Monte Carlo method are contained by that yielded by the interval analysis. So more possible solutions yielded by the interval analysis method are obtained. The Monte Carlo method can only get a part of solutions of frequencies. Compared with the traditional probability method, interval analysis method possesses the property of high accuracy and low calculated amount and can be easily realized in computer. Interval analysis method has a great potential role in the design and manufacture of rotors.

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