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Multi-channel Hybrid Wireless Network Capacity with Bottleneck Constraint

Lin Chen

College of Computer and Information,
Shanghai Second Polytechnic University, 201209, Shanghai, China

Abstract: Network capacity is one of basic problems of wireless networks, which reflects the asymptotic capacity when the number of nodes approaches infinite. Multi-channel hybrid wireless network, such as vehicular ad hoc network utilizes base station supports and multi-channel technologies. Due to randomly selecting the destination nodes, the performance of destination node may be constrained. For this kind of network sceneries, the mathematical analysis model is presented. Furthermore, the upper bound is estimated and the lower bound is constructed and calculated. The theoretical analysis results show that the time and channel resources are distributed according to the number of flows of nodes may improve the network performance. It provides one important reference for network system optimization.

Key words: Network capacity, multi-channel, hybrid, bottleneck constraint

INTRODUCTION

A basic attribute of Wireless network is the asymptotic capacity which reflects the network performance changing rule with increasing network size. Furthermore, the analysis and evaluation of wireless network is the fundamental base to plan and design the network. The network capacity is constrained by multiple factors, such as flow model, bandwidth, energy and network topology.

In the landmark paper, Gupta and Kumar (2000) presented that when there are n nodes uniformly distributed in an unit circle and each node randomly selects another nodes as its destination, then the capacity of each node is $\lambda(n) = \Theta\left(\frac{W}{\sqrt{n \log n}}\right)$. This is a disappointing result, for it shows that the large scale of ad hoc network is infeasible.

From then on, many work have been done on the wireless network capacity (Dai *et al.*, 2006; Han *et al.*, 2011; Guo and Shi, 2008).

Utilizing the node mobility is one method to improve capacity which has been studied in papers (Grossglauser and Tse, 2002; Gamal *et al.*, 2004). Add base stations, building hybrid network is another way and it is more feasible. Liu *et al.* (2003) studied hybrid wireless network model which assumes base stations are fixed in the hexagon lattices and ad hoc nodes are randomly distributed among the whole area. The results showed that when there are m base stations and n ad hoc nodes in the network and the growing rate of m is less than \sqrt{n} , the network capacity improves little; when the growing rate is greater than \sqrt{n} , it increases linearly with the

growth of m .area and the ratio of the number of base stations to ad hoc nodes is constant, then the capacity of each ad hoc node is $\Phi(W/\log n)$.

Another way to improve the network capacity is to adopt multi-channel technology. For the general ad hoc networks, Kyasanur and Vaidya (2005) studied the relation between the number of channels and that of interfaces of each nodes. In the model, the interfaces can switch freely among channels. The analysis results show that when interfaces are in correspondence with channels, the capacity of multi-channel multi-interface network is the same as that of single-channel single-interface ad hoc network addressed by Gupta and Kumar (2000). Furthermore, for stochastic network, when the ratio between the number of channels to that of interfaces is in the interval $O(\log n)$, there is no capacity loss; however, if the ratio is greater than the interval, then there has been capacity loss. Bhandari and Vaidya (2006) studied the network connectivity and capacity of multi-channel wireless network with channel switch constraints, that is the interfaces can switch among a part of channels.

In fact, many networks adopt both base stations and multi-channel technologies. For example, In Vehicular ad hoc networks, vehicles communicate with each other by ad hoc mode; on the other hand, vehicles can also communicate with base stations. Different communication modes adopts different interfaces and channels. Chen and Wei (2010) and Chen *et al.* (2011) studied multi-channel hybrid wireless network capacity, however, the bottleneck constraint has not been addressed. In the paper, we use MC-HB to represents multi-channel hybrid wireless network. Our main contributions are as follows:

Construct the analysis architecture for MC-HB:

- Analyze the upper bound of network capacity, obtain the quantitative results
- Build up the analysis model for lower bound of capacity of MC-HB, obtain the quantitative results
- Discuss the network optimization method according to the capacity analysis results

The remainder is organized as follows: The symbol and notations are presented in section 2; network model is given in section 3; the upper and lower bounds of network capacity of MC-HB are presented in section 4; discussions and conclusions are described in section 5.

DEFINITION AND NOTATION

We use the following notation to represent asymptotic bounds:

- $F(n) = O(g(n))$ means there exists some constant α and integer N such that α for $n > N$
- $F(n) = o(g(n))$ means that $\lim_{n \rightarrow \infty} f(n)/g(n) = 0$
- $F(n) = \alpha(g(n))$ means that $g(n) = O(f(n))$
- $F(n) = \alpha(g(n))$ means that $g(n) = \alpha(f(n))$
- $F(n) = o(g(n))$ means that $f(n) = O(f(n))$ and $g(n) = O(f(n))$

The traffic between a source-destination pair is referred to as a "flow". As in (Kysanur and Vaidya, 2005), we say that per flow capacity is λ if each flow in the network can be guaranteed a throughput of at least λ and the network capacity is defined to the aggregate throughput over all the flows in the network, $n\lambda$. High probability is when $n \rightarrow \infty$, the probability is 1.

MODEL

MC-HB networks consist of base stations and ad hoc nodes, the same as (Chen and Wei, 2010) All these nodes and stations are uniformly distributed on the surface of a unit torus. Ad hoc nodes alone form a connected topology graph with high probability. In other words, any pair of ad hoc nodes can communicate with each other along the paths that cross only the ad hoc nodes with probability close to one. Base stations are connected through wired and alternative wireless links with relatively high bandwidth. Moreover, base stations only relay traffic and do not generate their own data. Ad hoc nodes are also allowed to utilize the infrastructure network composed by base stations. Each ad hoc node randomly selects one destination node, so many ad hoc nodes may

select the same destination node which may become the traffic bottleneck and decrease the network capacity. References(Chen and Wei, 2010; Chen *et al.*, 2011) assume that each node should be the destination of at most $O(1)$ flows.

These ad hoc nodes are capable of transmitting and receiving Wbits/s via all c channels and the base stations can also communicate with ad hoc nodes by these common channels. Any node is equipped with m interfaces and each interface is assumed to be capable of transmitting or receiving data on any channel. We assume that the number of ad hoc nodes per base station is bounded and $\lim_{n \rightarrow \infty} (n/k) = \beta$ where $\beta \in (0, \infty)$.

Interference model: The protocol model proposed by Gupta and Kumar (2000) is adopted.

Suppose node X_i transmits over the m th channel to a node X_j . Then this transmission is successfully received by node X_j , for every other node X_k simultaneously transmitting over the same channel and the guard zone $\Delta > 0$, the following condition holds.

$$\{|X_k - X_j| \geq (1 + \Delta)|X_i - X_j|\}$$

X_i also denotes the location of a node.

Channel model: The total data rate is divided equally among the channels and then the data rate supported by any one of the c channels is W/c bits/s.

CAPACITY BOUNDS

The network capacity of MC-HB is constrained by a variety of factors, interference, bandwidth, the number of interfaces and other ones. The minimum value of upper bound caused by different factors is the upper bound of its capacity; the lower bound is obtained by construction method.

Upper bound: The network capacity of MC-HB is constrained by two factors (according to the relation of the number of channels c to that of interfaces m).

Constraint 1: Interference constraint: from the protocol model, each transmission consumes a certain area of region. Thus, if there are two simultaneous transmissions on the same channel, the two regions cannot intersect. So, we can determine the number of simultaneous transmissions through bounding that of regions. The number of disc regions is not greater than:

$$\frac{1}{\pi \Delta^2 r(n,k)^2}$$

where $r(n, k)$ is transmission radius. Furthermore, according to Reference (Kozat and Tassiulas, 2003), to maintaining connectivity of MC-HB:

$$r(n, k) \geq \sqrt{\frac{\log n}{\pi n}}$$

following the same steps in Ref[8], we can obtain the network capacity of MC-HB is $O(nW/\log n)$ bits/sec.

Constraint 2

Bottleneck constraint: Each ad hoc node has m interfaces and each interface can transmit on any channel, that is the transmit rate is w/c bits/sec, so the altogether traffic rate for each node is mW/c bits/sec. Because randomly selects the destination nodes, one ad hoc node is the destination of at most $D(n)$ flows, then the maximum rate of the minimum transmission rate of one flow of the node is $mW/cD(n)$ bits/sec. Therefore, the network capacity of MC-HB is $O(mW/cD(n))$ bits/sec. From Reference (Kysanur and Vaidya, 2005) with high probability:

$$D(n) = \Theta\left(\frac{\log n}{\log \log n}\right)$$

so the capacity is $O(m \log \log n W / c \log n)$ bits/sec.

According to the above discussion, the upper bound of MC-HB is:

Theorem 1: The upper bound of MC-HB is:

- When c/m is $O(\log \log n)$, the network capacity of MC-HB is $O(nW/\log n)$ bits/sec
- When c/m is $\Omega(\log \log n)$, the network capacity of MC-HB is $O(m \log \log n W / c \log n)$ bits/sec

Lower bound: We adopt the following steps to construct routing and scheduling scheme to implement the lower bound of MC-HB. The steps are similar with that in Reference (Gupta and Kumar, 2000; Kysanur and Vaidya, 2005). The network with each node equipped single interface is first dealt with and then expanded to the general condition with m interfaces.

Step 1: Cell construction: The surface of the unit torus is divided into square cells using a square grid and we denote each of area by $a(n)$, similar to that used in (Gupta and Kumar, 2000). In particular, we set:

$$a(n, k) = \max\left(\frac{100 \log(n)}{n}, \frac{cD(n)}{n}\right)$$

The transmission radii of each node are set to $\sqrt{8a(n)}$, thereby satisfying the requirement that any node in one cell can communicate with any other node in its neighboring cells and also to guarantee that ad hoc nodes form a connected graph with high probability.

Lemma 1: Base stations and ad hoc nodes in any cell is $\Theta(na(n, k))$ with high probability (Chen and Wei, 2010; Chen *et al.*, 2011).

Proof: The proof process is based on VC theory (Vapnik-Chervonenkis theory). According to the theory, if F is finite VC dimension set $VC-d(F)$, $\{X_i\}$ is independent and identically distributed random variables, has the same probability distribution P , then for any $\epsilon, \delta > 0$:

$$\Pr\left(\sup_{D \in F} \left| \frac{1}{N} \sum_{i=1}^N I(X_i \in D) - P(D) \right| \leq \epsilon\right) > 1 - \delta$$

Where:

$$N > \max\left(\frac{8VCdim(F)}{\epsilon} \log \frac{16e}{\epsilon}, \frac{4}{\epsilon} \log \frac{2}{\delta}\right)$$

Based on VC theory and the same process in Reference (Gupta and Kumar, 2000), lemma 1 can be proved.

Lemma 2: If some transmission in some cell interfere with one transmission in another cell, then call one cell as interference cell with another cell. The number of interference cell with any cell is a constant (dependent on Δ).

Proof: The scheme in Reference (Gupta and Kumar, 2000; Kysanur and Vaidya, 2005) can be adopted to prove the lemma. Given the transmission range of a given node can limit the interference area by the transmission. Obviously, the maximum number of cells it contains is a limited constant.

Step 2: Routing scheme: There are two routing strategy (Chen *et al.*, 2011): One, if destination node is located in the same cell as source node, then packets are transmitted to its destination node directly; otherwise, packets are firstly transmitted to the base station in its own cell that is with the least number of flows load. Next, the base station forwards the data to that base station that is located at the same cell with the destination node and with the least number of flows. Lastly, the base station transmits the data to the destination node directly

Lemma 3: The number of transmissions in any cell is $\Theta(n\alpha(n))$.

Proof: It is shown from lemma 1 that the number of *ad hoc* nodes in any cell is $\Theta(n\alpha(n))$. Each node is the source node of one flow and generate one transmission in the cell; if the destination node is at the same cell with the source node, the source node transmits directly data to its destination node; otherwise, the source node forward data to some base station in the same cell. So as source nodes, there are $\Theta(n\alpha(n))$ transmissions in any cell.

Next, we should address the number of nodes as flow destination in any cell. The problem can be converted to the famous “Ball into bins” problem. There are cells with unit area $a(n)$, then the number of cells is $1/a(n)$, so the problem is like to throw n balls into $1/a(n)$ bins. From (Raab and Steger, 1998), with high probability, the maximum number of balls in any bin is:

$$\frac{n}{\frac{1}{a(n)}} = na(n,k)$$

Thus, the number of destination nodes for flows in any cell is $O(na(n))$.

Based on the above analysis, The number of transmissions in any cell is $\Theta(n\alpha(n)) + O(na(n)) = \Theta(n\alpha(n))$.

If there are excessive flows through some base station, then it may become the bottleneck, so we should

balance the load among base stations. When some flow needs to through one base station in any cell, the base station with minimum load is always selected. With this load distribution scheme, the load of each base station can be guaranteed by the following Lemma.

Lemma 4: With high probability, the number of flows through any base station in any cell is $\Theta(1)$.

Proof: Based on lemma 1, the number of base stations in any cell is $\Theta(n\alpha(n))$. Moreover, according to lemma, the number of transmissions in any cell is $\Theta(n\alpha(n))$. Furthermore, the load balance scheme is adopted, so the lemma is obviously obtained.

Step 3: Transmission scheduling: The transmission scheduling strategy is shown in Fig. 1.

It can be known from lemma 2 that the interfered cell is a constant and we define its upper bound as k_1 . Then it is obviously that we can use time division method, by poll, let each cell get a fixed time slice to complete internal transmissions. The fixed time slice is defined as cell slot, its duration is $\Omega(1/k_1+1)$ bits/sec.

The cell slot can be further divided. There are $\Theta(n\alpha(n, k))$ *ad hoc* nodes in any cell, that is at most $k_2 na(n, k)$ (k_2 is some constant); the number of transmissions is $\Theta(n\alpha(n, k))$, that is at most $k_3 na(n, k)$. The whole transmissions in the cell is distributed among c channels of MC-HB (label from 1 to c), that is the number of transmissions on one channel is $\lfloor k_3 na(n, k)/c \rfloor$.

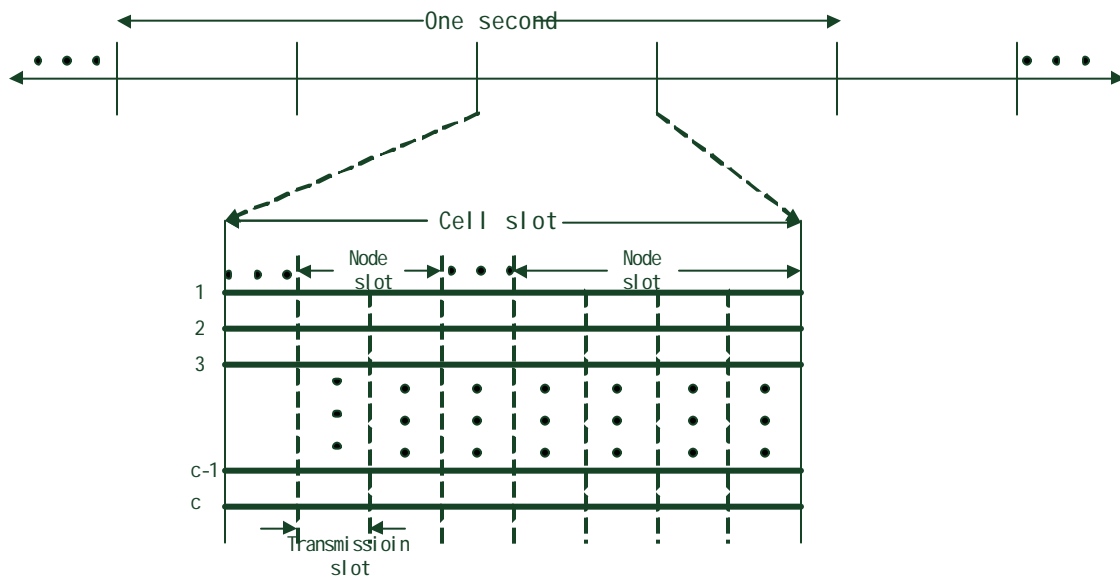


Fig. 1: Transmission scheduling strategy

Each ad hoc node has different lengths of node slot and the node having more transmissions will be allocated more transmission time. The number of transmissions of each ad hoc node is at most $D(n)+1$. Suppose on channel s , the number of transmissions of some ad hoc node is t , then the allocated node slot is:

$$\left\lceil \frac{t}{k_3 n a(n, k)} \right\rceil$$

So each transmission of the node has the transmission slot:

$$\Omega \left(\left\lceil \frac{t}{k_3 n a(n, k)} \right\rceil / t \right)$$

seconds. One channel support the transmission rate W/c bits/sec. Thus, each transmission is:

$$\Omega \left(\frac{W}{c \left\lceil \frac{t}{k_3 n a(n, k)} \right\rceil} \right)$$

bits/sec.

Moreover, each flow of MC-HB has at most two transmissions. Therefore, for each flow:

$$\lambda(n, k) = \Omega \left(\frac{W}{c \left\lceil \frac{t}{k_3 n a(n, k)} \right\rceil} \right)$$

bits/sec and the network capacity is:

$$n\lambda(n, k) = \Omega \left(\frac{nW}{c \left\lceil \frac{t}{k_3 n a(n, k)} \right\rceil} \right) \leq \Omega \left(\frac{nW}{k_3 n a(n, k) + c} \right)$$

bits/sec.

Furthermore, for each cell,:

$$a(n, k) = \max \left(\frac{100 \log(n)}{n}, \frac{cD(n)}{n} \right)$$

$$D(n) = \Theta \left(\frac{\log n}{\log \log n} \right)$$

according to lemma 2 in Kyasanur and Vaidya (2005), the results can be extended to the scene where each ad hoc node is equipped with m interfaces, then we get the following theorem:

Theorem 2: The lower bound of MC-HB is:

When c/m is $O(\log \log n)$, the network capacity is:

$$\Omega \left(\frac{nW}{\log n} \right)$$

bits/sec;

When c/m is $\Omega(\log \log n)$, the network capacity is:

$$\Omega \left(\frac{m \log \log n W}{c \log n} \right)$$

bits/sec.

The upper and lower bounds of MC-HB are the same which shows the capacity bounds are tight.

CONCLUSION

Network capacity is one of fundamental problems in wireless network. It reflects the scalability.

In the study, the multi-channel hybrid wireless network capacity with bottleneck constraint is dealt with. Vehicular ad hoc networks and many other networks belong to this kind. They can communicate with each other and also can exchange data with base stations. The previous studies suppose that each node to be the destination of one flow. In fact, due to randomly selection, many nodes may select the same node as their destination. Thus, the node may become the bottleneck of the network performance.

The theoretical analysis model is constructed; the upper and lower bounds of MC-HB are addresses. From the analysis process of lower bound, to improve the performance of this kind of network, the time and channel resources should be allocated according to the amount of traffic of every node which alleviates the bottleneck.

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