http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (15): 3412-3417, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.3412.3417

Mathematical Model Analysis of Intra-organisational Collaboration

[1]Anliang Ning, [2]Xiaojing Li and [3]Chunxian Wang
[1]Engineering Training Center,
[2]College of Electronic Engineering,
[3]College of Computer Sciences and Software, Tianjin Polytechnic University, Tianjin, China

Abstract: Collaboration means working together to achieve a common goal or to solve a problem. Grounded on complex network theory and collaborative design research, a mathematical model for analysing collaboration level in organisations is proposed. The concepts for characterising organisational structures for collaboration and indicators for assessing organisational behaviour were defined. The article concludes by discussing the limitations of the proposed model.

Key words: Complex network, social network analysis, collaboration design, organisational behaviour

INTRODUCTION

Collaboration, although not a new organisational characteristic, has become a critical factor that determines the success of businesses. It means working together in group(s) to achieve a common task or goal which is often beyond the capabilities of the participants involved in the collaboration. Collaboration studies in recent years has also been motivated by increasing informal interactions that causes companies to adopt flatter and flexible structures and the need to explore and integrate differences of team members and groups within an organisation (Zinnikus *et al*., 2013).

Complex networks in scientific research have proven to be useful paradigms/disciplines for delineating organisations. Complex network concepts have been used to analyse organisational characteristics such as hierarchies and decision making. Within complex network research, Social Network Analysis (SNA) is the main approach adopted by researchers to study and understand relationships, social roles and social structure in organisations (Kolberg *et al*., 2013). SNA is often associated with organisation theory and is used to identify clear patterns of relations and involvement (centralized and decentralized) based on gathered data such as the age, gender and race of actors. It makes use of techniques from sociology and mathematics for the representation and quantification of an organisation's information structure (Lavrac *et al*., 2007).

Rather studies within SNA have examined or proposed models that directly or indirectly influence the level of collaboration within an organisation. However, providing quantitative indicators for complex networks offers potentials for guiding researchers and industrial practitioners in monitoring the evolution of the organisational characteristics at intra-organisational (individual or group) and inter-organisational levels (Lavrac *et al*., 2007).

INTRA-ORGANISATIONAL COLLABORATION MATHEMATICAL MODEL CONCEPTUALIZATION

Organisation as a network: The mindset of 'an organisation as a network' is widely considered in research as a useful approach for promoting organisational flexibility and adaptability, particularly in the quality and sharing of information (Duque *et al*., 2013). It is for this reason that complex networks can offer useful insights into how people work together based on media choice (depending on the context and needs of information flow) and communication media that influence information sharing.

The indicators are derived as sums of existing SNA measures for clustering coefficient, closeness and degree centrality. These quantities were selected because they reflect interconnectedness within groups, individual connections for relationships and activity of individuals respectively(Tang *et al*., 2012).

The Degree Centrality (Dc_i) is a ratio of number of directly connected vertices to the number of possible vertices in a network and can be computed as:

$$Dc_i = \frac{[\deg]_i}{N-1}$$

where, N is the number of vertices in the network and $[\deg]_i$ is the number of vertices directly connected to i.

Corresponding Author: Anliang Ning, Engineering Training Center, Tianjin Polytechnic University, Tianjin, 300387, China

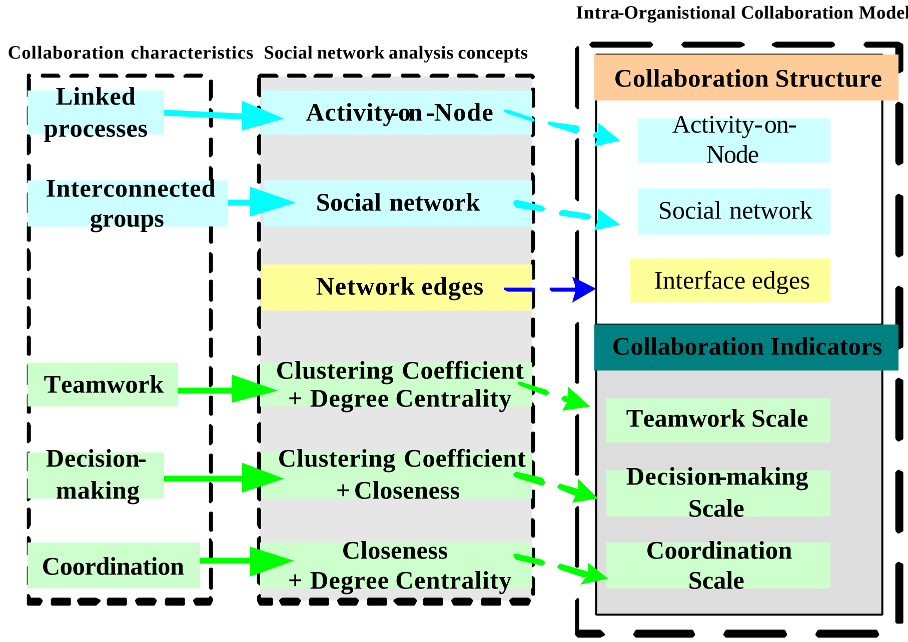

Fig. 1: Model conceptualisation

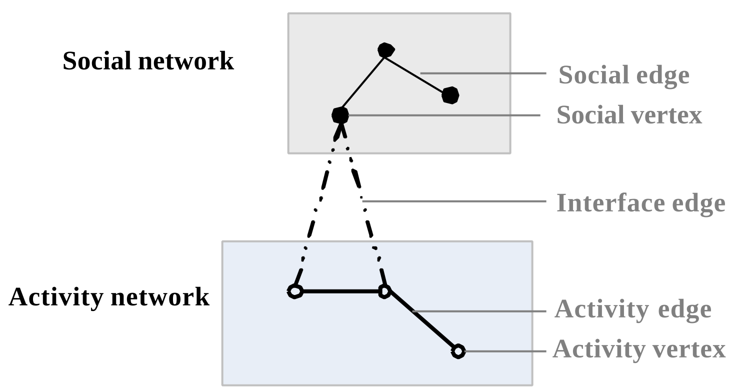

Fig. 2: IOC model as a hypergraph

The Clustering Coefficient (Cc_i) assesses the density between vertices and represents the tendency for vertices to cluster together. If a vertex i, connects to b_i neighbours and the number of possible edges between the vertices is given as $b_i (b_i-1)/2$, then Cc_i for i can be computed as:

$$Cc_i = \frac{2n_i}{b_i(b_i-1)}$$

where, n_i is the number of edges between b_i neighbours.

The closeness (c_{ij}) between vertices defines the order with which one vertex connects to another vertex. It is computed as the inverse of the geodesic distance (d_{ij}) between a pair of vertices I and j. d_{ij} is the number of edges along the shortest path between i and j. c_{ij} can be calculated as:

$$C_{ij} = \frac{1}{\sum_{i\neq j\in N} d_{ij}}$$

An intra-organisational collaboration model: Intra-organisational Collaboration (IOC), in this study, is modelled as a connected, partitioned, non-overlapping hypergraph G = (V, E) containing a graph for character-ising the collaborative social network of individuals/groups $G_s = (V_s, E_s)$ and a digraph for characterising the collaborative activity network of processes/tasks $G_p = (V_p, E_p)$, as shown in Fig. 2. V_s

represents social vertices of collaborating individuals, teams or organisations and V_p represents activity vertices for processes that are required to achieve a common goal that could not be achieved by the collaborating individ-uals. E_s and E_p correspond to edges between teams (or individuals) and processes.

For the proposed model, processes become part of a collaboration based on the set of interface edges T created by vertices within collaborators: T associates V_s with V_p.

Interface edges are connections between individuals/groups and tasks/processes for the exchange of resources. This interaction, related to formal work practise, can be enabled by edges (defined here as interface edges) for human-machine relationships. Each social vertex can be linked to as many as V_p activity vertices. The maximum number of possible interface edges in the model is given as $V_s \times V_p$ in which every social vertex is linked to every activity vertex. Consequently G is defined by $V = V_s \times V_p$ and $V_s \cap V_p = \emptyset$. Similarly, $E = E_s \cup E_p \cup T$ and $E_s \cap E_p \cap T = \emptyset$.

IOC information structure: social and activity networks: The main information for analysing collaboration were obtained as a combination of social vertices and edges for individuals/groups and activity vertices and edges for tasks/processes.

Topologies of the social network for collaboration, some possible configurations for the dictator, mutual and exclusive collaboration forms captured in (Reif *et al*., 2011) and were investigated and adopted to: (1) Illustrate the potential use of the model, (2) Simplify the model and (3) Align the model with existing collaborative design research.

The topologies of the activity network for collabor-ation were based on the Activity-on-node (AON), a traditional activity network employed in the widely used Project Evaluation and Review Technique and critical path method (Liptchinsky *et al*., 2013).

To conceptualise formal relationships that symbolise roles and responsibilities, the study have introduced a set of 'interface edges' which represent relationships that are associated with individuals, teams and organisations for involvement in linked processes that contribute to a common goal, for interfacing social vertices with activity vertices.

Collaboration Indicators: decision making, teamwork and coordination: Based on this derivative, a set of novel indicators for collaboration was proposed and for each indicator a constant is introduced to quantify the strength of network relationships and the availability of collaboration information. The introduced constants are as follows: Coordination constant (α_i), decision constant (β_i) and teamwork constant (γ_i). These constants are subjective probabilities that are based on the availability of a vertex i to: harmonise interactions (α_i), make choices (β_i) and pool resources (γ_i). The proposed collaboration indicators for a vertex i include: Decision-making scale (δ_i), coordination scale (χ_i) and teamwork scale (τ_i). These identified indicators are consistent with existing studies in complex network research where decision making measures have been introduced for agent-based systems and coordination quantities for edges between vertices have been investigated for hierarchical networks (Reif *et al*., 2011).

The proposed collaboration indicators are introduced because existing quantities identified in literature have been used in different contexts to those defined for decision making, teamwork and coordination. For instance, the coordination degree by (Tang *et al*., 2012) measures the ability of a vertex i to interchange information with another vertex j within a network and the coordination score by (Zinnikus *et al*., 2013) assesses the degree to which networks are concentrated around important vertices.

ANALYSIS AND EVALUATION OF COLLABORATION NETWORK

This section makes use of complex network concepts and properties to characterise the underlying topology, composed vertices and connected edges of the IOC model.

These formal relationships are defined by formal work practises for which tasks and events need to be defined particularly for process-intensive organisations (Liptchinsky *et al*., 2013) and information is usually stored in a more structured form. It is for this reason, that existing structures studied in SNA may not be enough to model collaboration. Nevertheless, the SNA is a flexible approach in which basic SNA concepts can be adapted by researchers to propose new attributes/indicators to characterise phenomena and systems. Consequently, for the model proposed in this study, the SNA approach has been augmented with adapted techniques from other domains and novel indicators for characterising collaboration.

Social collaboration evaluation: For f groups (each containing g social vertices) within the social network G_s, three different topologies for characterising IOC are proposed as shown in Fig. 3.

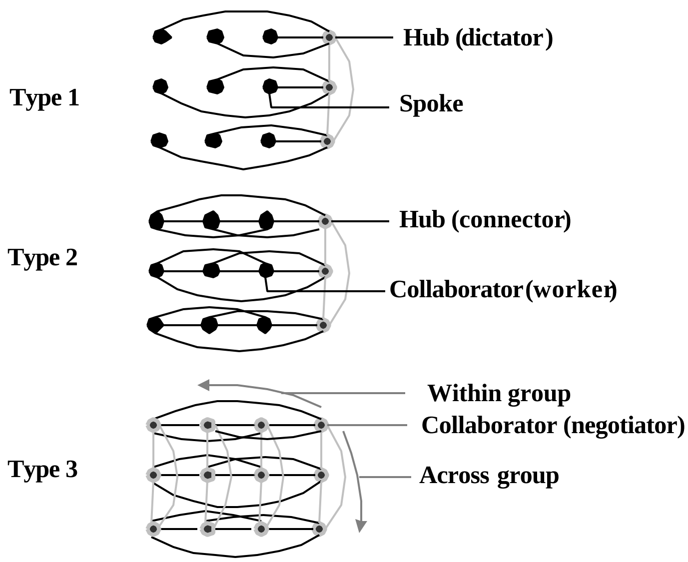

Fig. 3: Note how the caption is centered in the column

In all the forms of social network topologies proposed in the IOC model, the number of vertices within the social network (G_s) can be calculated as the sum of social vertices from each group i.e.:

$$|V_s| = \sum_{i=1}^{f} g_i$$

where $|V_s|$ is the cardinality of V_s, f is the number of groups involved in collaboration and g_i is the number of social vertices that form a group i.

Within G_s, as shown in Fig. 3, two forms of edges facilitate connections: Collaborative-and network-edges.

Collaborative-edges E'_s shown in Fig. 3 as gray coloured lines between vertices, are a subset of edges that form a sub-graph of the social network G'_s for enabling collaboration between groups. Within the type 1 and 2, f social vertices across teams (inter-team) acting as hubs can form f(f-1)/2 collaborative edges with each other. In the type 3, each social vertex exclusively collaborates (i.e., creates edges) across groups by establishing g×f(f-1)/2 edges based on factors such as common disciplines or pre-defined problems.

Network edges (E_s) on the other hand, are possible edges for the topologies shown in Fig. 3 and their cardinality $|E_s|$ are computed as follows:

Type 1:

$$\overbrace{\frac{f(f-1)}{2}}^{\text{hubs}} + \overbrace{f(g-1)}^{\text{spoke}}$$

Type 2:

$$\overbrace{\frac{f(f-1)}{2}}^{\text{hubs}} + \overbrace{\frac{fg(g-1)}{2}}^{\text{spoke}}$$

Type 3:

$$\overbrace{\frac{fg(g-1)}{2}}^{\text{within-groups}} + \overbrace{\frac{gf(f-1)}{2}}^{\text{across-groups}} = \frac{1}{2}[g^2 f - 2fg + f^2 g]$$

Activity network: The activity network G_p within the IOC model is derived from: serial topologies that impose precedence in dependencies for creating an additive chain of processes and parallel topologies that enforce multiple dependencies for concurrent processes. The parallel topology may involve multiple processes that are dependsocent on a single process (burst) or a single process that is dependent on multiple processes (merge).

For an activity network (G_p) contains I and J number of serial and parallel configurations for vertices, the number of vertices within G_p i.e., $|V_p|$ can be computed as:

$$|V_p| = \overbrace{\sum_{i\in I}^{I} s_i}^{\text{serial}} + \overbrace{\sum_{j\in J}^{J} p_j}^{\text{parallel}}$$

where s_i and p_j are the number of processes in each serial and parallel configuration respectively and $|V_p|$ is the cardinality of V_p. Suppose an intra-organisational collaboration is set up to carry out 4, 3, 5 and 2 processes with parallel dependencies and 9 serially dependent

Table 1: Evaluation of IOC model

Research modelling goals	IOC model focus	Implementations in the model
Inter-connected groups (C1)	Relationships between social actors in organisations adapted from SNA and interface edges to processes	1. Informal/formal relationships within the social network of human actors 2. Formal relationships, through interface edges, that symbolise roles and responsibilities during collaboration
Linked processes (C2)	AON from the widely used project evaluation and review technique and critical path method	Activity networks for serial and parallel configurations for sets of processes
Decision-making, Teamwork and Coordina-tion (C3 and C4)	Egocentric and sociocentric approaches for individual and group interactions	Decision scale: Measures the ease with which social vertices can make choices Teamwork scale: Measures the ease with which social vertices can pool resources Coordination scale: Measures the ease with which social vertices can harmonise interactions

processes and if the IOC makes use of 5 collaborating teams each containing 6 team members, then the number of vertices within the IOC will be 53, broken down as 5×6 = 30 social vertices for Gs and (4+3+5+2)+9 = 23 activity vertices for G_p.

If the activity network is made up of I serial, L parallel (burst) and M parallel (merge) then processes within G_p of the IOC are associated by E_p input and output edges in the formulation:

$$|E_p| = \overbrace{\sum_{i=1}^{I} a_i'}^{\text{serial}} + \overbrace{\sum_{l=1}^{L} b_l' + \sum_{m=1}^{M} c_m}^{\text{parallel}}$$

$|E_p|$ is the cardinality of E_p, a'_i and b'_l are inputs to I serial and L parallel (burst) sets of configured vertices and c_m is the output edge from M parallel (merge) sets of configured vertices where:

$$E'_p = a'_i \cup b'_l \cup c_m \text{ and } a'_i \cap b'_l \cap c_m = \varnothing$$

The maximum number of edges within G_p can be computed as $V_p(V_p-1)/2$. However, when L = 0 then the maximum number of edges within G_p can be simplified to $2V_p-2$ activity edges. Two edges are subtracted from the total number for terminal vertices – the start vertex that has no preceding vertices and the end vertex that has no following vertices.

Activity network collabortion indicators: In Table 1, the IOC model proposed in this study is evaluated based on the characteristics of collaboration identified in Section I, with regards to the information structure and behaviour for organisations. The table demonstrates coverage of the required characteristics for collaboration in organisations.

Within the IOC network (i.e., G_s and G_p), three collaboration indicators with values greater than or equal to zero and less than or equal to two are proposed.

The first indicator termed the 'teamwork scale' (τ_i) is introduced to assess the activity of a social vertex i and interconnectedness within a cluster for teamwork. To do this, the degree centrality and clustering coefficient of i are multiplied by a teamwork constant (γ_i) that is based on the availability and capability of i (i.e., the participant) to pool resources. The teamwork scale τ_i can be calculated as:

For a social vertex I:

$$\tau_i = \left[\overbrace{\frac{2E_s}{V_s'(V_s'-1)}}^{\text{clust\_coefficient}} + \overbrace{\frac{[\deg]_i}{V_s'-1}}^{\text{deg\_coefficient}} \right] \bullet \gamma_i$$

where $[degs]_i$ is the number of social vertices that are directly linked to i. For the overall IOC network, theaverage teamwork scale (τ) can be calculated as:

$$\tau = \frac{1}{V_s'} \sum_{i=1}^{V_s'} \tau_i$$

where, V'_s is a sub-graph consisting of social vertices at group, inter-group or organisational level.

The 'decision-making scale' (δ_i) is the second collaboration indicator introduced to assess the ease with which a social vertex i within the intra-organisational network can make decisions based on the interconnected-ness and connections for relationships. To do this, the clustering coefficient and closeness ofi in a defined sub-graph (group or overall organisation) of the collaboration social network are multiplied by a decision constant (β_i) that is dependent on the availability and capability of i to make choices. It is calculated as:

For a social vertex i:

$$\delta_i = \left[\overbrace{\frac{1}{\sum_{i \neq j \in V_s'} d_{ij}}}^{\text{closeness}} + \overbrace{\frac{2E_i}{V_s'(V_s'-1)}}^{\text{clust\_coefficient}} \right] \bullet \beta_i$$

where, d_{ij} is the distance between two vertices i and j, E_i is the number of edges created with directly connected vertices. The average decision-making scale (δ) for social vertices in the IOC network can then be computed as:

$$\delta = \frac{1}{V'_s}\sum_{i\in V'_s}^{V'_s}\delta_i$$

The third indicator, the 'coordination scale' (χ_i) assesses the connections and activity associated with which a social vertex i through which interactions can be harmonised. To do this, a coordination constant (α_i) that is dependent on the availability and capability of i for harmonising interactions, is multiplied by the sum of thecloseness and degree centrality of i towards the social and activity network. The activity network is included to take into account coordination theory that depicts dependencies as emerging from tasks [1]. The coordination scale χ_i can be calculated as:

For a social vertex i:

$$\chi_i = \left[\overbrace{\frac{1}{\frac{1}{2}\left(\sum_{i\neq j\in V'_s} d_{ij} + \sum_{i\neq k\in V'_p} d_{ik}\right)}}^{\text{closeness}} + \overbrace{\frac{1}{2}\left(\frac{[\deg]_i}{V'_s - 1} + \frac{[\deg]_{i(T)}}{V'_p - 1}\right)}^{\text{deg\_centrality}}\right] \bullet \alpha_i$$

where, V'_p is a sub-graph consisting of activity vertices and $[degs]_{i(T)}$ is the number of activity vertices that are directly linked to i through interface edges that constitute T. The average coordination scale (χ) for social vertices in the IOC network can then be computed as:

$$\chi = \frac{1}{V'_s}\sum_{i\in V'_s}^{V'_s}\chi_i$$

CONCLUSION

In this article, a sociological and technical (i.e., socio-technological) perspective has been applied to mathematically model an organisation as a network that collaborates to solve a problem or achieve a goal. Useful insights from the proposed 'intra-organisational collaboration model' in this study suggested that an organisation can be: Analysed as an amalgamation of social networks of human actors and activity networksof processes and assessed through indicators for teamwork-to tally the manner in which participants and groups pool resources to achieve a goal, purposely, or inadvertently, decision-making-to score the manner in which choices are made during collaborations through dictated decisions by a dictating entity, participatory decisions made by participating entities and democratic decisions based on collaborators who are individually responsible for decision making and coordination-to measure the ability of collaborators to harmonise interactions for maintaining and updating the flow of resources such as materials, funds and information.

Within the proposed IOC model, communication is enabled by social, activity and interface edges. For researchers and industrial practitioners, the presence of these different edges presents a wide range of communication roles for enabling human-to-human, human-to-process and process-to-process communications. Furthermore, initial or regular analysis of the information structure and behaviour for collaboration can be conducted to determine and review information flow factors such as group sizes, data storage roles and flow control policies. Also, as discussed in the study, the proposed model can serve as a benchmarking approach for improving the free flow and exchange of information within organisations.

REFERENCES

Duque, R., C. Bravo and M. Ortega, 2013. An ontological approach to automating collaboration and interaction analysis in groupware systems. Knowledge-Based Syst., 37: 211-229.

Kolberg, M., J.F. Buford, K.K. Dhara, X. Wu and V. Krishnaswamy, 2013. Feature interaction in a federated communications-enabled collaboration platform. Comput. Networks, 57: 2410-2428.

Lavrac, N., P. Ljubic, T. Urbancic, G. Papa, M. Jermol and S. Bollhalter, 2007. Trust modeling for networked organizations using reputation and collaboration estimates. IEEE Trans. Syst. Man Cybernetics Part C Applic. Rev., 37: 429-439.

Liptchinsky, V., R. Khazankin, S. Schulte, B. Satzger, H.L. Truong and S. Dustdar, 2013. On modeling context-aware social collaboration processes. Informat. Syst. (In Press). 10.1016/j.is.2013.05.007

Reif, J.H., S. Sahu and P. Yin, 2011. Complexity of graph self-assembly in accretive systems and self-destructible systems. Theoret. Comput. Sci., 412: 1592-1605.

Tang, J., B. Jiang, C.C. Chang and B. Luo, 2012. Graph structure analysis based on complex network. Digital Signal Proc., 22: 713-725.

Zinnikus, I., X. Cao and K. Fischer, 2013. Agent-supported collaboration and interoperability for networked enterprises: Modeling interactions and service compositions. Comput. Ind., 64: 938-950.