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Design of Exponential Stability Non-fragile Control of Discrete Delay Large-scale Systems

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Abstract: The problem of exponential stability non-fragile control for a class of discrete large-scale systems with delays is considered in this paper. Based on Lyapunov stability theorem and linear matrix inequality approach, a new approach is given to design the state feedback non-fragile controller. By introducing a new Lyapunov functional, a exponential stability condition is obtained in terms of linear matrix inequalities. The simulation is easier with non-fragile controller. The solutions of linear matrix inequality can be easily obtained by using linear matrix inequality Control Toolbox in MATLAB. Finally, a numerical example is given to demonstrate the validity of the results.

Key words: Large-scale systems, exponential stable, non-fragile control

INTRODUCTION

Time delay is frequently encountered in various engineering, communication and biological systems (Kau *et al*., 2005). The characteristics of dynamic systems are significantly affected by the presence of time delays, even to the extent of instability in extreme situations. Therefore, the study of delay systems has received much attention and various analysis and synthesis methods have been developed over the past years (Shyu *et al*., 2005).

Recently, the problems of decentralized stabilization for large-scale systems with delays have been studied (Krishnamurthy and Khorrami, 2003; Wu, 2002). Keel and Bhattacharyya (1997) considered the problems of the design non-fragile controller. But on the non-fragile control for discrete large-scale with delays, a few results have been present (Park, 2004).

This study presents the problem of exponential stability non-fragile control for discrete large-scale systems with delays. Based on Lyapunov stability theorem, a sufficient condition is given in terms of linear matrix inequality.

PROBLEM FORMULATION

Consider the following discrete large-scale systems with delays composed of N interconnected subsystems described by:

$$s_i : x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{j=1}^{N} A_{ij} x_j(k - h_{ij}) \tag{1}$$

$$x_i(k) = y_i(k) \qquad -h \pounds k \pounds 0,\ i = 1, 2, L, N$$

where, $x_i(k) \in R^{ni}$ are state vectors. $u_i(k) \in R^{ni}$ are control vectors. h_{ij} are positive integers representing the delays of systems. $\iota_i(k)$ are the given initial states on [-h, 0]. A_i, B_i and A_{ij} are real constant matrices with appropriate dimensions:

$$x(k) = [x_1^T(k), x_2^T(k), \cdots, x_N^T(k)]^T,\ h = \max_{i,j=1,2,\cdots,N}\{h_{ij}\}$$

$$\| \psi_i \|_h = \max_{-h \le k \le 0}\{\| \psi_i(k) \|\}$$

For the systems Eq. 1, we will design the decentralized non-fragile controller such as (Xu *et al*., 2004):

$$u_i(k) = (K_i + \Delta K_i) x_i(k) \quad I = 1, 2, \ldots N \tag{2}$$

where, $K_i \in R^{miHni}$ are constant matrices, $\Delta K_i \in R^{miHni}$ are unknown matrices representing time-varying parameter uncertainties satisfying:

$$\Delta k_i = D_i \varphi_i(k) G_i K_i$$

where, D_i, G_i are known constant matrices, $\varphi_i(k)$ are unknown matrices satisfying:

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$$\varphi^T_i(k)\varphi_i(k)\le I$$

Substituting controller Eq. 2 into systems 1 yields the closed-loop systems:

$$\begin{aligned} x_i(k+1) &= [A_i + B_i(I + D_i\varphi_i(k)G_i)K_i]x_i(k) \\ &+\sum_{j=1}^{N} A_{ij}x_j(k-h_{ij}) \\ &= \bar{A}_i x_i(k) + \sum_{j=1}^{N} A_{ij}x_j(k-h_{ij}) \end{aligned} \tag{3}$$

Where:

$$\bar{A}_i = A_i + B_i(I + D_i\varphi_i(k)G_i)K_i$$

MAIN RESULTS

Defintion 1: Hsien and Lee (1995) Large-scale systems Eq. 1 is exponentially stable, if there exist constants $0\ge\alpha<1$ and $c\ge 1$ such that:

$$\|x(k)\|\le c\|\iota\|h\ \alpha k\ \ k\ge 0$$

where, α is called exponential stable degree.

Lemma 1: Kwon and Park (2004). The LMI:

$$\begin{bmatrix} Y(x) & W(x) \\ W^T(x) & R(x) \end{bmatrix} > 0$$

is equivalent to:

$$R(x)>0,\ Y(x)\text{-}W(x)RG^{1}(x)WT\ (x)>0$$

where, $Y^T(x) = Y^T(x)$, $R(x) = R^T(x)$ depend on x.

Lemma 2: Yang *et al.* (2000) for known constant $\epsilon>0$ and matrices D, E, F which satisfying $F^TF\le I$, then the following matrix inequality is hold:

$$DEF+E^TF^TD^T\le \epsilon DD^T+\epsilon G^{1}E^TE$$

Theorem 1: For any given constant α $(0\le\alpha<1)$, if there exist matrices $K\in R^{miHni}$, n_iHn_i positive-definite matrices P_t, $R_i\in R^{miHni}$, such that the following linear matrix inequality holds:

$$\begin{bmatrix} J_{i0} & \bar{A}_i^T P_i A_{i1} & \bar{A}_i^T P_i A_{i2} & \cdots & \bar{A}_i^T P_i A_{iN} \\ A_{i1}^T P_i \bar{A}_i & J_{i1} & A_{i1}^T P_i A_{i2} & \cdots & \bar{A}_{i1}^T P_i A_{iN} \\ A_{i2}^T P_i \bar{A}_i & A_{i2}^T P_i A_{i1} & J_{i2} & \cdots & \bar{A}_{i2}^T P_i A_{iN} \\ \vdots & \vdots & \vdots & \ddots & M \\ A_{iN}^T P_i \bar{A}_i & A_{iN}^T P_i A_{i1} & A_{iN}^T P_i A_{i2} & \cdots & J_{iN} \end{bmatrix} < 0 \tag{4}$$

Where:

$$\begin{aligned} J_{i0} &= \bar{A}_i^T P_i \bar{A}_i - \alpha^2 P_i + \delta_i R_i \\ J_{in} &= A_{in}^T P_i A_{in} - \delta(A_{in})R_n \alpha^{2h_{in}} \qquad (n = 1, 2, \cdots, N) \end{aligned}$$

With non-fragile controller Eq. 2, the discrete time large-scale systems Eq. 1 with delays is exponentially stable, where, $\delta(.)$ is a real function satisfying:

$$\delta(0) = 0;\ \forall E\ne 0, \delta(E) = 1;\ \delta_i = \sum_{j=1}^{N}\delta(A_{ij})$$

Proof: Selecting a Lyapunov functional such as:

$$\begin{aligned} V(k) &= \sum_{i=1}^{N}[x_i^T(k)P_i x_i(k) \\ &+ \sum_{j=1}^{N}\sum_{m=1}^{h_{ij}} x_j^T(k-m)R_j\delta(A_{ij})\alpha^{2(m-1)}x_j(k-m)] \end{aligned} \tag{5}$$

where, $\alpha(0\le\alpha<1)$ is a constant, P_i, R_i are positive-definite matrices of theorem1.

Along the solution of systems Eq. 1, the forward difference of V(k) is obtainedw:

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= \sum_{i=1}^{N}\{[x_i^T(k+1)P_i x_i(k+1) - x_i^T(k)P_i x_i(k)] \\ &+ \sum_{j=1}^{N}\sum_{m=1}^{h_{ij}} x_j^T(k+1-m)R_j\delta(A_{ij})\alpha^{2(m-1)}x_j(k+1-m) \\ &- \sum_{j=1}^{N}\sum_{m=1}^{h_{ij}} x_j^T(k-m)R_j\delta(A_{ij})\alpha^{2(m-1)}x_j(k-m)\} \\ &= \sum_{i=1}^{N}\{[x_i^T(k+1)P_i x_i(k+1) - x_i^T(k)P_i x_i(k)] \\ &+ \sum_{j=1}^{N}\sum_{l=0}^{h_{ij}-1} x_j^T(k-l)R_j\delta(A_{ij})\alpha^{2l}x_j(k-l) \\ &- \sum_{j=1}^{N}\sum_{l=1}^{h_{ij}} x_j^T(k-l)R_j\delta(A_{ij})\alpha^{2(l-1)}x_j(k-l)\} \\ &= \sum_{i=1}^{N}\{[x_i^T(k+1)P_i x_i(k+1) - x_i^T(k)P_i x_i(k)] \\ &+ \sum_{j=1}^{N}[x_j^T(k)R_j\delta(A_{ij})x_j(k) \\ &- x_j^T(k-h_{ij})R_j\delta(A_{ij})\alpha^{2(h_{ij}-1)}x_j(k-h_{ij}) \\ &+ \sum_{l=1}^{h_{ij}-1}(1-\alpha^{-2})x_j^T(k-l)R_j\delta(A_{ij})\alpha^{2l}x_j(k-l)]\} \\ &= \sum_{i=1}^{N}\{[x_i^T(k+1)P_i x_i(k+1) - x_i^T(k)P_i x_i(k)] \\ &+ \sum_{j=1}^{N}[x_j^T(k)R_j\delta(A_{ij})x_j(k) \\ &- x_j^T(k-h_{ij})R_j\delta(A_{ij})\alpha^{2(h_{ij}-1)}x_j(k-h_{ij}) \end{aligned}$$

$$+\sum_{m=2}^{h_{ij}}(1-\alpha^{-2})x_j^T(k+1-m)R_j\delta(A_{ij})$$
$$\bullet\,\alpha^{2(m-1)}x_j(k+1-m)]$$
$$+\sum_{j=1}^{N}(1-\alpha^{-2})x_j^T(k)R_j(A_{ij})x_j(k)$$
$$-\sum_{j=1}^{N}(1-\alpha^{-2})x_j^T(k)R_j(A_{ij})x_j(k)$$
$$+(1-\alpha^{-2})x_i^T(k+1)P_ix_i(k+1)$$
$$-(1-\alpha^{-2})x_i^T(k+1)P_ix_i(k+1)\}$$
$$=(1-\alpha^{-2})V(k+1)+\sum_{i=1}^{N}\{\alpha^{-2}x_i^T(k+1)P_ix_i(k+1)$$
$$-x_i^T(k)P_ix_i(k)+\sum_{j=1}^{N}\alpha^{-2}x_j^T(k)R_j\delta(A_{ij})x_j(k)$$
$$-\sum_{j=1}^{N}x_j^T(k-h_{ij})R_j\delta(A_{ij})\alpha^{2(h_{ij}-1)}x_j(k-h_{ij})\}$$
$$=(1-\alpha^{-2})V(k+1)+\alpha^{-2}\sum_{i=1}^{N}\Omega_i$$

Where:

$$\Omega_i=x_i^T(k+1)P_ix_i(k+1)-\alpha^2x_i^T(k)P_ix_i(k)$$
$$+\sum_{j=1}^{N}x_j^T(k)\delta(A_{ij})R_jx_j(k)$$
$$-\sum_{j=1}^{N}x_j^T(k-h_{ij})\delta(A_{ij})R_j\alpha^{2h_{ij}}x_j(k-h_{ij})$$
$$=x_i^T(k)\overline{A_i}^TP_i\overline{A_i}x_i(k)+2\sum_{j=1}^{N}x_i^T(k)\overline{A_i}^TP_iA_{ij}x_j(k-h_{ij})$$
$$+\sum_{j=1}^{N}\sum_{l=1}^{N}x_j^T(k-h_{ij})\overline{A_{ij}}^TP_iA_{il}x_l(k-h_{il})$$
$$-\alpha^2x_i^T(k)P_ix_i(k)+\sum_{j=1}^{N}x_j^T(k)\delta(A_{ij})R_jx_j(k)$$
$$-\sum_{j=1}^{N}x_j^T(k-h_{ij})\delta(A_{ij})R_j\alpha^{2h_{ij}}x_j(k-h_{ij})$$

Therefore:

$$\Delta V(k)=(1-\alpha^{-2})V(k+1)+\alpha^{-2}\sum_{i=1}^{N}\Omega_i$$
$$=(1-\alpha^{-2})V(k+1)+\alpha^{-2}\sum_{i=1}^{N}\{x_i^T(k)[\overline{A_i}^TP_i\overline{A_i}$$
$$-\alpha^2P_i+\delta_iR_i]x_i(k)$$
$$-\sum_{j=1}^{N}x_j^T(k-h_{ij})\delta(A_{ij})R_j\alpha^{2h_{ij}}x_j(k-h_{ij})$$
$$+\sum_{j=1}^{N}2x_i^T(k)\overline{A_i}^TP_iA_{ij}x_j(k-h_{ij})$$
$$+\sum_{j=1}^{N}\sum_{l=1}^{N}x_j^T(k-h_{ij})A_{ij}^TP_iA_{il}x_l(k-h_{il})\}$$

$$=(1-\alpha^{-2})V(k+1)+\alpha^{-2}\sum_{i=1}^{N}\begin{bmatrix}x_i(k)\\x_1(k-h_{i1})\\x_2(k-h_{i2})\\\vdots\\x_N(k-h_{iN})\end{bmatrix}^T$$

$$\times\begin{bmatrix}J_{i0} & \overline{A_i}^TP_iA_{i1} & \overline{A_i}^TP_iA_{i2} & L & \overline{A_i}^TP_iA_{iN}\\A_{i1}^TP_i\overline{A_i} & J_{i1} & \overline{A_{i1}}^TP_iA_{i2} & L & \overline{A_{i1}}^TP_iA_{iN}\\A_{i2}^TP_i\overline{A_i} & A_{i2}^TP_iA_{i1} & J_{i2} & L & A_{i2}^TP_iA_{iN}\\M & M & M & 0 & M\\A_{iN}^TP_i\overline{A_i} & A_{iN}^TP_iA_{i1} & A_{iN}^TP_iA_{i2} & L & J_{iN}\end{bmatrix}$$

$$\times\begin{bmatrix}x_i(k)\\x_1(k-h_{i1})\\x_2(k-h_{i2})\\\vdots\\x_N(k-h_{iN})\end{bmatrix}$$

Where:

$$J_{i0}=\overline{A_i}^TP_i\overline{A_i}-\alpha^2P_i+\delta_iR_i$$
$$J_{in}=A_{in}^TP_iA_{in}-\delta(A_{in})R_n\alpha^{2h_{in}}\quad(n=1,2,\cdots,N)$$

With Eq. 4, we obtain:

$$\Delta V(k)<(1-\alpha G^2)V(k+1)$$

i.e:

$$V(k+1)<(\alpha G^2V(k)$$

Therefore:

$$V(k)<\alpha^2V(k-1)<\alpha^2\alpha^2V(k-2)<\ldots<\alpha^{2k}V(0)\tag{6}$$

With Eq. 5, we know:

$$V(0)\le[\overline{\lambda}_PN+\overline{\lambda}_RhN]\|\psi\|_h^2\tag{7}$$

$$V,k,\ge\underline{\lambda}_PN\|x(k)\|^2\tag{8}$$

Where:

$$\overline{\lambda}_P=\max_{i=1,2,\cdots,N}\{\lambda_{max}(P_i)\},\overline{\lambda}_R=\max_{i=1,2,\cdots,N}\{\lambda_{max}(R_i)\},$$
$$\underline{\lambda}_P=\min_{i=1,2,\cdots,N}\{\lambda_{min}(P_i)\}$$

With Eq. 6-8, we obtain:

$$\|x(k)\|<\sqrt{\frac{\overline{\lambda}_P+\overline{\lambda}_Rh}{\underline{\lambda}_P}}\|\psi\|_h\alpha^k\tag{9}$$

With definition1 and the inequality Eq. 9, we know that discrete large-scale systems Eq. 1 are exponentially stable.

Theorem 2: For any given constant $\alpha(0\leq\alpha<1)$, if there exist constant $\epsilon>0$, matrices $Y_i \in R^{miHni}$, positive-definite matrices Q_i, $T_0 \in R^{miHni}$ such that the following linear matrix inequality holds:

$$\begin{bmatrix} -Q_i+\varepsilon B_i D_i D_i^T B_i^T & A_iQ_i+B_iY_i & A_{i1}Q_1 & \cdots & A_{iN}Q_N & 0 \\ (A_iQ_i+B_iY_i)^T & -\alpha^2 Q_i+\delta_i T_i & 0 & \cdots & 0 & (G_iY_i)^T \\ Q_1^T A_{i1}^T & 0 & \Sigma_{i1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & & 0 \\ Q_N^T A_{iN}^T & 0 & 0 & \cdots & \Sigma_{iN} & 0 \\ 0 & G_iY & 0 & \cdots & & -\varepsilon I \end{bmatrix} < 0 \quad (10)$$

Where:

$$\Sigma_{in} = -\delta(A_{in})T_n\alpha^{2h_{in}} \qquad (i, n = 1, 2, \cdots, N)$$

With non-fragile controller:

$$u_i(k) = (I + D_i\varphi_i(k)G_i)Y_iQ_i^{-1}x_i(k)$$

The large-scale systems Eq. 1 is exponentially stable.

Proof: The matrix inequality Eq. 4 can be rewritten:

$$\begin{bmatrix} -\alpha^2 P_i+\delta_i R_i & 0 & \cdots & 0 \\ 0 & -\delta(A_{i1})R_1\alpha^{2h_{i1}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\delta(A_{iN})R_N\alpha^{2h_{iN}} \end{bmatrix} + \begin{bmatrix} \overline{A}_i^T \\ A_{i1}^T \\ M \\ A_{iN}^T \end{bmatrix} \times P_i \begin{bmatrix} \overline{A}_i & A_{i1} & L & A_{iN} \end{bmatrix} < 0$$

With lemma 1, the above inequality is equivalent to:

$$\begin{bmatrix} -P_i^{-1} & \overline{A}_i & A_{i1} & \cdots & A_{iN} \\ \overline{A}_i^T & -\alpha^2 P_i+\delta_i R_i & 0 & \cdots & 0 \\ A_{i1}^T & 0 & -\delta(A_{i1})R_1\alpha^{2h_{i1}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{iN}^T & 0 & 0 & \cdots & -\delta(A_{iN})R_N\alpha^{2h_{iN}} \end{bmatrix} < 0 \quad (11)$$

Where:

$$\overline{A}_i = A_i + B_i(I + D_i\varphi_i(k)G_i)K_i$$

Pre-and Post-multiplying the inequality Eq. 11 by:

$$\{I, P_i^{-1}, P_1^{-1}, P_2^{-1}, L, P_N^{-1}\}$$

and giving some transformations:

$$Q_i = P_i^{-1},\ Y_i = K_iQ_i,\ T_i = P_i^{-1}R_iP_i^{-1}$$

We obtain:

$$\begin{bmatrix} -Q_i & \Psi_{i0} & \Psi_{i1} & \cdots & \Psi_{iN} \\ \Psi_{i0}^T & -\alpha^2 Q_i+\delta_i T_i & 0 & \cdots & 0 \\ \Psi_{i1}^T & 0 & -\delta(A_{i1})T_1\alpha^{2h_{i1}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Psi_{iN}^T & 0 & 0 & \cdots & -\delta(A_{iN})T_N\alpha^{2h_{iN}} \end{bmatrix} < 0$$

Where:

$$\Psi_{i0} = A_iQ_i + B_iY_i + B_iD_i\varphi_i(k)G_iY_i$$
$$\Psi_{in} = A_{in}Q_n \qquad (n = 1, 2, \cdots, N)$$

With theorem2, for any $\epsilon>0$, we have:

$$\begin{bmatrix} \Pi_i & A_iQ_i+B_iY_i & A_{i1}Q_1 & L & A_{iN}Q_N \\ (A_iQ_i+B_iY_i)^T & \Pi_{i0} & 0 & L & 0 \\ (A_{i1}Q_1)^T & 0 & \Pi_{i1} & L & 0 \\ M & M & M & 0 & M \\ (A_{iN}Q_N)^T & 0 & 0 & L & \Pi_{iN} \end{bmatrix} < 0 \quad (12)$$

Where:

$$\Pi_i = -Q_i + \varepsilon B_iD_iD_i^TB_i^T$$
$$\Pi_{i0} = -\alpha^2 Q_i + \delta_i T_i + \tfrac{1}{\varepsilon} Y_i^T G_i^T G_i Y_i$$
$$\Pi_{in} = -\delta(A_{in})T_n\alpha^{2h_{in}} \qquad (n = 1, 2, \cdots, N)$$

With lemma1, it is easy to know that the inequality Eq. 12 is equivalent to Eq. 10.

CONCLUSION

A new approach of design non-fragile controller for discrete large-scale is given in this study. By introducing a new Lyapunov functional, a sufficient condition is obtained in terms of linear matrix inequality.

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