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CMM Measurement Error Model Based on High-order Lagrange Interpolation

Zhang Mei
College of Electrical Engineering and Automation,
Anhui University, Hefei, 230601, China

Abstract: Most scholars were committed to study the low-dimensional measurement error of CMM, few to research the high-dimensional model. In this study, the idea of dimension reduction is found from the derivational process of Lagrange interpolation formula, which is from one-dimension to two-dimension. High-dimension and high orders Lagrange interpolation formula is analogized by using the rule. Then apply the formula to the fields of the CMM measurement error models and get the better results.

Key words: CMM, high-dimensional model, lagrange interpolation

INTRODUCTION

When it comes to the modeling process of the error of CMM, most research are limited to the models with one or two dimensions. Considering the fact that the high dimensional models like three or higher are pretty sophisticated, few scholars are committed to them. Fei *et al*. (2004) have employed the one dimensional polynomial to fit the linear error of one certain dimension on the CMM which only one factor was taken into consideration. While Yang *et al*. (2010) used the Neutral Networks and SVM methods predicted certain plan of the CMM which considered two influences' effects. Wei and Chen (2011) focused on the deformation of the crossbeam. However, the error of CMM is very complex which is the results of multi-dimension variables' interactions, actually the low dimensional model can hardly indicates the characteristics of the CMM's error.

Fitting methods are the basics of low dimensional modeling and we are familiar with them such as one or two dimensional functions' interpolation.

How to put the multiple variables' high dimensional interpolation into practice is a problem in author's mind. According to the literatures available now which have referred the terms of "interpolation of high dimensional function", but most of them are just about the two dimensional interpolation ending up with the words of "the rest may be deduced by analogy".

Han (2005) said "It has been widely used in practical scenarios for the reason that the Lagrange Fitting has no requirement of the uniformity of the crunodes and has the advantage of giving out the apparent form of certain polynomials which is very suitable for the modeling process based on uneven data." In this study the rule of the deduction of the Lagrange Fitting process from one dimension to two dimensions-the idea of the dimension reduction was found out and deduced the high-dimensional Lagrange interpolation formulas demonstrated by the data of CMM error data which obtained favorable modeling result.

HIGH DIMENSIONAL MULTI-ORDER LAGRANGE INTERPOLATION

For general multi-variables condition, there have been theories demonstrated the existence of polynomial interpolation in study. Mo and Liu (2003) has discussed 1-D, 2-D functions' interpolation methods. In this study, the 1-D function's high order Lagrange interpolation is cited without its deduction process. In the following contents, the deduction process of Lagrange interpolation form 1-D to 2-D will be analyzed in detail to help find rules. As for the order of interpolation, we begin with a simple scenario: all the variables' interpolation orders are second, which will be generalized later.

The formula of 1-D function's Lagrange second order interpolation: Assumes that the interpolation spots are x_{k-1}, x_k, x_{k+1}, where the order n is 2, by Shi (2004) "basic function method" we obtain such Eq:

$$P_2(x) = y_{k-1}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})} + y_k\frac{(x-x_{k-1})(x-x_{k+1})}{(x_k-x_{k-1})(x_k-x_{k+1})} + y_{k+1}\frac{(x-x_{k-1})(x-x_k)}{(x_{k+1}-x_{k-1})(x_{k+1}-x_k)} \quad (1)$$

Generalize to obtain the 1-D interpolating formula (for x, mth interpolating order): Generalized to the 1-D high order interpolation Eq:

$$P_m(x) = \sum_{i=s}^{s+m} \prod_{\substack{k=s \\ k \neq i}}^{s+m} \frac{x - x_k}{x_i - x_k} y_i \quad (m, i, s, k \in R)$$

Lagrange multi-order interpolating Eq. of 2-D function z = f(x, y): Let z be the 2-D function where x and y are independent variables. Even though z is composed of x's and y's low order polynomials, the computation will be complex. The 1-D function's interpolation will be employed if the dimension can be reduced from 2 to 1, which is to say, let y be a constant. The deduction process of 2-D second order interpolation will be recalled as follows:

- **Step 1:** Let y be a constant (for the second order interpolation, let the values of y are y_{k-1}, y_k, y_{k+1}), for the given values of x (x_{k-1}, x_k, x_{k+1}), we obtain the one variable interpolation polynomials about x:

$$\text{For } y = y_{k-1},\ z_{(k-1,k-1)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k,k-1)} \frac{(x - x_{k-1})(x - x_{k+1})}{(x_k - x_{k-1})(x_k - x_{k+1})} + z_{(k+1,k-1)} \frac{(x - x_{k-1})(x - x_k)}{(x_{k+1} - x_{k-1})(x_{k+1} - x_k)} \tag{2}$$

$$\text{For } y = y_k,\ z_{(k-1,k)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k,k)} \frac{(x - x_{k-1})(x - x_{k+1})}{(x_k - x_{k-1})(x_k - x_{k+1})} + z_{(k+1,k)} \frac{(x - x_{k-1})(x - x_k)}{(x_{k+1} - x_{k-1})(x_{k+1} - x_k)} \tag{3}$$

$$\text{For } y = y_{k+1},\ z_{(k-1,k+1)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k,k+1)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k+1,k+1)} \frac{(x - x_k)(x - x_{k-1})}{(x_{k+1} - x_k)(x_{k+1} - x_{k-1})} \tag{4}$$

- **Step 2:** We obtain the Lagrange interpolation formula of y according to Eq. 1:

$$P_2(y) = g_{k-1} \frac{(y - y_k)(y - y_{k+1})}{(y_{k-1} - y_k)(y_{k-1} - y_{k+1})} + g_k \frac{(y - y_{k-1})(y - y_{k+1})}{(y_k - y_{k-1})(y_k - y_{k+1})} + g_{k+1} \frac{(y - y_{k-1})(y - y_k)}{(y_{k+1} - y_{k-1})(y_{k+1} - y_k)} \tag{5}$$

- **Step 3:** we can obtain the 2-D Lagrange interpolating polynomials by combining Eq. 5 with 2 and Eq. 3 and 4 where coefficients in 5 should correspond those in 2-4:

$$\begin{aligned}
&\frac{(y - y_k)(y - y_{k+1})}{(y_{k-1} - y_k)(y_{k-1} - y_{k+1})} \Bigg[z_{(k-1,k-1)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k,k-1)} \frac{(x - x_{k-1})(x - x_{k+1})}{(x_k - x_{k-1})(x_k - x_{k+1})} + z_{(k+1,k-1)} \frac{(x - x_{k-1})(x - x_k)}{(x_{k+1} - x_{k-1})(x_{k+1} - x_k)} \Bigg] \\
&+ \frac{(y - y_{k-1})(y - y_{k+1})}{(y_k - y_{k-1})(y_k - y_{k+1})} \Bigg[z_{(k-1,k)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k,k)} \frac{(x - x_{k-1})(x - x_{k+1})}{(x_k - x_{k-1})(x_k - x_{k+1})} + z_{(k+1,k)} \frac{(x - x_{k-1})(x - x_k)}{(x_{k+1} - x_{k-1})(x_{k+1} - x_k)} \Bigg] \\
&+ \frac{(y - y_{k-1})(y - y_k)}{(y_{k+1} - y_{k-1})(y_{k+1} - y_k)} \Bigg[z_{(k-1,k+1)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k,k+1)} \frac{(x - x_k)(x - x_{k+1})}{(x_{k-1} - x_k)(x_{k-1} - x_{k+1})} + z_{(k+1,k+1)} \frac{(x - x_k)(x - x_{k-1})}{(x_{k+1} - x_k)(x_{k+1} - x_{k-1})} \Bigg]
\end{aligned}$$

Then, generalize it to obtain 2-D multi-order interpolating formulas (for x, mth order; for y, nth order):

$$P_{m,n}(x, y) = \sum_{j=t}^{t+n} \sum_{i=s}^{s+m} \prod_{\substack{l=t \\ l \neq j}}^{t+n} \frac{y - y_l}{y_j - y_l} \prod_{\substack{k=s \\ k \neq i}}^{s+m} \frac{x - x_k}{x_i - x_k} z_{ij}$$
$$(m, n, i, j, s, t, l, k \in R)$$

Lagrange interpolating formulas of 3-D functione = (f, x, z): For the 3-D function the interpolation formulas can be deduced in analogy: First let two variables be constants, obtain a series of Lagrange interpolation polynomials of the third variable; then change one of the two constant into variable which will be interpolated which can result to a series of polynomials; lastly, combine the last one's Lagrange polynomials with the two above variables' interpolation polynomials to obtain the 3-D Lagrange polynomials.

According to the 2-D second order interpolation formulas, the 3-D third order interpolation formulas can be obtained directly:

$$
\begin{aligned}
&\frac{(z-z_k)(z-z_{k+1})}{(z_{k-1}-z_k)(z_{k-1}-z_{k+1})}\left\{
\begin{aligned}
&\frac{(y-y_k)(y-y_{k+1})}{(y_{k-1}-y_k)(y_{k-1}-y_{k+1})}\left[e_{(k-1,k-1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k-1,k-1)}\frac{(x-x_{k-1})(x-x_{k+1})}{(x_k-x_{k-1})(x_k-x_{k+1})}+e_{(k+1,k-1,k-1)}\frac{(x-x_{k-1})(x-x_k)}{(x_{k+1}-x_{k-1})(x_{k+1}-x_k)}\right]\\
&+\frac{(y-y_{k-1})(y-y_{k+1})}{(y_k-y_{k-1})(y_k-y_{k+1})}\left[e_{(k-1,k,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k,k-1)}\frac{(x-x_{k-1})(x-x_{k+1})}{(x_k-x_{k-1})(x_k-x_{k+1})}+e_{(k+1,k,k-1)}\frac{(x-x_{k-1})(x-x_k)}{(x_{k+1}-x_{k-1})(x_{k+1}-x_k)}\right]\\
&+\frac{(y-y_{k-1})(y-y_k)}{(y_{k+1}-y_{k-1})(y_{k+1}-y_k)}\left[e_{(k-1,k+1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k+1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k+1,k+1,k-1)}\frac{(x-x_k)(x-x_{k-1})}{(x_{k+1}-x_k)(x_{k+1}-x_{k-1})}\right]
\end{aligned}\right\}\\
&+\frac{(z-z_{k-1})(z-z_{k+1})}{(z_k-z_{k-1})(z_k-z_{k+1})}\left\{
\begin{aligned}
&\frac{(y-y_k)(y-y_{k+1})}{(y_{k-1}-y_k)(y_{k-1}-y_{k+1})}\left[e_{(k-1,k-1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k-1,k-1)}\frac{(x-x_{k-1})(x-x_{k+1})}{(x_k-x_{k-1})(x_k-x_{k+1})}+e_{(k+1,k-1,k-1)}\frac{(x-x_{k-1})(x-x_k)}{(x_{k+1}-x_{k-1})(x_{k+1}-x_k)}\right]\\
&+\frac{(y-y_{k-1})(y-y_{k+1})}{(y_k-y_{k-1})(y_k-y_{k+1})}\left[e_{(k-1,k,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k,k-1)}\frac{(x-x_{k-1})(x-x_{k+1})}{(x_k-x_{k-1})(x_k-x_{k+1})}+e_{(k+1,k,k-1)}\frac{(x-x_{k-1})(x-x_k)}{(x_{k+1}-x_{k-1})(x_{k+1}-x_k)}\right]\\
&+\frac{(y-y_{k-1})(y-y_k)}{(y_{k+1}-y_{k-1})(y_{k+1}-y_k)}\left[e_{(k-1,k+1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k+1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k+1,k+1,k-1)}\frac{(x-x_k)(x-x_{k-1})}{(x_{k+1}-x_k)(x_{k+1}-x_{k-1})}\right]
\end{aligned}\right\}\\
&+\frac{(z-z_{k-1})(z-z_k)}{(z_{k+1}-z_{k-1})(z_{k+1}-z_k)}\left\{
\begin{aligned}
&\frac{(y-y_k)(y-y_{k+1})}{(y_{k-1}-y_k)(y_{k-1}-y_{k+1})}\left[e_{(k-1,k-1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k-1,k-1)}\frac{(x-x_{k-1})(x-x_{k+1})}{(x_k-x_{k-1})(x_k-x_{k+1})}+e_{(k+1,k-1,k-1)}\frac{(x-x_{k-1})(x-x_k)}{(x_{k+1}-x_{k-1})(x_{k+1}-x_k)}\right]\\
&+\frac{(y-y_{k-1})(y-y_{k+1})}{(y_k-y_{k-1})(y_k-y_{k+1})}\left[e_{(k-1,k,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k,k-1)}\frac{(x-x_{k-1})(x-x_{k+1})}{(x_k-x_{k-1})(x_k-x_{k+1})}+e_{(k+1,k,k-1)}\frac{(x-x_{k-1})(x-x_k)}{(x_{k+1}-x_{k-1})(x_{k+1}-x_k)}\right]\\
&+\frac{(y-y_{k-1})(y-y_k)}{(y_{k+1}-y_{k-1})(y_{k+1}-y_k)}\left[e_{(k-1,k+1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k,k+1,k-1)}\frac{(x-x_k)(x-x_{k+1})}{(x_{k-1}-x_k)(x_{k-1}-x_{k+1})}+e_{(k+1,k+1,k-1)}\frac{(x-x_k)(x-x_{k-1})}{(x_{k+1}-x_k)(x_{k+1}-x_{k-1})}\right]
\end{aligned}\right\}
\end{aligned}
$$

Then, generalize to obtain the 3-D multi-order interpolating formulas, for x, mth interpolating order; for y, nth interpolating order; for z, qth interpolating order:

$$P_{m,n,q}(x,y,z)=\sum_{h=r}^{r+q}\sum_{j=t}^{t+n}\sum_{i=s}^{s+m}\prod_{\substack{w=r\\ w\neq h}}^{r+q}\frac{z-z_w}{z_h-z_w}\prod_{\substack{l=t\\ l\neq j}}^{t+n}\frac{y-y_l}{y_j-y_l}\prod_{\substack{k=s\\ k\neq i}}^{s+m}\frac{x-x_k}{x_i-x_k}e_{ijh}$$

$$(m,n,q,h,i,j,r,s,t,w,l,k\in R)$$

Similarly, the higher dimensional polynomial interpolation can be deducted.

EXPERIMENTAL VALIDATION

Data acquisition: Zhang and Fei (2012) said, the CMM's measurement error separation experiment system was based on the double frequency laser interferometer which does not include the probe shown in Fig. 1. Such system can only separate the measuring errors in one direction for one time run for the reason that the interferometer can only measure the values of positions in one direction. The CMM's grating signal was extracted and the grating's a-quad-b signal was used to stimulate laser interferometer HP5529's reading. So both the reading of grating ruler and that of the laser interferometer can be obtained synchronously. Their on-time difference values are the measuring errors in this direction. Take the absolute zero point as reference, measure the spatial positions values within the CMM's measuring ranges such as origin point, edges and middle point, we can obtain the errors in the whole measuring ranges.

Modeling of 3-D multi-order interpolation: The measuring error is high dimension function of three coordinates when the measuring velocity is a constant

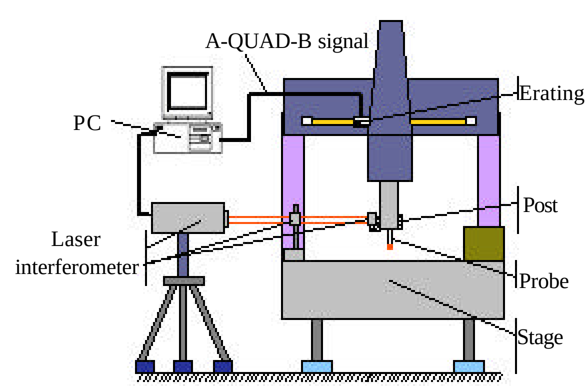

Fig. 1: Error measurement experiment

Table 1: 4×1×2 experimental data(e are the experimental value, unit is micrometer)

X	Y	z	E	X	y	z	e
1.2424	1	200	-0.0000850	400	400	200	-0.0532630
1.2424	1	350	-2.7112000	400	400	350	0.9908900
1.2424	1	500	0.3282800	400	400	500	1.2144000
1.2424	400	200	0.0027129	600	1	200	-0.7455400
1.2424	400	350	1.0997000	600	1	350	-0.5715200
1.2424	400	500	2.7984000	600	1	500	-0.4054000
200	1	200	-0.6427100	600	400	200	1.9441000
200	1	350	2.0863000	600	400	350	0.7021700
200	1	500	-0.4607700	600	400	500	0.6903000
200	400	200	3.4253000	800	1	200	-6.4449000
200	400	350	4.8946000	800	1	350	-2.2801000
200	400	500	2.4998000	800	1	500	0.1483200
400	1	200	-2.2702000	800	400	200	-3.7869000
400	1	350	-0.8187100	800	400	350	0.2750100
400	1	500	-0.6684200	800	400	500	0.0515480

Table 2: Contrast results of different interpolating orders (e means experimental values, ei means interpolating predictions, unit: micrometer)

x	y	z	e	ei (4×1×2)	ei (2×4×4)	ei (2×5×4)	ei (4×1×4)
1.2424	391	129.3769	-0.1019500	-0.2248	5.2239	6.2247	0.7056
200	560	600	0.2769700	0.2990	-4.9210	-9.0322	3.2005
400	447	200	-0.1765000	0.2079	0.1496	0.8813	0.2079
800	234	500	0.2896000	0.0918	0.2841	1.1476	0.0918
1.2424	250	129.3769	1.3258000	1.0378	16.1390	9.0363	0.0788
200	300	129.376	0.8630000	0.1049	4.1582	3.0080	3.6467
200	100	200	0.8914800	0.3666	0.8915	0.8915	0.3666

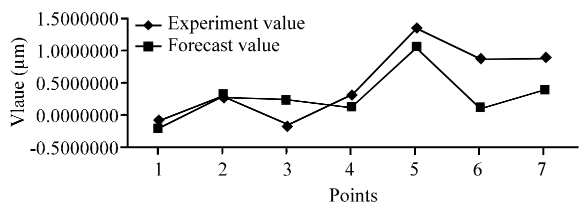

Fig. 2: Comparison of experimental results and interpolating predictions

whose formula is e = f(x, y, z) and how to predict the measuring errors in other positions just based on these scattered data? In that scenario, we can use the high dimensional multi-order interpolation polynomial model. In this study, we use the data which are cover the whole spatial measuring range were selected to interpolate. In tab.1, the data are means of errors which were obtained in 5 times reciprocating motion where the single spindle velocity was 120mm/sec. The influential extents are illustrated by the order of interpolation of variables x, y and z. In terms of this kind of single-actuated CMM, theory shows that: the closed the measuring position to actuating side (x way), the bigger the errors; the higher the measuring position to stage (z way), the bigger the errors; otherwise, the moving direction along with the vertical line of actuating will not affect the error that much. After many times of interpolation trails, we finally choose such interpolation form: for x, 4th order interpolation (i.e. 5 interpolating spots); for z, second order interpolation (i.e. 3 interpolating spots); for y, fist order interpolation (i.e. 2 interpolating spots) which means 4×1×2 interpolating order distribution form.

Jiang *et al*. (2003) introduced how to use MATLAB. In this study, the author predicted the high dimensional multi-order interpolation using MATLAB program and compared them with the experimental results shown in Fig. 2 while other interpolating distribution forms are not good as the mentioned one, illustrated in Table 2 specifically. From the interpolation results we can draw a conclusion that the theory matched well with the experiments. The apparent form of interpolating function can be obtained by substituting the data in Table 1 to Eq. 2.

We can safely draw a conclusion that the interpolating precision is influenced largely by the variables' interpolation orders, however, it is incorrect to suggest that the higher the order the better the results will be. It is unnecessary to pursue the higher interpolating order if the precision can be satisfied in practical.

CONCLUSION

The dimension reduction idea was utilized to analogize the high dimensional multi-order Lagrange interpolation formulas which were applied in the modeling process of CMM's measuring error and had high precision. This idea provided reference for the future study. However, the weakness of such method lies in the situation that the interpolating formulas are crossing crunodes whose interpolating precision is relatively low compared to other methods and the computation efficiency is not high, which needs further investigation.

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