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Research on Uncertain Multiple Attribute Decision Making with Preference Information on Alternatives

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Abstract: A kind of uncertain multiple attribute decision making problems with preference information on alternatives is studied, in which the information about the attribute weights is unknown and the attribute values are interval numbers. Based on the deviation degree between the comprehensive attribute value of alternatives and the preference of decision-maker for alternatives, an optimum model is constructed and a simple formula for obtaining the attribute weights is given, then a new method to get the priorities of alternatives is presented. An example of practical application is given to show the feasibility and effectiveness of the method.

Key words: Deviation degree, interval number, multiple attribute decision making, preference information

INTRODUCTION

Multiple attribute decision making is an important area in modern decision theory. The methods for the MADM with complete weight information have been widely studied (Cheng, 1987; Chen and Zhao, 1990; Hwang and Yoon, 1991). However, in fact, the decision maker may have uncertain knowledge about the attribute weight information and preference information on alternatives. So, the research on the MADM with preference information on alternatives has important theoretical significance and practical value.

Park and Kim (1997) and Kim *et al.* (1999) and Kim and Ahn (1999) have researched the MADM problem with incomplete weight information. Based on introducing the preference degree, Gao (2000) has studied the MADM problem with incomplete weight information expressed by interval number and preference information on alternatives. Under the situations where the decision maker has preference information on alternatives which takes the form of reciprocal judgement matrix and complementary judgement matrix, Xu (2004) has established two objective programming models to research the MADM problem under partial weight information expressed by interval number. Based on a projection method, Xu (2004) has studied the MADM problem with incomplete weight information and preference information expressed by real numbers on alternatives. Jiang and Fan (2005) have constructed an objective programming model to analyze the MADM problem with partial weight information and preference information expressed by interval numbers on alternatives. Based on the grey relational coefficients between objective preference and subjective preference, Wei and Wang (2008) have proposed a method to discuss the MADM problem with interval numbers and preference information on alternatives. Fan *et al.* (2002) have researched the MADM problem without weight information and the decision maker expresses his preference information on alternatives by a fuzzy relation. Based on the research of Fan *et al.* (2002) and Xu (2009) has constructed an optimal model by using a linear translation function to study the MADM problem, in which the weight information is completely unknown, the attribute values are real numbers and the decision maker has preference on alternatives. However, from the above researches, we can find few researches on the MADM problem, in which weight information is completely unknown, the attribute values are expressed by interval numbers and the decision maker has preference information expressed by interval numbers on alternatives. The aim of this study is to establish an optimal model to solve the above problem.

PRELIMINARIES

In the following, we will introduce important concepts and algorithms about interval numbers (Xu and Da, 2004).

Let $\tilde{a} = [a^-, a^+] = \{x | a^- = x = a^+, a^-, a^+ \in R\}$, $\tilde{a}$ is an interval number. Specially, if $a^- = a^+$, $\tilde{a}$ is a real number. The algorithms related to interval numbers are following:

If $\tilde{a} = [a^-, a^+]$ and $b = [b^-, b^+]$, $\beta \geq 0$, then:

- $\tilde{a} = \tilde{b}$ if and only if $a^- = b^-$, $a^+ = b^+$
- $\tilde{a} + \tilde{b} = [a^- + b^-, a^+ + b^+]$
- $\beta\tilde{a} = [\beta a^-, \beta a^+]$, specially, if $\beta = 0$, then $\beta\tilde{a} = 0$

Let $X = \{X_1, X_2, \cdots, X_n\}\ (n = 2)$ be the set of alternatives and $U = \{u_1, u_2, \ldots, u_m\}$ $(m \geq 2)$ be the set of attributes. Suppose the decision maker gives his subjective preference information on alternative $X_i \in X$ by $\tilde{\theta}_i = [\theta_i^-, \theta_i^+]\ (0 = \theta_i^- = \theta_i^+ \leq 1)$. Let $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ be the decision matrix, $\tilde{a}_{ij} = [a_{ij}^-, a_{ij}^+]$ is the attribute value of alternative X_i with respect to the attribute u_j, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

In order to avoid the influence of different dimensions on the decision making results, the decision matrix $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ should to be normalized into the dimensionless decision matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$, in which $\tilde{r}_{ij} = [r_{ij}^-, r_{ij}^+] = \{t \mid 0 \leq r_{ij}^- \leq t \leq r_{ij}^+ \leq 1\}\ (i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, m)$ Then following proportion transformations are employed (Goh *et al*., 1996).

If $\tilde{a}_{ij}$ is for the benefit, then:

$$r_{ij}^- = \frac{a_{ij}^-}{\sqrt{\sum_{i=1}^{n} (a_{ij}^+)^2}}, \quad r_{ij}^+ = \frac{a_{ij}^+}{\sqrt{\sum_{i=1}^{n} (a_{ij}^-)^2}} \tag{1}$$

If $\tilde{a}_{ij}$ is for the cost, then:

$$r_{ij}^- = \frac{1/a_{ij}^+}{\sqrt{\sum_{i=1}^{n} (1/a_{ij}^-)^2}}, \quad r_{ij}^+ = \frac{1/a_{ij}^-}{\sqrt{\sum_{i=1}^{n} (1/a_{ij}^+)^2}} \tag{2}$$

In order to compare the similarity degree of two interval numbers and realize the order of alternatives, we will introduce the concepts about deviation degree and possibility degree of interval number (Xu and Da, 2003).

Definition 1: Suppose $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+]$ are two interval numbers, let:

$$d(\tilde{a}, \tilde{b}) = \|\tilde{a} - \tilde{b}\| = \sqrt{(b^- - a^-)^2 + (b^+ - a^+)^2} \tag{3}$$

be the deviation degree between $\tilde{a}$ and $\tilde{b}$.

Definition 2: Suppose $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+]$ are two interval numbers and $l_{\tilde{a}} = a^+ - a^-$, $l_{\tilde{b}} = b^+ - b^-$, let:

$$p(\tilde{a} = \tilde{b}) = \frac{\min\{l_{\tilde{a}} + l_{\tilde{b}}, \max(a^+ - b^-, 0)\}}{l_{\tilde{a}} + l_{\tilde{b}}} \tag{4}$$

be the possibility degree of $\tilde{a} \geq \tilde{b}$ and $\tilde{a} \geq \tilde{b}$ be the order relationship between $\tilde{a}$ and $\tilde{b}$.

In this study, we will research how to rank X_i $(i = 1, 2, \ldots, n)$ according to the decision matrix $\tilde{A}$ and the preference information $\tilde{\theta}_i\ (\tilde{\theta}_i = [\theta_i^-, \theta_i^+])$.

MODEL AND METHOD

According to the normalized matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$ the attributes weight vector $w = (w_1, w_2, \ldots, w_m)$ and the algorithms related to interval numbers, the comprehensive attribute value of X_i is:

$$\tilde{z}_i = \sum_{j=1}^{m} \tilde{r}_{ij} w_j = [\sum_{j=1}^{m} r_{ij}^- w_j, \sum_{j=1}^{m} r_{ij}^+ w_j] \quad (i = 1, 2, \cdots, n) \tag{5}$$

where, w_j is the weight of the attribute u_j and:

$$\sum_{j=1}^{m} w_j = 1 (w_j \geq 0\, j = 1, 2, \cdots, m)$$

The preference information of the decision maker is subjective judgment to the comprehensive attribute values of alternatives. But because of the limitations of many real factors, there are deviations between the preference information of the decision maker and the comprehensive attribute values of alternatives. In view of the rationality of the decision making, the attributes weight vector $w = (w_1, w_2, \ldots, w_m)$ will minimize the total deviation between the preference information of the decision maker and the comprehensive attribute values of alternatives. So, we will give the following optimal model:

$$\begin{cases} \min D(w) = \sum_{i=1}^{n} d_i^2(\tilde{z}_i, \tilde{\theta}_i) = \sum_{i=1}^{n} [(\sum_{j=1}^{m} r_{ij}^- w_j - \theta_i^-)^2 + (\sum_{j=1}^{m} r_{ij}^+ w_j - \theta_i^+)^2] \\ \text{s.t} \quad \sum_{j=1}^{m} w_j = 1 \quad w_j \geq 0\ j = 1, 2, \cdots, m \end{cases}$$

Where:

$$d_i(\tilde{z}_i, \tilde{\theta}_i) = \sqrt{(\sum_{j=1}^{m} r_{ij}^- w_j - \theta_i^-)^2 + (\sum_{j=1}^{m} r_{ij}^+ w_j - \theta_i^+)^2} \quad (i = 1, 2, \cdots, n) \tag{6}$$

be the deviation degree between the subjective preference information of X_i and the comprehensive attribute values of X_i.

We construct the Lagrange function:

$$L(w, \lambda) = \sum_{i=1}^{n} [(\sum_{j=1}^{m} r_{ij}^- w_j - \theta_i^-)^2 + (\sum_{j=1}^{m} r_{ij}^+ w_j - \theta_i^+)^2] + 2\lambda(\sum_{j=1}^{m} w_j - 1) \tag{7}$$

Let:

$$\frac{\partial L(w, \lambda)}{\partial w_k} = 0 \quad (k = 1, 2, \cdots, m)$$

we have:

$$\sum_{i=1}^{n}[(\sum_{j=1}^{m} r_{ij}^{-}w_j - \theta_i^{-})r_{ik}^{-} + (\sum_{j=1}^{m} r_{ij}^{+}w_j - \theta_i^{+})r_{ik}^{+}] + \lambda = 0 (k = 1, 2, \cdots, m) \quad (8)$$

i.e.:

$$\sum_{j=1}^{m}[\sum_{i=1}^{n}(r_{ij}^{-}r_{ik}^{-} + r_{ij}^{+}r_{ik}^{+})]w_j = \sum_{i=1}^{n}(\theta_i^{-}r_{ik}^{-} + \theta_i^{+}r_{ik}^{+}) - \lambda \quad (k = 1, 2, \cdots, m) \quad (9)$$

Let:

$$e_m = (1, 1, \cdots, 1)^T, \eta = (\eta_1, \eta_2, \cdots, \eta_m)^T, Q = (q_{kj})_{m\times m}$$

Where:

$$\eta_k = \sum_{i=1}^{n}(\theta_i^{-}r_{ik}^{-} + \theta_i^{+}r_{ik}^{+}) \quad (k = 1, 2, \cdots, m) \quad (10)$$

$$q_{kj} = \sum_{i=1}^{n}(r_{ij}^{-}r_{ik}^{-} + r_{ij}^{+}r_{ik}^{+}) \quad (k, j = 1, 2, \cdots, m) \quad (11)$$

Then Eq. 9 will transform to the matrix formation:

$$Qw = \eta - \lambda e_m \quad (12)$$

Theorem 1: The matrix Q is a positive definite matrix.

Proof: From Eq. 11, we know $q_{kj} = q_{jk}(k, j = 1, 2, \cdots, m)$, i.e., the matrix Q is a symmetric matrix. Suppose $Y = (y_1, y_2, ..., y_m)$ is a nonzero vector, then:

$$Y^TQY = \sum_{k=1}^{m}\sum_{j=1}^{m}[\sum_{i=1}^{n}(r_{ij}^{-}r_{ik}^{-} + r_{ij}^{+}r_{ik}^{+})]y_k y_j = \sum_{i=1}^{n}\sum_{k=1}^{m}[(r_{ik}^{-})^2 + (r_{ik}^{+})^2]y_k^2$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{\substack{k=1\\k\neq j}}^{m}(r_{ij}^{-}r_{ik}^{-} + r_{ij}^{+}r_{ik}^{+})y_k y_j = \sum_{i=1}^{n}[(\sum_{k=1}^{m} r_{ik}^{-}y_k)^2 + (\sum_{k=1}^{m} r_{ik}^{+}y_k)^2] > 0$$

So, the matrix Q is a positive definite matrix.

From the theorem 1, we know the matrix Q^{-1} is existent, so:

$$w = Q^{-1}(\eta - \lambda e_m) \quad (13)$$

Because $e^T_m.w = 1$, so, we can conclude:

$$\lambda = \frac{e_m^T Q^{-1}\eta - 1}{e_m^T Q^{-1} e_m} \quad (14)$$

So:

$$w = Q^{-1}(\eta - \frac{e_m^T Q^{-1}\eta - 1}{e_m^T Q^{-1} e_m} e_m) \quad (15)$$

The optimal weight vector $w = (w_1, w_2, ..., w_m)$ can be calculated by Eq. 15 and then the comprehensive attribute values $\tilde{z}_i(i = 1, 2, ..., n)$ of all the alternatives can be calculated by Eq. 5. Because $\tilde{z}_i(i = 1, 2, ..., n)$ are still interval numbers, it is inconvenient to rank the alternatives, so we will calculate the possibility degree of $\tilde{z}_i(i = 1, 2, ..., n)$ by using Eq. 4 and establish the possibility degree matrix $p = (p_{il})_{n\times n}$, where $p_{il} = p(\tilde{z}_i \geq \tilde{z}_l)\ (i, l = 1, 2, \cdots, n)$.

The matrix $p = (p_{il})_{n\times n}$ is a complementary judgement matrix, the priority vector $\omega = (\omega_1, \omega_2, ..., \omega_m)$ of P can be given by using the following equation (Da and Xu, 2002):

$$\omega_i = \frac{1}{n(n-1)}\left(\sum_{l=1}^{n} p_{il} + \frac{n}{2} - 1\right) \quad (i = 1, 2, \cdots, n) \quad (16)$$

And further, the best alternative will be given if we rank all the alternatives based on the components of the priority vector $\omega = (\omega_1, \omega_2, ..., \omega_m)$.

Based on the above discussion, we develop a new method to analyze the MADM problem, in which weight information is unknown, the attribute values are expressed by interval numbers and the decision maker has preference information expressed by interval numbers on alternatives. There are six steps in the new method:

Step 1: Let $X = \{X_1, X_2, ..., X_n\}(n \geq 2)$ be the set of alternatives, $U = \{u_1, u_2, ..., u_n\}(m \geq 2)$ be the set of attributes and $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ be the decision matrix where $\tilde{a} = [a^-_{ij}, a^+_{ij}]$ is the attribute value of alternative X_i with respect to the attribute $u_j(i = 1, 2, ..., n, j = 1, 2, ..., m)$

Step 2: According to Eq. 1-2, the decision matrix $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ normalized into the dimensionless decision matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$

Step 3: According to Eq. 15, we obtain the optimal weight vector $w = (w_1, w_2, ..., w_m)$

Step 4: Utilize Eq. 5 to get the comprehensive attribute values $\tilde{z}_i(i = 1, 2, ..., n)$

Step 5: Utilize Eq. 4 to get the possibility degree of $\tilde{z}_i(i = 1, 2, ..., n)$ and establish the possibility degree matrix $P = (p_{il})_{n\times n}$

Step 6: We calculate the priority vector $\omega = (\omega_1, \omega_2, ..., \omega_m)^T$ of the possibility degree matrix $P = (p_{il})_{n\times n}$ and rank all the alternatives based on the components of the priority vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$, then get the best alternative

ILLUSTRATIVE EXAMPLE

When the decision maker select cadres, on one hand, he wants to select capable cadres, on the other hand, he also wants to select his preferred cadres. Hence, there is preference information on alternatives. Now, suppose some company faces how to select cadres. Firstly, the company built an index system with six attributes: u_1-morality, u_2-attitude to work, u_3-style of work, u_4-levels of culture and knowledge structure, u_5-ability of leadership and u_6-ability of innovation; secondly, the company determines five candidates $x_i(i = 1, 2,..., 5)$ based on the recommendation and evaluation of the masses and statistical treatment. Because the evaluation results about the same candidate are different, so, the attribute values after statistical treatment are expressed by interval numbers, as listed in the following Table 1.

Table 1: Decision matrix $\tilde{A}$

	u_1	u_2	u_3	u_4	u_5	u_6
x_1	0.85, 0.90	0.90, 0.92	0.91, 0.94	0.93, 0.96	0.90, 0.91	0.95, 0.97
x_2	0.90, 0.95	0.89, 0.91	0.90, 0.92	0.90, 0.92	0.94, 0.97	0.90, 0.93
x_3	0.88, 0.91	0.84, 0.86	0.91, 0.94	0.91, 0.94	0.86, 0.89	0.91, 0.92
x_4	0.93, 0.96	0.91, 0.93	0.85, 0.88	0.86, 0.89	0.87, 0.90	0.92, 0.93
x_5	0.86, 0.89	0.90, 0.92	0.90, 0.95	0.91, 0.93	0.90, 0.92	0.85, 0.87

To select the best cadre, the following steps are included:

Step 1: We utilize Eq. 1-2 to transform the decision matrix $\tilde{A}$ into the dimensionless decision matrix:

$$\tilde{R} = \begin{pmatrix} [0.378, 0.405] & [0.394, 0.414] & [0.398, 0.423] & [0.407, 0.432] & [0.394, 0.410] & [0.415, 0.437] \\ [0.394, 0.429] & [0.389, 0.410] & [0.394, 0.415] & [0.394, 0.415] & [0.411, 0.438] & [0.394, 0.419] \\ [0.395, 0.420] & [0.377, 0.396] & [0.408, 0.433] & [0.408, 0.433] & [0.386, 0.410] & [0.408, 0.424] \\ [0.385, 0.405] & [0.413, 0.433] & [0.385, 0.410] & [0.390, 0.414] & [0.395, 0.419] & [0.417, 0.433] \\ [0.384, 0.410] & [0.402, 0.414] & [0.402, 0.414] & [0.407, 0.419] & [0.402, 0.414] & [0.380, 0.391] \end{pmatrix}$$

Step 2: Suppose the decision maker's subjective preference value (after normalized) on the five candidates $x_i(i = 1, 2,..., 5)$ as follows:

$$\tilde{\theta}_1 = [0.3, 0.5], \tilde{\theta}_2 = [0.5, 0.6], \tilde{\theta}_3 = [0.3, 0.4], \tilde{\theta}_4 = [0.4, 0.6], \tilde{\theta}_5 = [0.4, 0.5]$$

Utilize Eq. 15 to get the optimal weight vector:

$$w = (0.1639, 0.1658, 0.1666, 0.1681, 0.1674, 0.1682)^T$$

Step 3: Utilize Eq. 5 to get the comprehensive attribute values of the five candidates $x_i(i = 1, 2,..., 5)$ as follows:

$$\tilde{z}_1 = [0.3978, 0.4202], \ \tilde{z}_2 = [0.3960, 0.4210], \ \tilde{z}_3 = [0.3970, 0.4194],$$
$$\tilde{z}_4 = [0.3975, 0.4190], \ \tilde{z}_5 = [0.3962, 0.4103]$$

Step 4: Utilize Eq. 4 to calculate the possibility degree of $\tilde{z}_i(i = 1, 2,..., n)$ and built the possibility degree matrix:

$$P = \begin{pmatrix} 0.5 & 0.5105 & 0.5179 & 0.5171 & 0.6575 \\ 0.4895 & 0.5 & 0.5063 & 0.5054 & 0.6343 \\ 0.4821 & 0.4937 & 0.5 & 0.4989 & 0.6356 \\ 0.4829 & 0.4946 & 0.5011 & 0.5 & 0.6405 \\ 0.3425 & 0.3657 & 0.3644 & 0.3595 & 0.5 \end{pmatrix}$$

Step 5: Utilize Eq. 16 to get the priority vector of the possibility degree matrix P:

$$\omega = (0.2102, 0.2068, 0.2055, 0.2059, 0.1716)^T$$

Step 6: Rank all the alternatives $x_i(i = 1, 2,..., 5)$ and select the best one according to the $\omega_i(i = 1, 2,..., 5)$:

$$x_1 \succ x_2 \succ x_4 \succ x_3 \succ x_5$$

Thus the most desirable candidate is x_1.

CONCLUSION

In this study, a new method is proposed to discuss the MADM problem with expert's preference information expressed by interval numbers on alternatives, in which the attribute weights information is unknown completely and the attribute values are interval numbers. Considering the preference information of the decision maker is subjective judgment to the comprehensive attribute values of alternatives, hence the weight vector of attributes will minimize the total deviation between the preference information values of the decision maker and the comprehensive attribute values of alternatives. In order to get the attribute weights, in according to the above idea, we have established an optimal model based on the deviation degree between the comprehensive attribute values of alternatives and the preference information values of the decision maker. Solving the above model, we can obtain the attribute weights and we utilize the priority vector of the possibility degree matrix to compare the comprehensive attribute values of the alternatives and then rank the alternatives. The method is practical and effective because it organically combines the subjective information and objective information. Finally, an illustrative example is given to show the application of the method.

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