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A New Optimization Approach for Emergency Facilities Locations Based on Fuzzy Information

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Abstract: The location problem in emergency service facilities is a complicated, systemic and cross-subject problem. The scientific and reasonable location decision-making for emergency facilities will affect the disposal results of emergency. In this study, we constructed a multi-objective programming model for locating emergency service facilities under fuzzy environment and investigated the model's solution method. On the basis of studying the process for determining the upper and lower bounds of the objective functions, the membership functions of the goals is constructed. a new approach for determining the preferred compromise solution of the proposed problem is presented.

Key words: Fuzzy multi-objective programming, multi-objective optimization, emergency service facility, facility location

INTRODUCTION

In recent years, larger scale emergencies have occurred more frequently. These catastrophic events have incurred huge casualties and property damage. Therefore, it is critical for the emergency management to establish a valid emergency logistics management system. The emergency service facilities play an important role in emergency logistics management, which provide high service level to ensure public safety. The number and placement of emergency facilities have a strong influence on the quality of emergency service. So, the efficient deployment of emergency service facilities becomes a crucial issue.

The problem of location planning for emergency service facilities can be classified as a special case of more general facility location problem. The literature on emergency facility location optimization is limited. Jia *et al.* (2007) presented an uncapacitated version of the covering model to locate staging areas in the event of a large-scale emergency. The location of the facilities and the allocation of the demand points to the open facilities are primarily based on distance constraints. Araz *et al.* (2007) considered a multi-objective fuzzy goal programming for covering-based emergency vehicle location model. The objective of Araz *et al.* (2007) is to maximize the population with backup coverage and increasing the service level by minimizing the total travel distance from locations at a distance larger than a pre-specified distance standard. Benman and Gavius (2007) presented

competitive location models to locate facilities that contain resources required for response to a terrorist attack. They consider the worst-case scenario where the terrorist has knowledge of the location of the facilities and the State needs to take this into account. Awasthi *et al.* (2011) present a multi-criteria decision making approach for location planning for urban distribution centers under fuzzy environment. While there exists a number of literature in the area of emergency facility location, there have not been papers on multi-objective covering models that use fuzzy sets theory to deal with parameter uncertainty. This paper will construct a multi-objective location model for emergency service facility under fuzzy environment and develop a optimization algorithm for the proposed problem.

The remainder of this study is organized as follows. In Section 2, we presented the preliminaries of fuzzy set theory. Section 3 described the problem of emergency service facilities location and constructed the mathematical model of this problem. A new optimization algorithm based on the (Bellman and Zadeh, 1970) minimum operator is introduced in section 4. Conclusions are provided in section 5.

PRELIMINARIES

In this section, we state some notions related to the considered problem (Zadeh, 1965; Bellman and Zadeh, 1970; Zimmermann, 1978, 1991; Luhadjula, 1984).

Definition 1: A fuzzy set \tilde{A} defined on a universe X may be give as:

$$A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$$

where, $\mu_A: X \rightarrow [0, 1]$ is the membership function of A . The membership value $\mu_A(x)$ describes the degree of the element $x \in X$ in A .

Definition 2: A fuzzy number $\tilde{A} = [a, b, c, d]$ is said to be a trapezoidal fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \leq x < b \\ 1, & b \leq x \leq c \\ (x-d)/(c-d), & c < x \leq d \\ 0, & \text{otherwise} \end{cases}$$

Assume that \tilde{A} and \tilde{B} are two generalized trapezoidal fuzzy numbers, where $\tilde{A} = [a_1, b_1, c_1, d_1; \omega_1]$, $\tilde{B} = [a_2, b_2, c_2, d_2; \omega_2]$ then:

- $\tilde{A} + \tilde{B} = [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(\omega_1, \omega_2)]$
- $\tilde{A} - \tilde{B} = [a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min(\omega_1, \omega_2)]$
- $k\tilde{A} = [ka_1, kb_1, kc_1, kd_1], k > 0$

Definition 3: $[a, b]$ is called an interval number, where $a, b \in \mathbb{R}, a \leq b$

For $[a, b]$ and $[c, d]$, we define:

- $[a, b] + [c, d] = [a + c, b + d]$
- $[a, b] - [c, d] = [a - d, b - c]$
- The order relation “ \leq ” is defined by:

$$[a, b] \leq [c, d] \text{ if and only if } a \leq c, b \leq d$$

Definition 4: Assume that $\tilde{A} = [a, b, c, d]$ is a trapezoidal fuzzy number. Let \tilde{A}_α be its intervals of confidence at the level of presumption α (i.e., α -cut):

$$(\tilde{A})_\alpha = \{ x \mid \mu_{\tilde{A}} \geq \alpha \}, \alpha \in [0, 1]$$

then:

$$(\tilde{A})_\alpha = \{ x \mid \mu_{\tilde{A}} \geq \alpha \}$$

is a interval number and $(\tilde{A})_\alpha = [a + (b-a)\alpha, d + (c-d)\alpha]$.

Let $(\tilde{A})_\alpha^L = a + (b-a)\alpha$, $(\tilde{A})_\alpha^R = d + (c-b)\alpha$, then α -cut of \tilde{A} becomes:

$$(\tilde{A})_\alpha = [(\tilde{A})_\alpha^L, (\tilde{A})_\alpha^R]$$

where, $(\tilde{A})_\alpha^L = a + (b-a)\alpha$ is the left points of $(\tilde{A})_\alpha$, $(\tilde{A})_\alpha^R = d + (c-b)\alpha$ is the right points of $(\tilde{A})_\alpha$.

PROBLEM FORMULATION

In this selection, we propose fuzzy multi-objective linear programming formulation of locating emergency service facilities.

Problem definition: Fuzzy multi-objective programming represents the natural extension of deterministic multi-objective programming to fuzzy context. The aim of this research was to conceive a fuzzy multi-objective programming modeling approach aimed at determining the number and locations of emergency service facilities.

Consider that there are m demands points and n candidate location points in an emergency system. The model parameters, such as travel distance, freight charge, emergency demand, etc., are fuzzy variables that can be presented by trapezoidal fuzzy number. To address this problem, we formulate the location problem of emergency service facility, as a fuzzy multi-objective linear programming model. Two objectives are considered: (1) minimization of the total travel distance and (2) minimization of the total operation cost.

The fuzzy multi-objective programming model designed here is based on the following assumptions:

- All of the objective functions and constrains are linear equations
- All of the objective functions are fuzzy with imprecise parameters
- The constructing cost of the candidate location points is different
- The pattern of trapezoidal fuzzy number is adopted to represent the imprecise travel distance, emergency demand, operation cost and so on
- The capacity of each emergency service facility is limited and the total capacity of all emergency service facilities is greater than or equal to the total demands of all emergency demand points

Mathematical model: Indices:

- $i = 1, 2, \dots, m$: index of demand points;
- $j = 1, 2, \dots, n$: index of Emergency service facilities.

Model parameters:

- \tilde{d}_i : Demand of i demand point
- \tilde{s}_j : Capacity of j facility point
- \tilde{c}_{ij} : Freight charge from i to j
- \tilde{f}_j : Fixed cost of j facility point
- \tilde{d}_{ij} : Distance from i to j

Decision variables:

- x_{ij} : Equals to 1 if j facility point provide service to i demand point otherwise it equals to 0
- x_j : Equals to 1 if a facility is established in j candidate location point otherwise it equals to 0

The problem of emergency service facility location under fuzzy environment can be stated as following:

$$(P1): \text{Minimize } S = \sum_{j=1}^n \sum_{i=1}^m \tilde{d}_{ij} x_{ij} \quad (1)$$

$$\text{Minimize } C = \sum_{j=1}^n \sum_{i=1}^m \tilde{c}_{ij} x_{ij} + \sum_{j=1}^n \tilde{f}_j x_j \quad (2)$$

$$\text{Subject to } \sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, m \quad (3)$$

$$x_{ij} - x_j \leq 0, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \quad (4)$$

$$\sum_{i=1}^m \tilde{d}_i x_{ij} - \tilde{s}_j x_j \leq 0, \quad j = 1, 2, \dots, n \quad (5)$$

$$x_{ij}, x_j \in \{0, 1\}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n \quad (6)$$

The objective function (1) minimizes the total distance from facility points to demand points, which indicates the availability for the emergency service facilities to each demand point. The objective function (2) minimizes the total operation costs of the emergency service facilities, which indicates the utilization effect of utilizing the fund. Constrain (3) assures that a demand point is assigned to a single service facility point. Constrain (4) ensures that a demand point is only assigned to a point that located service facility. Constrain (5) maintains the total capacity of all service facilities is greater than or equal to the total demands of all demand points. Constrain (6) assures binary integer values for the decision variables x_{ij}, x_j .

SOLUTION METHODOLOGY

In this selection, we combine the minimum operator (Bellman and Zadeh, 1970), fuzzy programming and the

membership function to develop an interactive fuzzy programming approach to determine the preferred compromise solution for emergency service facility location problem.

For any $\alpha \in [0, 1]$, the α -cuts of trapezoidal fuzzy parameters $\tilde{d}_{ij}, \tilde{c}_{ij}, \tilde{f}_j, \tilde{d}_i, \tilde{s}_j$ are stated as following:

$$(\tilde{d}_{ij})_\alpha = [(\tilde{d}_{ij})_\alpha^L, (\tilde{d}_{ij})_\alpha^R]; \quad (\tilde{c}_{ij})_\alpha = [(\tilde{c}_{ij})_\alpha^L, (\tilde{c}_{ij})_\alpha^R]$$

$$(\tilde{f}_j)_\alpha = [(\tilde{f}_j)_\alpha^L, (\tilde{f}_j)_\alpha^R]; \quad (\tilde{d}_i)_\alpha = [(\tilde{d}_i)_\alpha^L, (\tilde{d}_i)_\alpha^R]$$

$$(\tilde{s}_j)_\alpha = [(\tilde{s}_j)_\alpha^L, (\tilde{s}_j)_\alpha^R]$$

Introducing these α -cuts into problem (P1), we reformulate the proposed problem as follows:

$$P2): \text{Minimize } S = \sum_{j=1}^n \sum_{i=1}^m (\tilde{d}_{ij})_\alpha^L x_{ij}$$

$$\text{Minimize } C = \sum_{j=1}^n \sum_{i=1}^m (\tilde{c}_{ij})_\alpha^L x_{ij} + \sum_{j=1}^n (\tilde{f}_j)_\alpha^L x_j$$

$$\text{Subject to } \sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, m$$

$$x_{ij} - x_j \leq 0, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n,$$

$$\sum_{i=1}^m (\tilde{d}_i)_\alpha^L x_{ij} - (\tilde{s}_j)_\alpha^L x_j \leq 0, \quad j = 1, 2, \dots, n,$$

$$\sum_{i=1}^m (\tilde{d}_i)_\alpha^R x_{ij} - (\tilde{s}_j)_\alpha^R x_j \leq 0, \quad j = 1, 2, \dots, n$$

In order to confirm the upper and lower bounds of the objective functions, two single objective programming problem (P3) and (P4) are formulated as follows:

$$(P3): \text{Minimize } S = \sum_{j=1}^n \sum_{i=1}^m (\tilde{d}_{ij})_\alpha^L x_{ij}$$

$$\sum_{i=1}^m (\tilde{d}_i)_\alpha^L x_{ij} - (\tilde{s}_j)_\alpha^L x_j \leq 0, \quad j = 1, 2, \dots, n,$$

$$\sum_{i=1}^m (\tilde{d}_i)_\alpha^R x_{ij} - (\tilde{s}_j)_\alpha^R x_j \leq 0, \quad j = 1, 2, \dots, n,$$

$$x_{ij}, x_j \in \{0, 1\}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n$$

$$(P4): \text{Minimize } C = \sum_{j=1}^n \sum_{i=1}^m (\tilde{c}_{ij})_\alpha^L x_{ij} + \sum_{j=1}^n (\tilde{f}_j)_\alpha^L x_j$$

$$\text{Subject to } \sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m (\tilde{d}_i)_\alpha^L x_{ij} - (\tilde{s}_j)_\alpha^L x_j \leq 0, \quad j = 1, 2, \dots, n,$$

$$\sum_{i=1}^m (\tilde{d}_i)_\alpha^R x_{ij} - (\tilde{s}_j)_\alpha^R x_j \leq 0, \quad j = 1, 2, \dots, n,$$

$$x_{ij}, x_j \in \{0, 1\}, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

The problem (P3) and (P4) are ordinary linear programming problem which optimal solutions can be get by simplex method. let S^0 and C^0 present their optimum solution, respectively. Introducing S^0 and C^0 into the proposed problem, we can formulate two auxiliary linear programming problem (P5) and (P6) as follows:

$$(P5): \text{Minimize } S = \sum_{j=1}^n \sum_{i=1}^m (\tilde{d}_{ij})_{\alpha}^R x_{ij}$$

$$\text{Subject to } \sum_{j=1}^n \sum_{i=1}^m (\tilde{d}_{ij})_{\alpha}^R x_{ij} \geq S^0$$

$$\sum_{i=1}^m (\tilde{d}_i)_{\alpha}^L x_{ij} - (\tilde{s}_j)_{\alpha}^L x_j \leq 0, \quad j=1, 2, \dots, n$$

$$\sum_{i=1}^m (\tilde{d}_{ij})_{\alpha}^R x_{ij} - (\tilde{s}_j)_{\alpha}^R x_j \leq 0, \quad j=1, 2, \dots, n$$

$$x_{ij}, x_j \in \{0, 1\}, \quad i=1, 2, \dots, m, \quad j=1, 2, \dots, n$$

$$(P6): \text{Minimize } C = \sum_{j=1}^n \sum_{i=1}^m (\tilde{c}_{ij})_{\alpha}^R x_{ij} + \sum_{j=1}^n (\tilde{f}_j)_{\alpha}^R x_j$$

$$\text{Subject to } \sum_{j=1}^n \sum_{i=1}^m (\tilde{c}_{ij})_{\alpha}^R x_{ij} + \sum_{j=1}^n (\tilde{f}_j)_{\alpha}^R x_j \geq C^0$$

$$\sum_{j=1}^n x_{ij} = 1, \quad i=1, 2, \dots, m$$

$$\sum_{i=1}^m (\tilde{d}_i)_{\alpha}^L x_{ij} - (\tilde{s}_j)_{\alpha}^L x_j \leq 0, \quad j=1, 2, \dots, n$$

$$\sum_{i=1}^m (\tilde{d}_{ij})_{\alpha}^R x_{ij} - (\tilde{s}_j)_{\alpha}^R x_j \leq 0, \quad j=1, 2, \dots, n$$

$$x_{ij}, x_j \in \{0, 1\}, \quad i=1, 2, \dots, m, \quad j=1, 2, \dots, n$$

Let S^1 and C^1 present the optimum solution of problem (P5) and (P6), respectively, the upper and lower bounds of each objective function can be written as follows:

$$S^0 \leq S \leq S^1; \quad C^0 \leq C \leq C^1$$

We can define the membership functions of the fuzzy goals as follows:

$$\mu_s(S) = \begin{cases} 1, & S < S^0 \\ 1 - \frac{S - S^0}{S^1 - S^0}, & S^0 \leq S \leq S^1 \\ 0, & S > S^1 \end{cases}$$

$$\mu_c(C) = \begin{cases} 1, & C < C^0 \\ 1 - \frac{C - C^0}{C^1 - C^0}, & C^0 \leq C \leq C^1 \\ 0, & C > C^1 \end{cases}$$

By introducing an auxiliary variable β and the (Bellman and Zadeh, 1970) minimum operator, we transform problem (P3) into the following single objective integer linear programming problem as follows:

$$(P7): \text{Maximize } \beta$$

$$\text{Subject to } \mu_s \geq \beta$$

$$\mu_c \geq \beta, \quad \sum_{j=1}^n x_{ij} = 1, \quad i=1, 2, \dots, m$$

$$x_{ij} - x_j \leq 0, \quad i=1, 2, \dots, m; \quad j=1, 2, \dots, n$$

$$\sum_{i=1}^m (\tilde{d}_i)_{\alpha}^L x_{ij} - (\tilde{s}_j)_{\alpha}^L x_j \leq 0, \quad j=1, 2, \dots, n$$

$$\sum_{i=1}^m (\tilde{d}_{ij})_{\alpha}^R x_{ij} - (\tilde{s}_j)_{\alpha}^R x_j \leq 0, \quad j=1, 2, \dots, n$$

$$x_{ij}, x_j \in \{0, 1\}, \quad i=1, 2, \dots, m; \quad j=1, 2, \dots, n$$

This problem is a ordinary linear programming problem, which can be solve by simplex method.

CONCLUTIONS

In this study, we have constructed a fuzzy multi-objective programming model for locating emergency service facilities and investigated the model's solution method. By using the concept of α -cuts the above fuzzy programming problem is reformed to ordinary multi-objective programming problem. We have also studied the process for determining the upper and lower bounds of the objective functions and obtained the membership functions of the goals. By introducing the (Bellman and Zadeh, 1970) minimum operator, we transform the above problem into a single objective integer linear programming problem, which can determine the preferred compromise solution for emergency service facility location problem. This research can provide theoretical reference for optimizing an emergency service facilities system.

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