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Survey of Rapidly Calculating the Shortest Path of Large-scale Graph

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Abstract: The shortest path algorithm has developed for decades, there are Dijkstra algorithm, A* algorithm and Floyd-Warshall algorithm which are all less information and smaller size, so, they have certain limitations in the efficiency of processing large-scale graph data. This paper summarized this field from landmark, hierarchical technology, index and decomposition of the tree, the bidirectional search and the heuristic method etc, especially it emphasized on some latest and the most representative algorithms under the background of large-scale graph data.

Key words: Heuristic, bidirectional search, landmark, hierarchical, index, decomposition of the tree

INTRODUCTION

The data can use the graph to build a model in Biological networks, social network, traffic network, GIS and computer network and so on, because these data has a lot of nodes and there is a certain connection among some of the nodes, for example, in the highway traffic map, the side can not only describe the node of the corresponding graph of each city but also describe the highway among the neighboring cities.

From the weighted graph, if and are two nodes of graph G, the path length from node to node will be the sum of weight of each side. Maybe there will be multiple paths, the smallest path of path length is called the shortest path from node to node.

The problem of the shortest path about graph is a research hotspot in the operations research, the computer science, the geographic information science and the transportation (Bertsekas, 1998; Tarjan, 1983). Many practical problems can became calculation problems of the shortest path of the graph by means of the abstract, such as selection problems of travelling route in traffic network, information flow in the best transmission problem between routers in Computer network and calculating problems of the Contact tightness between the two strangers in social relation network (Song and Wang, 2012).

With the tremendous growth of information, the calculation of the shortest path faces new challenges, GPS navigation systems, fire control, disaster relief and so on, all need to get the best path in shorter time. Hence, the shortest path plays an important role in transportation, network planning and logistics

management; however, all of these depend on Higher performance of the shortest path algorithm.

SHOREST PATH ALGORITHM

After decades of development, there are more and more methods of calculating the shortest path, such as the classical method and the method of facing large-scale network.

In the classical algorithm, (Dijkstra, 1959) introduces Dijkstra algorithm of the shortest path of calculating unisources without negative rights, (Bellman, 1958) refers to Bellman-Ford algorithm of the shortest path of calculating unisources with rights (Floyd, 1962) is the Floyd-Warshall algorithm that finds out the shortest distance between any two node.

Algorithm of landmark: The design idea of road sign algorithm follows the daily life road mark, that is to say it tells pedestrians the way of each road in order that the people can get to the destination in time.

Generally speaking, Selection of landmarks are usually those are most likely to appear on the node or edge of the optimal path. The shortest path problem in the road network usually refers to select important transport hub, at the same time, according to user's preferences it also selects some landmark buildings, transit point and so on. Tretyakov *et al.* (2011) this paper uses the method of common node between the landmark to calculate the shortest path: firstly, set up landmark on the basis of node degree and degree of intimacy; secondly, improve the shortest distance between two node, namely if two paths have public node before reaching the landmark

nodes, the public node of connecting two node will improve the shortest distance. Thirdly, if the edge has a dynamic change-adding and removing of the edge, this will conclude the distance from relevant node to the landmark again. (Sarma et al., 2010; Gubichev et al., 2010) realize the query of the shortest path on the basis of the method of sketches and landmarks. The main idea is: 1, Produce sample and calculate the recent distance of the sample and each point and set the landmark (forward and reverse); 2,Find positive search sketches and the intersection of reverse search sketch of the target point; 3, Use d(s, d) = dist(s, 1) + dist(1, d) to find out the path, in addition, use the intersection path of the forward and reverse the paths so as to optimize the shortest path search.

Hierarchical algorithm: Hierarchical is a kind of abstract problem and solving mechanism. The basic idea is to solve the complicated problem, in the meantime seize the key to the problem and ignore other minor details, then, improve the details step by step. Such as: (Jin et al., 2012). The article proposed a collection of similar to build highway for improving the speed of query the shortest distance; (Geisberger et al., 2008; Rice and Tsotras, 2010) these paper put forward the sorting via the node of importance and creating a profile graph that ensure the characteristic of the shortest distance, after shrinking the node of importance order, the new shortest path store two nodes it have higher importance than the deleting node and the node of minor importance of two node deposited the relation about deleting node, so as to use in the query (Blondel et al., 2008) this study use the method of gaining to produce levels, on the one hand, the gain is that the point of certain community moves to the community of adjacent node; Calculate the gain, if it is positive it will continue to stay at new communities; on the other hand, after finishing the above process, this community will be combined with a new point, repeat the above steps and produce the largest community in order to inquire (Song and Wang, 2011) This paper designed the layering and heuristic combination technology for calculating the shortest distance, in addition it also designed the concept of community in terms of hierarchical, the edge of boundary node of two adjacent communities called a inter-community node, by means of this it also adjust the distance between node in the community. In this way, a large number of calculations between the source and target are abstracted and reduced, the query depend largely on the inter-community node it will improve the computational efficiency (Chan and Yang, 2009) this study solve the dynamic change of renewal process of the minimum spanning tree very well, detailed process is as follows: Firstly, delete the value of weight change edge and positioning weights of abnormal node; Secondly, judging the change node was internal or external, for the weight increase, the interior is designed to be infinite, the exterior was chosen as the candidate edge; for the weight decrease, keep changing node directly it determined again whether other node need to be updated; Thirdly, according to the change of weight, the shortest path of local change node would be determined whether they needed to update and generate a new minimum spanning tree

In a word, hierarchical technology is effective means of the shortest path calculation problems in large-scale network, by means of reduction and simplifying. The heuristic technology, data pretreatment technology and bidirectional search technology combined with it. The efficiency of hierarchical search algorithm was affected by various factors-preprocessing phase generated and saved how much amount of data were, the topology of the network layer and search rules and so on. In general, to some extent, the efficiency of the algorithm will be at the cost of precision and storage space. Storage consumption and real-time efficiency of the algorithm, real-time efficiency and precision of the algorithm is to find a good compromise, that is to say it has become the focus of most of the hierarchical algorithm.

Tree decomposition: Definition 1(the decomposition of the tree) G (V, E) is a simple undirected graph (without the loop, without multiple edge). Then the decomposition of tree of the graph G associated with a subset of each node T and T subsets (these subsets are called the fragment of tree decomposition). Tree T and section set {} satisfy the following three conditions:

- () = V, the nodes of all fragment set cover all the nodes of the graph G, Or each node of graph G belongs to certain fragment X_i:t∈T
- Each edge e of the graph G, e∈E, there are at least a fragment, it include two end node of e X; t∈T
- If isthree nodes of tree T is on the parth of $t_k, \ t_{k\!+\!1}$ and $t_{k\!-\!1}$
- Then: t_k-1, t_k, t_k+1 the node of graph G belongs to X_{tk-1}, X_{tk+1}And, meanwhile it also belongs to X_{tk}. The width of tree decomposition is equal to max(|X_t-1:t∈T):

$$d_{G}(v, u) = d_{G}(v, \omega) + d_{G}(\omega, u)$$

Tree width of Graph G is the width of the smallest tree decomposition of G. All the tree decomposition of graph G, the minimum width of the tree decomposition was called tree width of graph G (Gao and Li, 2012).

(Wei, 2010) This article uses tree decomposition to solve the query problem of the shortest path it main idea is to create the index of the tree, steps are as follows: Firstly, setting a KEY value. If a node degree is less than the KEY node, Put this node and neighboring node into one package, until the set is empty or does not meet the key value; Secondly, each package pre-calculate the shortest distance among each point; Thirdly, whether to find the shortest distance in a package, if it is it will directly get the answer, on the contrary, first of all find the two youngest ancestor nodes, then, calculate the shortest distance through simple path; (Akiba et al., 2012)the article puts forward the calculation method of the shortest path in complex network. The main contributions are two: in the first place, according to the width of the tree W, decompose the complicated graph, the decomposition of graph is called a package, then calculate a package of the shortest distance, in the second place, use the concept of landmarks to calculate the shortest distance of interpackage.

Akiba et al. (2013) The article demonstrates the mixture of tree decomposition and other methods-Mainly calculate the distance of the vertex label in advance, the key is pruning by breadth-first search, Thus, reduce the search space and size, Pruning strategy: S is a vertex set, if there is the shortest path between two nodes through a vertex of S and there is a node- $\omega \in S$, then prune u. Akiba et al. (2013) mainly combined with the advantages of three methods that are landmark based, tree decomposition and labeling based (Cheng and Jeffrey, 2009; Potamias et al., 2009) landmark based approximation method in solving the shortest path is found the central vertex of height in the complex network its key step is to choose a subset as landmarks. Calculate the distance from each point to landmarks, the main steps are: 1, Estimates of the approximate distance: It includes the upper bound of the distance, Lower bound of the distance, the average distance and the average distance about the set and so on. From the experimental contrast, selecting of the upper bound of the distance is better than before. 2,The selection of the landmark: The largest degree and the minimum value of other node.

Index algorithm: Index structure can actually input the characteristics of the data and can quickly find out the data. In the study of recent years, the index method on the graph database can be roughly divided into two parts, on the one hand the index method focus on the data characteristic, that is to say, the index method revolved around the characteristics; on the other hand, the index method pay more attention to the data structure. In the shortest path search, in order to accelerate query

execution, the researchers set up the index structure and use offline to establish index price for graph so as to cut down the price of online query execution.

Virgilio *et al.* (2013) mainly introduces the similar technology of the graph to solve the problem of graph query. The main methods are:1,Create index; 2,The index with the width of the limited traversal; 3,Calculate factor score in terms of similarity; finding the factor score is the key, the process is: a, the public portion of finding out each query in pretreatment; b, Cluster each query; c, Find out the query results (find out the matching path of the highest score); (Yuan *et al.*, 2011). This article introduces the query problem of uncertain graph, one is that solve subgraph isomorphism problem with upper and lower boundary, the other is that use the inverted index technology to solve the problem of graph traversal.

The efficiency of graph index is critical to the query of large-scale graph data. Good structure index is a feasible way to improve the efficiency of graph database queries.

Bidirectional search algorithm: The search is a single direction in the classic shortest path algorithm. The execution time of the algorithm depends on the number of node traversal in the search process. If the shortest path tree are made from start s and target t at the same time, the scale of the problem will accordingly reduce, bidirectional search technology decomposed into two sub-problems through the counterpart of the original problem so as to reduce the search space and the purpose of accelerating algorithm implementation.

Gao et al. (2011) Using the bidirectional search of greedy algorithm to search the shortest path its main steps are: Find the minimum value node as an extension, extension found that the minimum node adjacent nodes, then, the adjacent nodes are been merged into search list for the new iteration, the purpose is to find the shortest path. Gubichev et al. (2010) use sample sketches and landmark technology to achieve the query of the shortest path, first of all, Produce sample and calculate the sample and the closest distance of each point about the node set. In the meantime, set up positive and reverse landmarks; what's more, find out the intersection of forward search of starting node and reverse search of the target node, in order that it can quickly find out the shortest path. Goldberg and Harrelson (2004) Make full use of A* algorithm and the bidirectional search technology to achieve the shortest path search, the choice of the landmark is a more important technology in this kind of combination method.

Bidirectional search algorithm is usually used to solve the shortest path, especially the path optimization

problem of the dynamic change node, in this way it can be helpful to reduce the time complexity and improve the execution efficiency of the algorithm, in other words it is convenient to solve the large-scale network structure.

Heuristic algorithm: Heuristic search evaluates the position of each search in the state space search, the goal is to get the best position, thereby, search this position again so as to achieve the goal. So, many useless search path can be omitted, meanwhile it can improve the efficiency. In heuristic search, the evaluation of the location is very important. Adopted different evaluate method have different effects.

A* algorithm is the most wide method for calculating the shortest distance in heuristic techniques, A* algorithm is all the most effective methods in calculating the shortest path.

The equation is as follows:

$$f(n) = g(n) + h(n), f(n)$$

is a kind of evaluation function from the start node to the target node through the node n, g (n) is the actual cost from the start node to n nodes in the state space, h(n) is the estimate cost of the best path from n to the target node, In order to find the shortest path (optimum) conditions, The key selection is evaluation function h (n): Value h (n) less than or equal to actual value of the distance of n to the target node. In this case, the search node are much, the search range is bigger, efficiency is low but it can get the optimal solution. On the contrary, this will not promise to get the optimal solution.

Goldberg and Harrelson (2004) The article puts forward two methods of choosing landmarks: The first one chooses the farthest point of the source point as landmarks and then add one by one; The second method is to select the center of the package, in this package, find the farthest point near the center as landmarks. A* algorithm combined with the bidirectional search technique with the selection of two methods so as to find the shortest path. (Adler, 1998) put forward a kind of algorithm that is to get the optimal path: The first step uses A* search algorithm to calculate the distance from the start node s to the target node t and use it as estimated distance limit.

Euclidean distance approximation is similar to the estimated distance between nodes. The second step further improve the calculation efficiency through the pruning algorithm.

Colomi et al. (1994); Dorigo et al. (1996), Duan et al. (2004), Wang and Ye (2012) ant colony algorithm is described the implementation process of the shortest path problem: Assume that put m ants into n selected cities at

random. According to certain probability, an ant choose the next city that it hasn't visited. An ant choose the main basis of next target city in terms of the following two points: (1) Refers to the pheromone concentration which city i connects with the city j in distance. Every path pheromone amount is equal at the initial time. (2) Means the visibility of city i transferred to city j, it is also known as heuristic information.

Solving the problem can give out the heuristic information and achieve it via certain algorithm. At some point, ant k chooses cities j as the $h_{i,j}$ probability of the objective city in the city i:

$$p_{i,j}^{k} = \frac{[h_{i,j}]^{x}[see_deg\,ree_{i,j}]^{\beta}}{\sum_{i \in N_{i}^{y}} [h_{i,j}]^{x}[see_deg\,ree_{i,j}]^{\beta}}, j \in N_{i}^{k}$$
(1)

$$see _deg ree_{i,j} = 1/d_{i,j}$$

shows the distance between city i and city j in the shortest path problem. Ant k that located in the city i can directly get to the collection of neighboring cities and does not include the city that the ant k has visited. α and β are two parameters, they respectively determine the influence of pheromone and heuristic information. In order to avoid excessive residual pheromone and submerge the information, The residual information will be updated after each ant finish one step or visit n city. This updated strategy is similar to the characteristics of the human memory. The new pheromones are deposited in the brain, as time goes on, the old pheromones gradually fade away, even it is forgotten. Finally, pheromone concentration on the (i,j) is gotten, as follows:

$$\mathbf{h}_{i,j} \leftarrow (1 - \rho)\mathbf{h}_{i,j}, \forall (i,j) \in \mathbf{L}$$
 (2)

$$h_{i,j} \leftarrow h_{i,j} + \sum_{k=1}^{m} \Delta h_{i,j}^{k}, \forall (i,j) \in L$$
 (3)

(2) demonstrates the evaporation of the pheromone, (3) shows the release of the pheromone, ρ is evaporation rate of the pheromone, $0 < \rho \le 1$. Parameters ρ is to avoid the infinite accumulation of pheromone and make the algorithm forget selecting lower path. Here are defined as follows:

$$\Delta h_{i,j}^{k} = \begin{cases} Q/C, & \text{if the kth ant pass on the path } \left(i, \ j\right) \\ & \text{0, other} \end{cases} \tag{4}$$

Q is a constant value C_k, indicates that if the kth ant release the amount of pheromone on the path (i, j) in the

loop, shows the kth ant set up the path length in the loop, the ant builds the path better (that is to say, path length is even shorter), it will get more pheromone on the path side. Then the next round of solution of the corresponding edges will get greater access probability in the process of building.

Wang and Ye (2012) The article improves ant colony algorithm of calculating the shortest path from three aspects: (1) The kth ant can determine the path by means of routing probability formula from city i to city j but parameter setting is constant in the entire circulation process. The parameters with achieving maximum cycle times 1/4 in the experiment are adjusted dynamically. (2) Considering whether to optimize local search method, the goal is to make it find the global optimal solution as soon as possible. According to the revelation of genetic algorithm, the mutation of genetic algorithm has been introduced in local search algorithm, So that local search algorithm with variation characteristics can be designed. (3) In order to avoid increasing the pheromone indefinitely, pheromone evaporation his introduced in the basic ant colony algorithm but this is too absolute, pheromone evaporation in each path are the same but the real factors are not taken into consideration, so consider introducing evaporation factor classification parameters, then makes different path pheromones evaporate. If the product of $\omega \Box \rho$ evaporates the pheromones, the problem will be solved very soon.

Heuristic method use the basic idea of controlling knowledge to provide guidance for the search process. The search direction can be close to the direction of finishing solving the problem quickly. Based on the heuristic intelligent method, the design of the heuristic function is very complex problems. A good heuristic function can guide the search algorithm to finish a lot of difficult problems to solve. According to the heuristic function provides status information, Specific search algorithm of the shortest path determines the process of visiting the state space. The difference of search algorithm of different short paths lie in storing and reserving the state node. and in the face of the candidate state which has not yet visited, make a decision, Search process with reasonable decision can reach the target along the shorter path and save the price of searching process.

CONCLUSION

Current shortest path algorithms are close to all kinds of the direction of the integration of different technologies and methods. Although the algorithm for the shortest path problem research has made some breakthrough and progress and is accompanied by a large-scale application in the real network, the research is still in a stage of development on the whole. There's a lot of work which needs to be further explored and understood.

Firstly, At present, the shortest path algorithm have advantages and disadvantages in search efficiency, computation accuracy and storage consumption. The improvement of Many algorithm efficiency is all at the expense of a large number of pretreatment or search accuracy, thus how to reduce the dependence on storage space, improve the precision of the search and design a more efficient algorithm is still worth exploring further.

Secondly, In the network segmentation constructing layered structure algorithm, network segmentation quality has a certain impact on the execution efficiency of subsequent search algorithm. At present, in order to achieve segmentation degradation of network, the research generally uses the segmentation technology of plane structure. The network in the real world usually display non-planar structure, so quick and efficient segmentation technology for this kind of networks has yet to be further researched.

Finally, the heuristic algorithm research focused on solving the shortest path problem in static network. However, in fact, the network is usually dynamic and uncertain. So how will all kinds of heuristic strategy and the actual network combine? The research designs fast path algorithm for the dynamic network, this is also a trend in the future.

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