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Synchronizability Optimization for the Edge Iteration Based Deterministic Small-world Network with the Modified Simulated Annealing Algorithm

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Abstract: Recently, researchers have presented several deterministic small-world networks (DSWNs) which can be generated in a special iteration process without randomness. However, to the best of our knowledge, no one has studied the synchronizability of DSWNs up to now. In this study, we focus on the synchronizability of the edge iteration based deterministic small world network (EIB-DSWN) that was presented in 2006. Our testing results show that the EIB-DSWN has very poor synchronizability. To improve the synchronizability, we propose using the Modified Simulated Annealing (MSA) algorithm to optimize the EIB-DSWN. After MSA-based optimization, to check if the optimized network is still a kind of small-world network, we calculate its three main characteristics. It turns out that the MSA algorithm can significantly optimize the synchronizability of the EIB-DSWN under the premise of ensuring small world characteristics.

Key words: Complex network, deterministic small world network, simulated annealing algorithm, synchronizability, optimization

INTRODUCTION

Today, complex networks have attracted increasing attention from various field of science and engineering. The pioneering work from Watts and Strogatz (1998) introduced the concept of small-world networks which exhibit a high degree of clustering and a small average distance between two nodes. Since then, a number of small-world models have been proposed (Newman and Watts, 1999; Kasturirangan, 1999; Kleinberg, 2000; Ozik *et al*., 2004) which show good small-world effect. However, these models are all random. As mentioned by Barabasi *et al*. (2001) the randomness while in line with the major features of real-life networks, makes it harder to gain a visual understanding of how networks are shape and how do different nodes relate to each other (Barabasi *et al*., 2001). In addition, the probability analysis method and the random connection of the edge are not suitable for those networks with fixed connectivity between nodes (Comellas *et al*., 2000), such as neural networks, computer networks and circuit networks. To provide deterministic network models in line with the characteristics of real systems not only has some important theoretical significance but also has potential practical value (Barabasi *et al*., 2001). Thus, many researchers turn to constructing small-world networks in a deterministic manner (Comellas *et al*., 2000; Boettcher *et al*., 2008; Zhang *et al*., 2006; Lu and Guo, 2012; Guo *et al*., 2012). Among them, Zhang *et al*. (2006) presented a deterministic small world network created by edge iterations (EIB-DSWN) and gave analytical results of the main static network properties, including degree distribution, clustering coefficient and diameter (Zhang *et al*., 2006). However, they did not discuss any dynamic network property such as synchronizability.

As an emerging phenomenon of the population consisting of dynamically interacting units, synchronization has fascinated humans from ancient times. Synchronization processes are ubiquitous in nature and play a very important role in many different fields such as biology, ecology, climatology, sociology, technology, or even in arts (Pikovsky *et al*., 2001). Thus, studying the synchronizability of a network and further how to enhance its synchronizability is of significance (Lindsey *et al*., 1985). Nishikawa *et al*. (2003) first systematically explored the relationship between each structural feature and the synchronizability for a network. McGraw and Menzinger (2005) examined the synchronization of networks composed of nonidentity coupled phase oscillators and studied the effect of varying the clustering coefficient without changing the degree distribution on the synchronization. Zhao *et al*. (2006) respectively investigated the effects of average distance and standard deviation of degree distribution on the synchronizability of complex networks by using the random interchanging algorithm. Donetti *et al*. (2005) used

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global optimization algorithms to find the optimal or approximate optimal synchronization network under the given conditions.

This study first introduces the original EIB-DSWN and tests its synchronizability, finding out the fact that its synchronizability becomes worse with the growth of the network size. Then we optimize the synchronizability of the EIB-DSWN with the Modified Simulated Annealing (MSA) algorithm proposed by Donetti *et al.* (2005). Finally, we examine the three main features of the optimized EIB-DSWN to check if the network after MSA optimization is still a small-world network.

THE SYNCHRONIZABILITY OF THE ORIGINAL EIB-DSWN

The synchronizability of probabilistic small-world networks such as WS and NW networks (Watts and Strogatz, 1998; Newman and Watts, 1999) has been investigated by many researchers. However, to the best of our knowledge, no one has studied the synchronizability of DSWNs. In this study, we focus on the edge iteration based deterministic small world network (EIB-DSWN) proposed (Zhang *et al.*, 2006). The generation process of the EIB-DSWN can be described as follows. Suppose N(t) represents the network after t iterations. For t = 0, N(0) is a triangle whose three nodes connect one another. For t = 1, N(t) is obtained from N(t-1) by adding for each edge created at step t-1 a new node and attaching it to both end nodes of the edge, as shown in Fig. 1.

In general, small-world networks are featured by three main properties. First, their Average Path Length (APL) or diameter grows logarithmically with the number of nodes or slower. Second, the average node degree of the network is small, i.e., the network should be a sparse one. Third, the network has a high average clustering compared to an ER random network with equal size and equal average node degree. The analytic solution by Zhang *et al.* (2006) shows that: (1) the degree distribution P(k) is an exponential of a power of degree k, i.e., the EIB-DSWN is an exponential network; (2) the clustering coefficient approaches to a constant value 0.6931 when t approaches infinite, so the clustering coefficient is high; (3) the diameter grows logarithmically with the number of nodes. Thus the original EIB-DSWN is a small-world network because it is a sparse one with high clustering and short average path length which satisfy the three main necessary properties for small-world networks.

There are several criteria to judge whether a network can reach synchronization. Here we use the criterion established by Barahona and Pecora (2002).

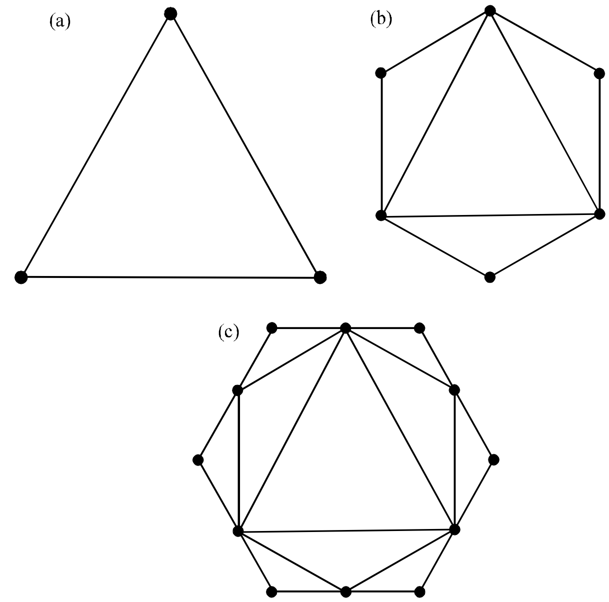

Fig. 1(a-c): The first three iterations (a) t = 0, (b) t = 1 and (c) t = 2 for EIB-DSWN construction

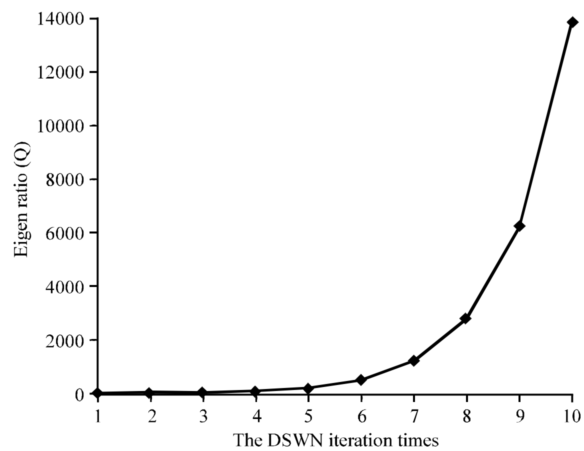

Fig. 2: The eigenratio versus the number of iteration times for the EIB-DSWN

They give the conclusion that a network exhibits better synchronizability if the ratio $Q = \lambda_N/\lambda_2$ is as small as possible, where the λ_N and λ_2 is, respectively the minimum and the second largest eigenvalues of the Laplace matrix of the network. The Laplace matrix $L = \{L_{ij}\}$ is defined as follows: let $L_{ii} = k_i$ (i.e., the connectivity degree of node i) and let $L_{ij} = -1$ if nodes i and j are directly connected and let $L_{ij} = 0$ otherwise.

Based on the above descriptions, we calculate the eigenratio of the EIB-DSWN from t = 1 to t = 10 as shown in Fig. 2. It shows that with the growth of the network size, the eigenratio increases exponentially. When t is

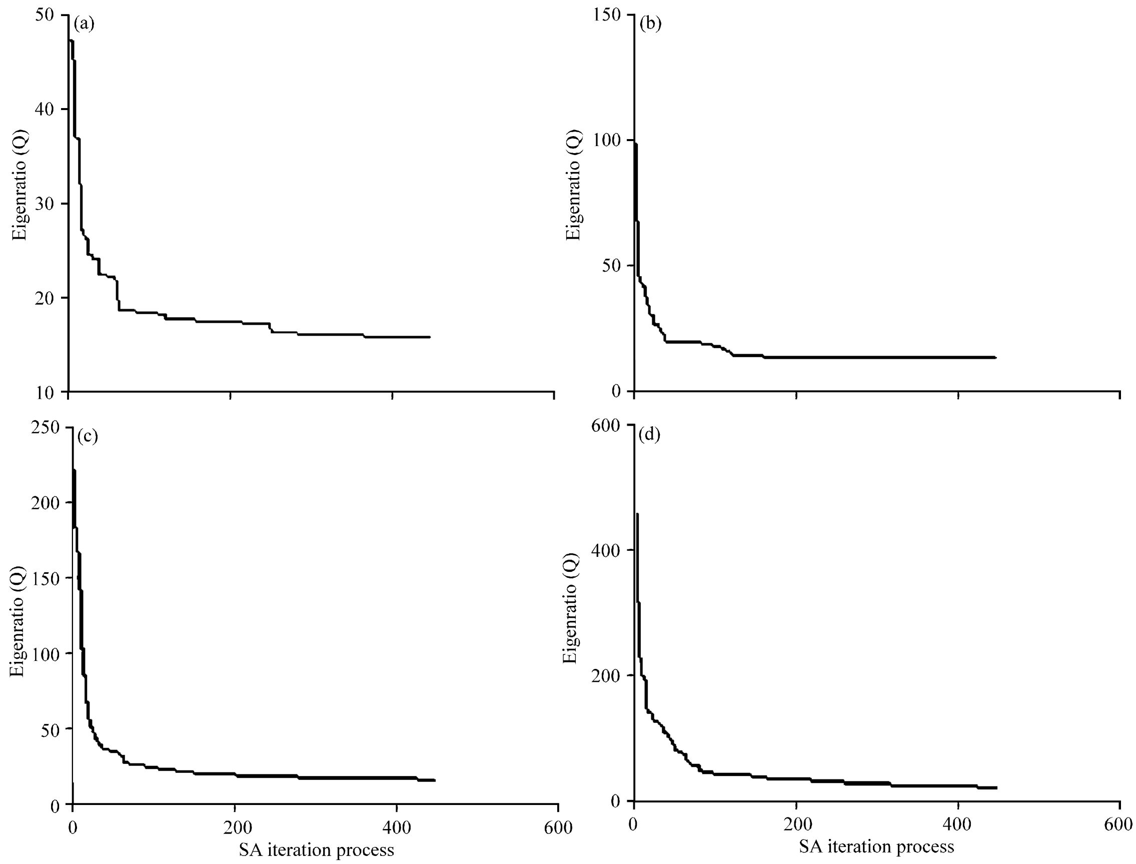

Fig. 3(a-d): The MSA optimization results under different EIB-DSWN size, (a) N (3), (b) N (4), (c) N (4) and (d) N (6)

large enough (i.e., t→+∞), the ratio is approaching +∞, thus the synchronizability of the EIB-DSWN is very poor.

MSA-BASED SYNCHRONIZABILITY OPTIMIZATION

It is known that the network synchronizability can be measured by the eigenratio of the Laplace matrix of network, thereby increasing the synchronizability of DSWN can be converted to minimizing the eigenratio Q continually. To find the global optimal eigenratio Q, here we use the modified SA algorithm which was first used by Donetti *et al.* (2005) to find the entangled network.

The modified SA algorithm for synchronizability optimization can be described as follows:

Step 0: Set the initial temperature T_0 and the termination temperature T_{min}, set the current temperature $T = T_0$

Step 1: Randomly select an edge from the network and remove this edge, then re-generate an edge in the network (this operation is called one re-connection)

Step 2: If the re-connection makes the network disconnected, refuses this re-connection, return to step 1

Step 3: Calculate the eigenratio after the re-connection. If the eigenratio decreases which says the network synchronizability has been strengthened, accept the re-connection. Otherwise, if the eigenratio increases, accept the re-connected with the probability (Penna, 1995) expressed as follows:

$$p = \min\{1, e^{\frac{Q_{final}-Q_{initial}}{T}}\}$$

Step 4: Update the current temperature with $T = r \times T$ (where, r is the annealing rate), if $T \le T_{min}$, terminate the program; else return to step 1

Taking both the program efficiency and the convergence speed of the eigenratio into consideration, the parameters are set as follows: The initial temperature T_0 is set to 0.85, the annealing rate r is set to 0.98 and the termination temperature T_{min} is set to 0.001.

We apply the above modified SA to the synchronizability optimization of the EIB-DSWN.

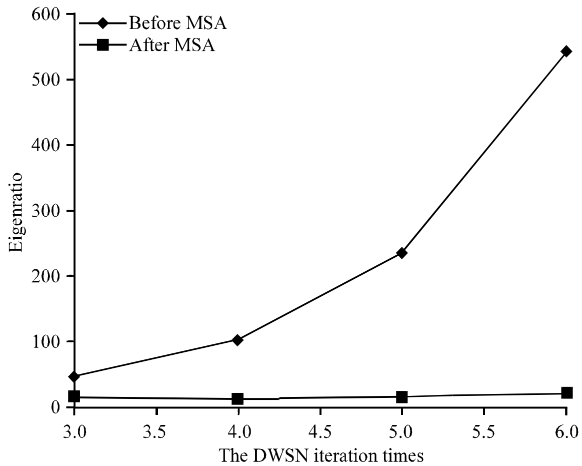

Fig. 4: The comparison of network synchronizability before and after the MSA optimization

Figure 3 gives four examples to show the numerical simulation results of the MSA optimization for networks N(3), N(4), N(5) and N(6). It shows that for each network, in the early stages of the MSA iteration process, the eigenratio Q drops rapidly, then it almost approaches to a constant. That is to say, for a given EIB-DSWN, the MSA can optimize the network synchronizability with a good level.

To find out how the network synchronizability changes after the MSA optimization, we compare the eigenratio before and after the MSA optimization as shown in Fig. 4. It shows that after the MSA optimization, with the network size growing, the eigenratio approaches to a nearly constant level which proves that the network can obtain the synchronization state easily after the MSA optimization.

THE CHARACTERISTICS OF THE OPTIMIZED EIB-DSWN

To check if the network after the MSA optimization is still a small-world network, we analyze the three main network characteristics, including degree distribution, diameter and clustering coefficient which are the most important features to judge small-world networks. Figure 5 gives an example to show how the network topology changes after the MSA optimization. The left figure shows the original EIB-DSWN N(5) (i.e., the network generated after 5 iterations) while the right one shows the optimized EIB-DSWN.

Degree distribution: Degree distribution P(k) is defined as the probability that a randomly selected node has exactly k edges directly connected with it. From the above section, we can easily know that the modified SA algorithm does not change the number of nodes and

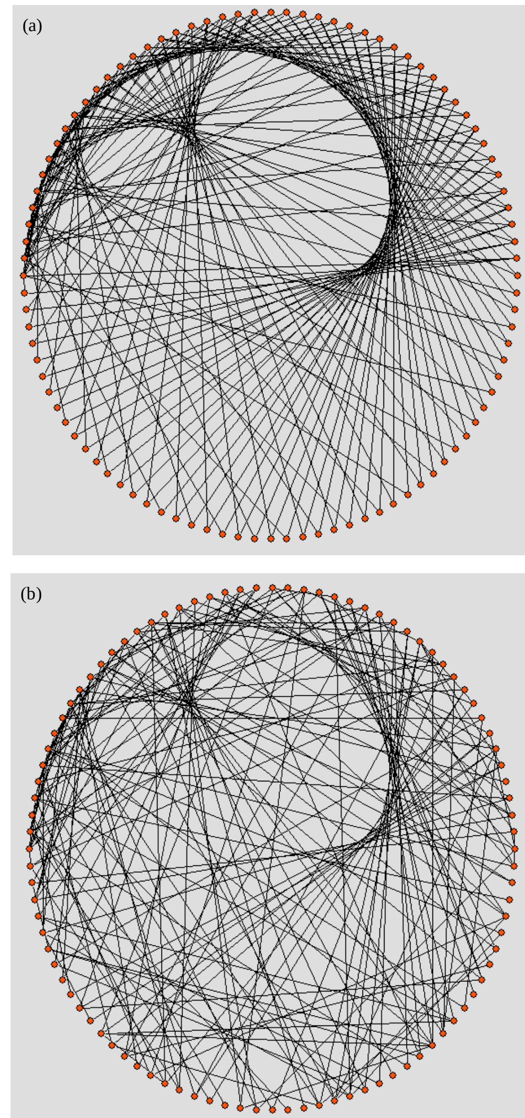

Fig. 5(a-b): The EIB-DSWN N(5) (a) Before and (b) After the MSA optimization

edges, so the average degree of the whole network remains the same, namely, 4. To find more details about the changes in degree distribution, we calculate the degree distribution of the EIB-DSWN N(5) before and after the MSA optimization. The results are shown in Fig. 6. We can obviously see that before the MSA optimization, the distribution is a kind of power-law distribution but after the MSA optimization the distribution is more like a Poisson distribution which can also be intuitively seen in Fig. 5. In summary, the optimized network is still a sparse network which corresponds to the features of small world networks.

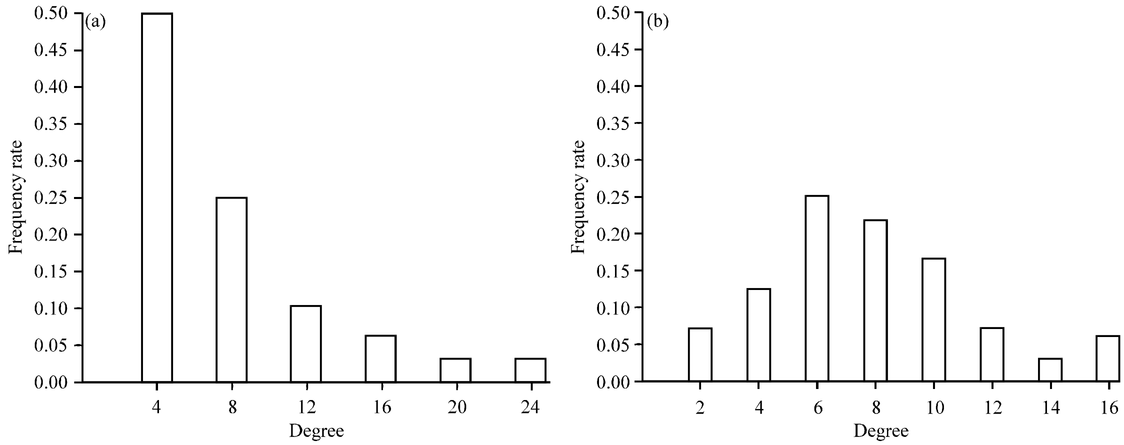

Fig. 6(a-b): The comparison of the degree distribution between the (a) Original and (b) Optimized EIB-DSWNs

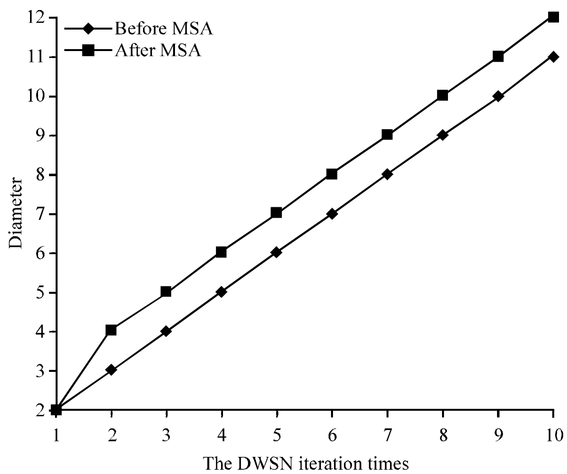

Fig. 7: The comparison of the diameter between the original and optimized EIB-DSWNs

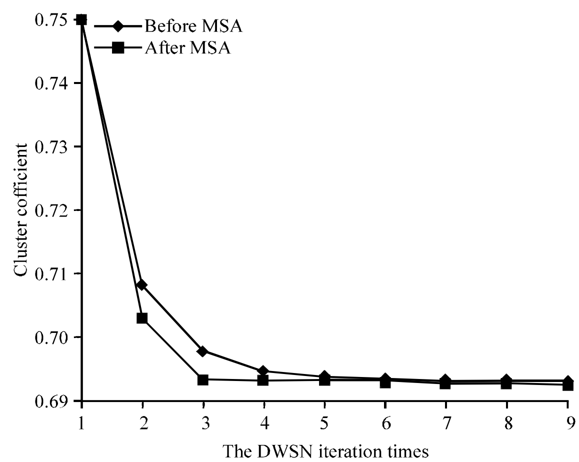

Fig. 8: The comparison of the clustering coefficient between the original and optimized EIB-DSWNs

Diameter: Since, the network after the MSA optimization is no longer a deterministic network, it is hard to find the analytical solution to the network diameter, thus we give the numerical simulation in Fig. 7. It shows that after the MSA optimization, the network diameter increases by 1 compared with the original EIB-DSWN under the same network size. Thus, the optimized network still has a linear relationship between the diameter and the number of iteration times. Similar to the results by Zhang *et al.* (2006), the diameter of the optimized network still has a logarithmic relationship with the number of network nodes which also corresponds to the features of small world networks.

Clustering coefficient: Clustering coefficient provides a measure of the local structure within the network. To show how the clustering coefficient changes before and after the MSA optimization, we give the simulation results in Fig. 8. We can see that compared with the original EIB-DSWN, the clustering coefficient of the optimized network does not change much. Thus, the clustering coefficient is still high which also corresponds to the features of small world networks.

CONCLUSIONS

This study introduces how to use the modified SA algorithm to improve the synchronizability of the EIB-DSWN. We find out that: (1) For a given EIB-DSWN, the MSA algorithm can significantly reduce the eigenratio of the EIB-DSWN, namely, it can significantly improve the network synchronizability, (2) When the EIB-DSWN is large enough, it cannot reach synchronization. However, after applying the MSA algorithm to the EIB-DSWN, the eigenratio of the optimized network basically does not vary with the network size. This shows that the network can reach synchronization easily after the MSA optimization and (3) After the MSA optimization, the network is still sparse with a high clustering coefficient and a smaller diameter, satisfying the three necessary properties of small-world networks.

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REFERENCES

Barabasi, A.L., E. Ravasz and T. Vicsek, 2001. Deterministic scale-free networks. Phys. A: Stat. Mech. Appl., 299: 559-564.

Barahona, M. and L.M. Pecora, 2002. Synchronization in small-world systems. Phys. Rev. Lett., Vol. 89.

Boettcher, S., B. Gongalves and H. Guclu, 2008. Hierarchical regular small-world networks. J. Phys. A: Math. Theor., Vol. 41.

Comellas, F., J. Ozon and J.G. Peters, 2000. Deterministic small-world communication networks. Inf. Process. Lett., 76: 83-90.

Donetti, L., P.I. Hurtado and M.A. Munoz, 2005. Entangled networks, synchronization and optimal network topology. Phys. Rev. Lett., Vol. 95, No. 18.

Guo, S.Z., Z.M. Lu, G.Y. Kang, Z. Chen and H. Luo, 2012. A tree-structured deterministic small-world network. IEICE Trans. Inform. Syst., 95: 1536-1538.

Kasturirangan, R., 1999. Multiple scales in small-world graphs. http://arxiv.org/pdf/cond-mat/9904055.pdf

Kleinberg, J.M., 2000. Navigation in a small world. Nat., 406: 845-845.

Lindsey, W.C., F. Ghazvinian, W.C. Hagmann and K. Dessouky, 1985. Network synchronization. Proc. IEEE, 73: 1445-1467.

Lu, Z.M. and S.Z. Guo, 2012. A small-world network derived from the deterministic uniform recursive tree. Phys. A: Stat. Mech. Appl., 391: 87-92.

McGraw, P.N. and M. Menzinger, 2005. Clustering and the synchronization of oscillator networks. Phys. Rev. E, Vol. 72.

Newman, M.E.J. and D.J. Watts, 1999. Scaling and percolation in the small-world network model. Phys. Rev. E, 60: 7332-7342.

Nishikawa, T., A.E. Motter, Y.C. Lai and F.C. Hoppensteadt, 2003. Heterogeneity in oscillator networks: Are smaller worlds easier to synchronize? Phys. Rev. Lett., Vol. 91.

Ozik, J., B.R. Hunt and E. Ott, 2004. Growing networks with geographical attachment preference: Emergence of small worlds. Phys. Rev. E, Vol. 69.

Penna, T.J.P., 1995. Traveling salesman problem and Tsallis statistics. Phys. Rev. Lett., Vol. 51

Pikovsky, A., M. Rosenblum and J. Kurths, 2001. Synchronization: A Universal Concept in Nonlinear Sciences. Cambridge University Press, Cambridge, MA., USA., ISBN-13: 9780521592857, Pages: 432.

Watts, D.J. and S.H. Strogatz, 1998. Collective dynamics of small-world networks. Nature, 393: 440-442.

Zhang, Z.Z., L.L. Rong and C.H. Guo, 2006. A deterministic small-world network created by edge iterations. Phys. A: Stat. Mech. Appl., 363: 567-572.

Zhao, M., T. Zhou, B.H. Wang, G. Yan, H.J. Yang and W.J. Bai, 2006. Relations between average distance, heterogeneity and network synchronizability. Phys. A: Stat. Mech. Appl., 371: 773-780.