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Monte-Carlo SURE for Image Reconstruction Based on TV Regularization

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Abstract: Many image reconstruction tasks, such as image denoising, deblurring, interpolation and super- resolution, are ill-posed inverse problems which can be solved by adding a regularization term. One of the well known regularization is the Total Variation (TV) regularization which is employed in this study. In addition, for the image reconstruction based on regularization, the tuning of regularization parameter is very difficult and nontrivial. In this study, a method for choosing the regularization parameter is proposed which combined the Stein's Unbiased Risk Estimate (SURE) and Monte-Carlo techniques. It only depends on the given data and the SURE can be used to replace the true Mean Squared Error (MSE) to obtain the optimal regularization parameter. The main contribution of the study is to extend the Monte-Carlo SURE method to determine the optimum TV regularization parameters in image deblurring, interpolation and super-resolution besides image denoising. Experimental results demonstrate the effectiveness and power of the proposed method.

Key words: Image reconstruction, total variation (TV) regularization, stein's unbiased risk estimate (SURE), monte-carlo

INTRODUCTION

In many image reconstruction applications, there is a need for solving an inverse problem, such as image denoising, image deblurring, image interpolation and image super-resolution etc. It is well known that these inverse problems are all ill-posed and must be regularized to get the meaningful and reasonable solution (Hansen and O'Leary, 1993; Molina *et al*., 1999). One of the popular regularization is the Total Variation (TV) regularization which uses the smooth prior knowledge measured by L1 norm of the image gradient (Goldstein and Osher, 2009; Aubert and Kornprobst, 2001). On the other hand, for the image reconstruction based on regularization, the tuning of regularization parameter is a very difficult and open problem (Hansen and O'Leary, 1993; Ramini *et al*., 2008; Gilboa *et al*., 2006).

In this study, the TV regularization is employed to solve the ill-posed image reconstruction problems and the Monte-Carlo and Stein's unbiased risk estimate (SURE) methods (Stein, 1981; Ramini *et al*., 2008; Elder, 2009; Giryes *et al*., 2011) are used to determine the optimum regularization parameter.

The study is organized as follows. In Section 2, we put forward the image degradation and the TV regularization models and discuss the iterative algorithm for image reconstruction. In Section 3, after a briefly introduction of the MSE and SURE theories, a Monte-Carlo SURE method is developed to find the optimal regularization parameter for the image reconstruction based on TV regularization. Experimental results are presented in Section 4 and conclusions are drawn in Section 5.

IMAGE DEGRADATION AND TV REGULARIZATION

Image degradation model: Suppose there is an observed degraded imageY which is generated from the original high quality image via following formula:

$$Y = HX+N \tag{1}$$

where, X represents the samples of the ideal unknown deterministic noise free image, N denotes the zero-mean white Gaussian noise with variance $C = \sigma^2 I$. H is the deterministic part of the degraded model which represents any kinds of distortion, blurring and down sampling in the process of image acquisition. Our goal is to find the estimation of ideal image X from the observation Y as correct as possible. Obviously, when H is an identity matrix I, it denotes image denoising; when H is a space-invariant blur matrix B, it denotes image deblurring; when H stands for a decimation operation D, it denotes image interpolation and when H = DB, it denotes super-resolution reconstruction.

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Total variation (TV) regularization: According to the degraded model, estimating X from Y is obviously an ill-posed problem, there exist an infinite number of solutions for the under-determined case or the solution for square and over-determined cases is not stable (Aubert and Kornprobst, 2001). Therefore, considering regularization in image reconstruction algorithm as a means for picking a stable solution is very useful. Also, regularization can help the algorithm to remove artifacts from the final solution and improve the rate of convergence. The general regularization model can be given by:

$$\hat{X} = \arg\min_{X}\{\|Y - HX\|_2^2 + \lambda R(X)\} \tag{2}$$

where, $\|Y - HX\|_2^2$ is the data fidelity term that measures the consistency of X to the given data, R(X) is a suitable regularization function that often penalizes a lack of smoothness in X. λ denotes regularization parameter which plays an important role in balancing the regularization item and the data item.

Essentially, regularization aims at finding a solution that not only fully fits the observed data but makes some kind of singularity minimal. There are three problems to be addressed. The first is designing the regularization model R(X); the second is finding the solution by an iterative algorithms; the last is selecting the regularization parameter λ. For the regularization model, unless noted otherwise, we adopt the well known robust regularizer:

$$R(X) = \|\nabla X\|_1 = \iint_{\Omega} |\nabla X|\,dx\,dy$$

which is called Total Variation (TV) and Ω is the image domain. So, we have:

$$\hat{X} = \arg\min_{X}\{\|Y - HX\|_2^2 + \lambda\|\nabla X\|_1\} \tag{3}$$

Certainly, other regularization methods are also available to find the solution. For the iterative algorithm for TV regularization, we applied the split Bregman method in this study which is an efficient algorithm proposed by Goldstein and Osher (2009) to solve the L1 regularization optimization problems. The split Bregman algorithm mainly contains two steps in each iteration such that:

$$U = X^{(n)} + ?H^T(Y - HX^{(n)}) \tag{4}$$

$$X^{(n+1)} = \arg\min_{X}\left\{\frac{1}{?}\|X - U\|_2^2 + \lambda\|\nabla X\|_1\right\} \tag{5}$$

where n is the iteration time. γ is a parameter which should be greater than the maximum eigenvalue of H^TH to ensure convergence. For the problem 5, it can be solved by using an efficient TV denoising method. In this study, we mainly talk about how to choose an optimal regularization parameter λ.

MONTE-CARLO SURE FOR CHOOSING THE REGULARIZATION PARAMETER

As stated in introduction, the optimization of regularization parameter is very important and nontrivial. Generally, the MSE of the signal estimate is the preferred measure of quality to optimize λ. Unfortunately, the MSE depends on the noise-free signal which is usually unavailable or unknown a priori. A practical approach, therefore, is needed to replace the true MSE by some estimate in the scheme of things. A theoretical result due to Stein makes this possible in the Gaussian scenario, that is the Stein's Unbiased Risk Estimate (SURE) which is a well-established technique in the statistical literature but not so widely known in signal processing (Stein, 1981; Blu and Luisier, 2007). SURE, as it is called, provides a means for unbiased estimation of the true MSE without ever requiring knowledge of the noise-free signal, solely depends on the given data. Moreover, the closeness of SURE to the true MSE is aided by the law of large numbers for large data size.

Stein's unbiased risk estimate: Here, we first introduce the MSE theory and then discuss the SURE technique. Suppose the pixels of the observed image and the original image are L and M, thus, the probability density of the observed image Y can be expressed as an exponential distribution:

$$f(Y|X) = b(Y)\exp\{X^T\varphi(Y) - g(X)\} \tag{6}$$

Where:

$$b(Y) = \frac{1}{\sqrt{(2\pi)^L \det(C)}}\exp\left\{-\frac{1}{2}Y^TC^{-1}Y\right\}$$

$$\varphi(Y) = H^TC^{-1}Y$$

$$g(X) = \frac{1}{2}X^TH^TC^{-1}HX$$

Obviously, u = φ(Y) is a sufficient statistics for estimating X. Any reasonable estimate of X will be a function only of u. On the other hand, we can see from

Eq. 4 and 5that the split Bregman iterative algorithm for the regularization model Eq. 3 only depends on the observed data Y via H^TY, that means it is a function only of sufficient statistics u. Let $\hat{X}_\lambda = h_?(u)$ be a split Bregman iterative solution with the regularization parameter λ, the MSE between $\hat{X}_\lambda$ and X is defined as:

$$\frac{1}{M}E\left\{\left\|X-\hat{X}_?\right\|_2^2\right\}=\frac{1}{M}E\{\|X-h_?(u)\|_2^2\} \tag{7}$$

The criteria of the regularization parameter selection is to make this MSE minimal. From the definition of MSE, we get:

$$E\left\{\left\|X-\hat{X}_?\right\|_2^2\right\}=\|X\|_2^2+v(h_\lambda,X) \tag{8}$$

where:

$$v(h_?,X)=E\left\{\left\|\hat{X}_?\right\|_2^2\right\}-2E\{\hat{X}_?^TX\}$$

The regularization parameter can be chosen by minimizing v(h_λ), X. The challenge is to compute $E\{h_\lambda^T(u)X\}=E\{\hat{X}_\lambda^TX\}$ because X is unknown. We try to construct an unbiased estimation g(h_λ(u)) to replace $E\{h_?^T(u)X\}$, that is $E\{g(h_?(u))\}=E\{h_?^T(u)X\}$ then we have $v(h_?,X)=E\{\|h_?(u)\|_2^2\}-2E\{g(h_?(u))\}$. So, the unbiased risk estimation of MSE is $\|X\|_2^2+\|h_?(u)\|_2^2-2g(h_?(u))$ the optimal parameter is chosen to minimize $\|h_?(u)\|_2^2-2g(h_?(u))$.

According to the work of (Elde, 2009), when H is full rank, the unbiased risk estimation of the MSE can be represented by:

$$S(h_\lambda(u))=\|X\|_2^2+\|h_\lambda(u)\|_2^2+2\mathrm{Tr}\left(\frac{\partial h_\lambda^T(u)}{\partial u}\right)-2h_\lambda^T(u)\hat{X}_{ML} \tag{9}$$

where, $\hat{X}_{ML}=(H^TC^{-1}H)^{-1}H^TC^{-1}Y$ denotes the maximum likelihood estimate. So the optimal parameter λ can be chosen by minimizing (9). When H is rank-deficient, the sufficient statistics u = HTCG1Y lies in the range space $\Re(H^T)$ of H^T, so $\hat{X}_? = h_?(u)$ also belongs to this space. Denote by P = H^T (HH^T),H the orthogonal projection on the space $\Re(H^T)$, the regularization parameter can be decided by minimizing:

$$E\left\{\left\|PX-P\hat{X}_\lambda\right\|_2^2\right\}$$

Similar to the derivation in (Elder, 2009), the SURE estimate of:

$$E\left\{\left\|PX-P\hat{X}_\lambda\right\|_2^2\right\}$$

can be expressed by:

$$\begin{aligned}S(h_?(u))=&\|PX\|_2^2+\|Ph_?(u)\|_2^2\\&+2\mathrm{Tr}\left(P\frac{\partial h_?^T(u)}{\partial u}\right)-2h_?^T(u)\hat{X}_{ML}\end{aligned} \tag{10}$$

where, $\hat{X}_{ML}=(H^TC^{-1}H)^{+}H^TC^{-1}Y$ denotes the maximum likelihood estimate and $(\cdot)^+$ is the pseudo inverse of a matrix. So the optimal parameter λ can be chosen by minimizing (10) when H is rank deficient.

Monte-Carlo estimation of SURE: According to Eq. 9 and 10, the trace of:

$$\mathrm{Tr}\left(\frac{\partial h_?^T(u)}{\partial u}\right)$$

or:

$$\mathrm{Tr}\left(P\frac{\partial h_?^T(u)}{\partial u}\right)$$

must be calculated when evaluating the SURE formulation. However, calculating these traces present a bigger difficulty because h_λ(u)is not available explicitly for most reconstruction algorithms. Here the Monte-Carlo method is employed to overcome this difficulty (Ramini *et al*., 2008) which can be expressed by:

$$\mathrm{Tr}\left(P\frac{\partial h_?^T(u)}{\partial u}\right)=\lim_{e\to 0}E_b\left\{b^T\left(\frac{Ph_?(u+eb)-Ph_?(u)}{e}\right)\right\}$$

where:

$$\mathrm{Tr}\left(\frac{\partial h_?^T(u)}{\partial u}\right)=\lim_{e\to 0}E_b\left\{b^T\left(\frac{h_?(u+eb)-h_?(u)}{e}\right)\right\}$$

$b\in R^N$ is a zero-mean i.i.d. random vector with unit variance and bounded higher order moment. When P = I, we have:

$$\mathrm{Tr}\left(\frac{\partial h_?^T(u)}{\partial u}\right)=\lim_{e\to 0}E_b\left\{b^T\left(\frac{h_?(u+eb)-h_?(u)}{e}\right)\right\}$$

Therefore, the overall algorithm for image reconstruction can be summarized as two steps: (1) Use the Monte-Carlo SURE algorithm to find the optimal regularization parameter. (2) Apply the split Bregman iterative algorithm to solve the TV-based image reconstruction.

EXPERIMENTAL RESULTS

We now present some numerical results for Monte-Carlo SURE-based optimization in image denoising, deblurring, interpolation and super-resolution based on TV regularization. The standard test images of size 256×256 (Cameraman, Lena, Baboon, Boats, Peppers and Elaine) have been chosen for simulations. The original images are blurred by [5×5] Gaussian kernel with standard deviation 0.8, decimated using 2:1 decimation ratio on each axis and added by zero mean Gaussian white noise with standard deviation 0.05, where the original images have been normalized to [0,1]. In all simulations, we choose the periodic boundary condition and apply the FFT to calculate the convolution and its transpose. The performance of reconstruction results are quantified by the Improved Signal-to-noise Ratio (ISNR) of $\hat{X}_\lambda$ compared to the reference image X_0. It is noted that the reference image X_0 is the degraded image Yin the case of image denoising and deblurring and is the bicubic interpolation image of Ywhen we do the image interpolation and super-resolution.

Regularization parameter and ISNR comparisons: Here we compared the optimal regularization parameter chosen based on MSE and MC-SURE and their corresponding ISNR. The results are shown in Table 1-4. The second column shows a comparison of the optimal regularization parameters, the first value in each cell means the optimal λ chosen based on true MSE (oracle value) while the second one means the optimal λ chosen based on MC-SURE. It demonstrates that in all cases the optimum regularization parameter obtained by MC-SURE optimization is almost perfect agreement with the oracle solution (minimum MSE). In the third column, we provide the corresponding ISNR with the optimum regularization parameters. It is noted that, in Table 3 and 4, the ISNR values are compared between the bicubic interpolation and the TV regularization reconstruction results. We can again observe that our method has greatly improvement in terms of ISNR in all cases.

Visual comparisons: To further validate the effectiveness of our method in different cases, we compare the results visually. Figure 1-4 show the image reconstruction results for different test images based on TV regularization with the optimum regularization parameters chosen by MC-SURE method. Figure 1 is the image denoising for

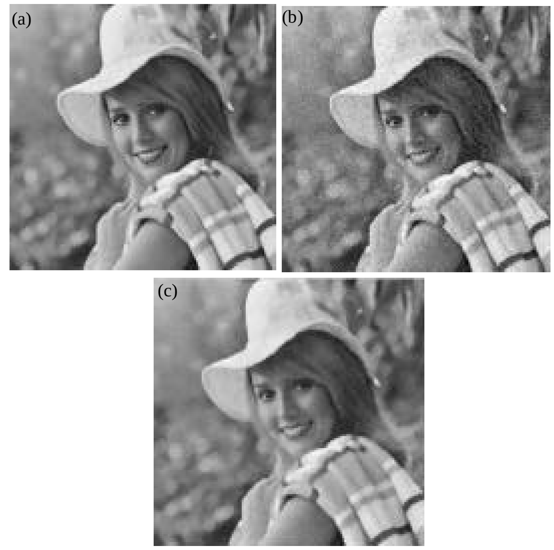

Fig. 1(a-c): Image denoising for Elaine, (a) Original image, (b) Degraded image and (c) TV denoising image

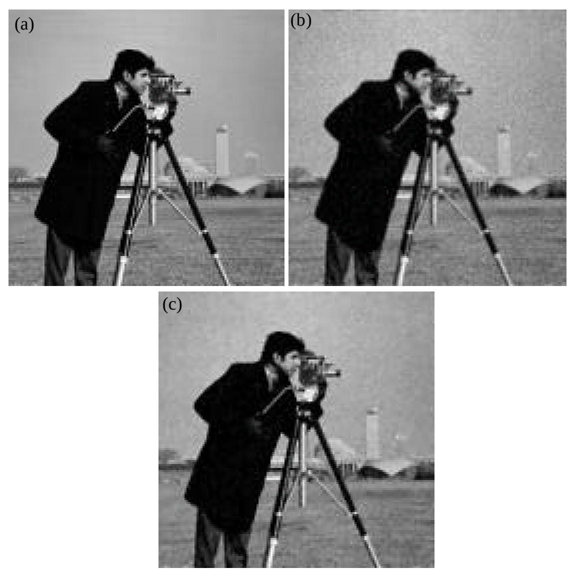

Fig. 2(a-c): Image deblurring for Cameraman. (a) Original image, (b) Degraded image and (c) TV deblurring image

Table 1: Comparisons of image denoising in terms of regularization parameter and ISNR

Image	Regularization parameter	ISNR(dB)
Cameraman	(0.06, 0.06)	(4.915, 4.915)
Lena	(0.08, 0.06)	(5.702, 5.696)
Baboon	(0.04, 0.04)	(2.070, 2.070)
Boats	(0.06, 0.06)	(4.344, 4.344)
Peppers	(0.08, 0.08)	(6.418, 6.418)
Elaine	(0.08, 0.08)	(6.798, 6.798)

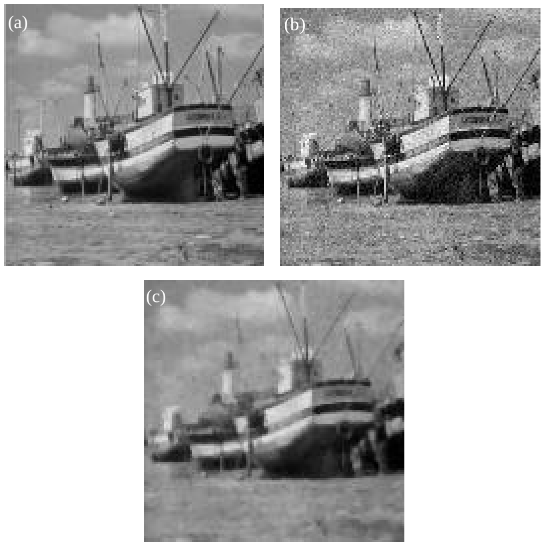

Fig. 3(a-c): Image interpolation for Boats. (a) Original image, (b) Degraded image and (c) TV interpolation image.

Fig. 4(a-b): Image super-resolution for Lena. (a) Original image, (b) Degraded image, (c) TV super-resolution image

Elaine; Fig. 2 is the image deblurring for Cameraman; Fig. 3 is the image interpolation for Boats and Fig. 4 is the image super-resolution for Lena. Fig. 1a, 2a, 3a and 4a are the original images; Fig. 1b, 2b, 3b and 4b are the degraded images and Fig. 1c, 2c, 3c and 4c are the reconstructed images with the optimum TV regularization parameters chosen by MC-SURE optimization. From Fig. 1-4, we can observe that in all cases the proposed algorithm plays a good effect. The reconstructed results are closer to the original images rather than the degraded ones in visual inspection.

Table 2: Comparisons of image deblurring in terms of regularization parameter and ISNR

Image	Regularization parameter	ISNR(dB)
Cameraman	(0.025, 0.025)	(3.266, 3.266)
Lena	(0.04, 0.0350)	(4.531, 4.528)
Baboon	(0.02, 0.0200)	(1.139, 1.139)
Boats	(0.03, 0.0300)	(3.063, 3.063)
Peppers	(0.048, 0.048)	(5.408, 5.408)
Elaine	(0.05 0.0400)	(6.248, 6.134)

Table 3: Comparisons of image interpolation in terms of regularization parameter and ISNR

Image	Regularization parameter	ISNR(dB)
Cameraman	(0.04, 0.04)	(1.531, 1.531)
Lena	(0.04, 0.04)	(2.283, 2.283)
Baboon	(0.02, 0.02)	(1.423, 1.423)
Boats	(0.03, 0.04)	(1.442, 1.363)
Peppers	(0.04, 0.05)	(2.280, 2.144)
Elaine	(0.04, 0.04)	(1.627, 1.627)

Table 4: Comparisons of image super-resolution in terms of regularization parameter and ISNR

Image	Regularization parameter	ISNR(dB)
Cameraman	(0.02, 0.0200)	(2.236, 2.236)
Lena	(0.025, 0.020)	(2.847, 2.838)
Baboon	(0.015, 0.015)	(0.682, 0.682)
Boats	(0.02, 0.0200)	(1.780, 1.730)
Peppers	(0.025, 0.025)	(3.453, 3.453)
Elaine	(0.025, 0.025)	(3.653, 3.653)

CONCLUSION

Some image reconstruction inverse problems, including denoising, deblurring, interpolation and super-resolution, are involved in this study. The Total Variation (TV) regularization is employed to solve these ill-posed reconstruction problems. In addition, the MC-SURE has been developed to determine the optimum regularization parameter. Experimental results demonstrate the applicability of the MC-SURE technique to these ill-posed image reconstruction problems.

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