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Multiple Criteria Decision Making Method Based on Triangular Intuitionistic Fuzzy Information

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Abstract: Triangular Intuitionistic Fuzzy Numbers (TIFNs) is a generalization of the intuitionistic fuzzy numbers which are more suitable to represent uncertainties than intuitionistic fuzzy sets. this study proposes an approach for solving fuzzy multiple criteria decision making (MCDM) problems, where evaluation values of alternatives on considered criteria are TIFNs. We first introduce the Hamming distance between two TIFNs, some operational laws of TIFNs and its properties. Based on the proposed operational laws of TIFNs, we present the Triangular Intuitionistic Fuzzy Weighed Geometric (TIFWG) operator. Then, a new approach for handling fuzzy Multiple Criteria Decision Making (MCDM) problems based on triangular intuitionistic fuzzy information is proposed.

Key words: Multiple criteria decision making; fuzzy number; triangular intuitionistic fuzzy number; aggregation operator

INTRODUCTION

Multiple Criteria Decision Making (MCDM) problems are widely spread in real life decision situation (Park *et el.*, 2011). MCDM handles decision situations where a set of alternatives has to be assessed against multiple attributes or criteria before a final choice is selected (Hwang and Yoon, 1981). The purpose of MCDA is to find a best alternative among a set of feasible alternatives from a set of alternatives by means of evaluating multiple attributes of the alternatives (Behnam Vahdani *et al.*, 2013). Thus, MCDM was described as the most well known branch of decision making (Triantaphyllou, 2000). Due to ambiguity and incomplete information in many decision problems, it is often difficult for a Decision-Maker (DM) to give his/her assessments on attribute values and weights in crisp values. Instead, it has become increasingly common that these assessments are provided as Fuzzy Numbers (FNs) or Intuitionistic Fuzzy Numbers (IFNs), leading to a rapidly expanding body of literature on MADM under the fuzzy or intuitionistic fuzzy framework (Wang *et al.*, 2011).

Since, Zadeh (1965) introduced fuzzy set theory, many research achievements have been made to enrich the fuzzy set theory (K.T. Atanassov, 1986; Wu and Mendel, 2007). Owing to the advantage of dealing with uncertain information, Fuzzy sets (FNs) has been proven to be a very suitable tool to be used to describe the imprecise or uncertain decision information. Many theories and methods have been presented for handling fuzzy multiple attributes decision-making problems based on Fns. Tan and Zhang (2006) presented a novel method for multiple attribute decision making based on IVIFS and TOPSIS method in uncertain environment. Lin and Wu (2008) presented a causal analytical method for group decision-making under fuzzy environment. Xu (2007) developed some geometric aggregation operators and gave an application of the operators to multiple criteria group decision making with interval-valued intuitionistic fuzzy information. Mitchell (2004) interpreted an IFN as an ensemble of fuzzy numbers and introduced a ranking method. Ye (2010) investigated interval-valued intuitionistic fuzzy multi-criteria group decision making based on the extended TOPSIS. Chen *et al.* (2011) studied the multi-criteria group decision-making approach under interval-valued intuitionistic fuzzy environment. Xia and Xu (2013) introduced Some basic operations based on intuitionistic multiplicative aggregation operators. Yu *et al.* (2012) studied the IVIF prioritized operators and their application in group decision making.

The above fuzzy multiple criteria decision making methods are based on intuitionistic fuzzy sets. As a generalization of the intuitionistic fuzzy numbers, Triangular Intuitionistic Fuzzy Numbers (TIFNs) are more suitable to represent uncertainties than intuitionistic fuzzy sets. In this study, we extend the TOPSIS method to propose a new method for handling fuzzy multiple criteria decision making problems based on triangular intuitionistic fuzzy numbers.

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PRELIMIDARIES

In this selection, the definitions of Intuitionistic Fuzzy Sets (IFSs) and the Triangular Intuitionistic Fuzzy Numbers (TIFNs) are introduced(Atanassov, 1999; Szmidt and Kacprzyk, 2000; Li, 2010; Wu and Cao, 2013).

Definition 1: An intuitionistic fuzzy sets, A, in the universe of discourse X is defined with the form [21]:

$$A=\{<x,\mu_A(x),\nu_A(x)>|x\in X\}$$

where $\mu_A x$:→ [0,1], $v_A x$→: [0,1] and with the condition:

$$0\le\mu_A(x)+\nu_A(x)\le 1,\ \forall x\in X.$$

The values μ_A (x) and v_A (x) denote membership and non-membership degree of x with respect to A, respectively. In addition, we call $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ the degree of hesitancy of x with respect toA.

Especially, if π_A (x) μ_A (x) = 1,v_A (x) for all χ∈×, then the IFSs A is reduced to a fuzzy set.

Definition 2: A triangular intuitionistic fuzzy number $\tilde{\alpha}=<(a,b,c);u_{\tilde{\alpha}},v_{\tilde{\alpha}}>$ is a special intuitionistic fuzzy set on a real number set *R*, where membership function $\mu_{\tilde{\alpha}}(x)$ and non-membership function $v_{\tilde{\alpha}}(x)$ are defined as follows:

$$\mu_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{x-a}{b-a}u_{\tilde{\alpha}}, & \text{if } a\le x<b,\\ u_{\tilde{\alpha}} & ,\ \text{if } x=b,\\ \dfrac{c-x}{c-b}u_{\tilde{\alpha}}, & \text{if } b<x\le c,\\ 0 & ,\ \text{otherwise },\end{cases}$$

$$v_{\tilde{\alpha}}(x)=\begin{cases}\dfrac{(b-x)+(x-a)v_{\tilde{\alpha}}}{b-a}, & \text{if } a\le x<b,\\ v_{\tilde{\alpha}} & ,\ \text{if } x=b,\\ \dfrac{(x-b)+(c-x)v_{\tilde{\alpha}}}{c-b}, & \text{if } b<x\le c,\\ 1 & ,\ \text{otherwise },\end{cases}$$

The numbers u $\dot{\alpha}$ and v$\dot{\alpha}$ represent the maximum membership degree and minimum membership degree, respectively. They satisfy the conditions: $0\le u_{\tilde{\alpha}}\le 1$, $0\le v_{\tilde{\alpha}}\le 1$, and $u_{\tilde{\alpha}}+v_{\tilde{\alpha}}\le 1$ Let $\pi_{\tilde{\alpha}}(x)=1-u_{\tilde{\alpha}}(x)-\nu_{\tilde{\alpha}}(x)$ which is called an intuitionistic fuzzy index of an element x in$\dot{\alpha}$. The smaller of π$\dot{\alpha}$ (x) is, the clear of the fuzzy number is.

It is easily seem that $u_{\tilde{\alpha}}(x)+v_{\tilde{\alpha}}(x)=1$ for any χ∈*R* if u$\hat{\alpha}$ and v$\hat{\alpha}$ = 0. Therefore, the TIFN $\tilde{\alpha}=<(a,b,c);u_{\tilde{\alpha}},v_{\tilde{\alpha}}>$ degenerates to $\tilde{\alpha}=<(a,b,c);1,0>$ which is just about triangular fuzzy number. Hence, the concept of the TIFN is a generalization of that of the triangular fuzzy number.

Definition 3: Let, $\tilde{\alpha}_1=<(a_1,b_1,c_1);u_{\tilde{\alpha}_1},v_{\tilde{\alpha}_1}>$, $\tilde{\alpha}_2=<(a_2,b_2,c_2);u_{\tilde{\alpha}_2},v_{\tilde{\alpha}_2}>$ be two TIFNs, the Hamming distance between them are defined as follows:

$$d(\tilde{\alpha}_1,\tilde{\alpha}_2)=\frac{1}{3}(|a_1-a_2|+|b_1-b_2|+|c_1-c_2|)$$

$$+\max(|u_{\tilde{\alpha}_1}-u_{\tilde{\alpha}_2}|,|v_{\tilde{\alpha}_1}-v_{\tilde{\alpha}_2}|)$$

If, $u_{\tilde{\alpha}_1}=u_{\tilde{\alpha}_2}=1$ and $v_{\tilde{\alpha}_1}=v_{\tilde{\alpha}_2}=0$ $\tilde{\alpha}_2=<(a_2,b_2,c_2);u_{\tilde{\alpha}_2},v_{\tilde{\alpha}_2}>$ then TIFNs and $\tilde{\alpha}_2=<(a_2,b_2,c_2);u_{\tilde{\alpha}_2},v_{\tilde{\alpha}_2}>$, respectively degenerate to $\tilde{\alpha}_1=<(a_1,b_1,c_1);u_{\tilde{\alpha}_1},v_{\tilde{\alpha}_1}>$, $\tilde{\alpha}_2=<(a_2,b_2,c_2);1,0>$ and $\tilde{\alpha}_1=<(a_1,b_1,c_1);1,0>$ which are just triangular fuzzy numbers.

Theorem1: The Hamming distance of TIFNs have the properties as follows:

- (Non-negativity) $d(\tilde{\alpha}_1,\tilde{\alpha}_2)\ge 0$
- (Symmetry) $d(\tilde{\alpha}_1,\tilde{\alpha}_2)=d(\tilde{\alpha}_2,\tilde{\alpha}_1)$
- (Triangular inequality)If $\tilde{\alpha}_3=<(a_3,b_3,c_3);u_{\tilde{\alpha}_3},v_{\tilde{\alpha}_3}>$ is any TIFN, then $d(\tilde{\alpha}_1,\tilde{\alpha}_3)\le\ d(\tilde{\alpha}_1,\tilde{\alpha}_2)+d(\tilde{a}_2+\tilde{a}_3)$

Proof: According to Definition1, it's clear that the Hamming distance $d(\tilde{\alpha}_1,\tilde{\alpha}_2)$ meet non-negativity and symmetry properties. Therefore, only Triangular inequality property need to proved.

Since:

$$|a_1-a_2|+|a_2-a_3|\ge|a_1-a_3|$$

$$|b_1-b_2|+|b_2-b_3|\ge|b_1-b_3|$$

$$|c_1-c_2|+|c_2-c_3|\ge|c_1-c_3|$$

We have:

$$|a_1-a_3|+|b_1-b_3|+|c_1-c_3|\le$$

$$(|a_1-a_2|+|b_1-b_2|+|c_1-c_2|)+(|a_2-a_3|+|b_2-b_3|+|c_2-c_3|)$$

In addition:

$$\left|u_{\tilde{\alpha}_1} - u_{\tilde{\alpha}_2}\right| + \left|u_{\tilde{\alpha}_2} - u_{\tilde{\alpha}_3}\right| \geq \left|u_{\tilde{\alpha}_1} - u_{\tilde{\alpha}_3}\right|$$

$$\left|v_{\tilde{\alpha}_1} - v_{\tilde{\alpha}_2}\right| + \left|v_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_3}\right| \geq \left|v_{\tilde{\alpha}_1} - v_{\tilde{\alpha}_3}\right|$$

We get:

$$\max\left\{\left|u_{\tilde{\alpha}_1} - u_{\tilde{\alpha}_2}\right|, \left|v_{\tilde{\alpha}_1} - v_{\tilde{\alpha}_2}\right|\right\} + \max\left\{\left|u_{\tilde{\alpha}_2} - u_{\tilde{\alpha}_3}\right| + \left|v_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_3}\right|\right\}$$

$$\geq \max\left\{\left|u_{\tilde{\alpha}_1} - u_{\tilde{\alpha}_3}\right|, \left|v_{\tilde{\alpha}_1} - v_{\tilde{\alpha}_3}\right|\right\}$$

Hence:

$$d(\tilde{\alpha}_1, \tilde{\alpha}_3) \leq d(\tilde{\alpha}_1, \tilde{\alpha}_2) + d(\tilde{a}_2 + \tilde{a}_3)$$

OPERATION LAWS AND GEOMETRIC AGGREGATION OPERATORS FOR TIFNS

Definition 4: Let:

$$d(\tilde{\alpha}_1, \tilde{\alpha}_3) \leq d(\tilde{\alpha}_1, \tilde{\alpha}_2) + d(\tilde{a}_2 + \tilde{a}_3)$$

$$d(\tilde{\alpha}_1, \tilde{\alpha}_3) \leq d(\tilde{\alpha}_1, \tilde{\alpha}_2) + d(\tilde{a}_2 + \tilde{a}_3)$$

Be two TIFNs and:

$$\|\tilde{\alpha}_1\| = \frac{|a_1| + 2|b_1| + |c_1|}{4}$$

$$\|\tilde{\alpha}_2\| = \frac{|a_2| + 2|b_2| + |c_2|}{4}$$

$$\|\tilde{\alpha}_1\| \neq 0, \|\tilde{\alpha}_2\| \neq 0$$

Then:

- $\tilde{\alpha}_1 + \tilde{\alpha}_2 =$

$$< (a_1 + a_2, b_1 + b_2, c_1 + c_2); \frac{\|\tilde{\alpha}_1\| u_{\tilde{\alpha}} + \|\tilde{\alpha}_2\| u_{\tilde{\alpha}_2}}{\|\tilde{\alpha}_1\| + \|\tilde{\alpha}_2\|}, \frac{\|\tilde{\alpha}_1\| v_{\tilde{\alpha}_1} + \|\tilde{\alpha}_2\| v_{\tilde{\alpha}_2}}{\|\tilde{\alpha}_1\| + \|\tilde{\alpha}_2\|} >$$

- $\tilde{\alpha}_1 - \tilde{\alpha}_2 =$

$$< (a_1 - a_2, b_1 - b_2, c_1 - c_2); \frac{\|\tilde{\alpha}_1\| u_{\tilde{\alpha}} + \|\tilde{\alpha}_2\| u_{\tilde{\alpha}_2}}{\|\tilde{\alpha}_1\| + \|\tilde{\alpha}_2\|}, \frac{\|\tilde{\alpha}_1\| v_{\tilde{\alpha}_1} + \|\tilde{\alpha}_2\| v_{\tilde{\alpha}_2}}{\|\tilde{\alpha}_1\| + \|\tilde{\alpha}_2\|} >$$

- $\lambda\tilde{\alpha}_1 =< (\lambda a_1, \lambda b_1, \lambda c_1); u_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_1} >$
- $\tilde{\alpha}_1 \times \tilde{\alpha}_2 =< (a_1 a_2, b_1 b_2, c_1 c_2); u_{\tilde{\alpha}_1} u_{\tilde{\alpha}_2}, v_{\tilde{\alpha}_1} + v_{\tilde{\alpha}_2} - v_{\tilde{\alpha}_1} v_{\tilde{\alpha}_2} >$
- $\tilde{\alpha}_1{}^{\lambda} =< (a_1{}^{\lambda}, b_1{}^{\lambda}, c_1{}^{\lambda}); u_{\tilde{\alpha}_1}{}^{\lambda}, 1 - (1 - v_{\tilde{\alpha}_1})^{\lambda} >$

From definition 4, the following properties are proven:

- $\tilde{\alpha}_1 + \tilde{\alpha}_2 = \tilde{\alpha}_2 + \tilde{\alpha}_1$
- $\tilde{\alpha}_1 \times \tilde{\alpha}_2 = \tilde{\alpha}_2 \times \tilde{\alpha}_1$
- $\lambda(\tilde{\alpha}_1 + \tilde{\alpha}_2) = \lambda\tilde{\alpha}_1 + \lambda\tilde{\alpha}_2 \quad (\lambda \geq 0)$
- $\lambda_1\tilde{\alpha}_1 + \lambda_2\tilde{\alpha}_1 = (\lambda_1 + \lambda_2)\tilde{\alpha}_1 \quad (\lambda_1 \geq 0, \lambda_2 \geq 0)$
- $\tilde{\alpha}_1{}^{\lambda_1}\tilde{\alpha}_1{}^{\lambda_2} = \tilde{\alpha}_1{}^{\lambda_1 + \lambda_2} \quad (\lambda_1 \geq 0, \lambda_2 \geq 0)$
- $(\tilde{\alpha}_1{}^{\lambda_1})^{\lambda_2} = \tilde{\alpha}_1{}^{\lambda_1\lambda_2} \quad (\lambda_1 \geq 0, \lambda_2 \geq 0)$

Definition 5: [26] Let $\tilde{\alpha}_j \ (j = 1, 2, \cdots, n)$ be a collection of triangular intuitionistic fuzzy numbers and let TIFWG: $\Omega_n \rightarrow \Omega$ if:

$$\text{TIFWG}_{\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = \tilde{\alpha}_1{}^{\omega_1} \times \tilde{\alpha}_2{}^{\omega_2} \times \cdots \times \tilde{\alpha}_n{}^{\omega_n} \quad (5)$$

Then TIFWG is called triangular intuitionistic fuzzy weighed geometric operator of dimension *n* where $\omega = (\omega_1\ \omega_2 \ldots \omega_n)$ is the weight vector of $\tilde{\alpha}_j \ (j = 1, 2, \cdots, n)$ with $\omega \in [0,1]$ and:

$$\sum_{j=1}^{n} \omega_j = 1$$

Especially, if:

$$\omega = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)^T$$

Then the TIFWG operator is reduced to a Triangular Intuitionistic Fuzzy Geometric Averaging (TIFGA) operator of dimension *n* which is defined as follows:

$$\text{TIFGA}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = (\tilde{\alpha}_1 \times \tilde{\alpha}_2 \times \cdots \times \tilde{\alpha}_n)^{\frac{1}{n}} \quad (6)$$

Theorem 2: Let $\tilde{\alpha}_j$ (j = 1, 2...n) be a collection of triangular intuitionistic fuzzy numbers, then the aggregated value by using TIFWG operator is also a triangular intuitionistic fuzzy number and:

$$TIFWG_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_n)=\prod_{j=1}^{n}\tilde{\alpha}_j^{\omega_j} \quad (7)$$

$$=<(\prod_{j=1}^{n}a_j^{\omega_j},\prod_{j=1}^{n}b_j^{\omega_j},\prod_{j=1}^{n}c_j^{\omega_j});\prod_{j=1}^{n}u_{\tilde{\alpha}_j}^{\omega_j},1-\prod_{j=1}^{n}(1-v_{\tilde{\alpha}_j})^{\omega_j}>$$

From definition 5, we can see that the aggregated value TIFWGω ($\tilde{\alpha}_1$, $\tilde{\alpha}_2$,...$\tilde{\alpha}_n$) is also a TIFN. In the following, we prove Eq. 7 by using mathematical induction on n.

We first prove that Eq. 7 holds for *n* = 2. Since:

$$\tilde{\alpha}_1^{\omega_1}=<(a_1^{\omega_1},b_1^{\omega_1},c_1^{\omega_1});u_{\tilde{\alpha}_1}^{\omega_1},1-(1-v_{\tilde{\alpha}_1})^{\omega_2}>$$

$$\tilde{\alpha}_2^{\omega_2}=<(a_2^{\omega_2},b_2^{\omega_2},c_2^{\omega_2});u_{\tilde{\alpha}_2}^{\omega_2},1-(1-v_{\tilde{\alpha}_2})^{\omega_2}>$$

Then, we have:

$$<(a_1^{\omega_1}a_2^{\omega_2},b_1^{\omega_1}b_2^{\omega_2},c_1^{\omega_1}c_2^{\omega_2});u_{\tilde{\alpha}_1}^{\omega_1}u_{\tilde{\alpha}_2}^{\omega_2},1-(1-v_{\tilde{\alpha}_1})^{\omega_1}$$

$$+1-(1-v_{\tilde{\alpha}_2})^{\omega_2}-(1-(1-v_{\tilde{\alpha}_1})^{\omega_1})(1-(1-v_{\tilde{\alpha}_2})^{\omega_2})>$$

$$=<(a_1^{\omega_1}a_2^{\omega_2},b_1^{\omega_1}b_2^{\omega_2},c_1^{\omega_1}c_2^{\omega_2});u_{\tilde{\alpha}_1}^{\omega_1}u_{\tilde{\alpha}_2}^{\omega_2},$$

$$1-(1-v_{\tilde{\alpha}_1})^{\omega_1}(1-v_{\tilde{\alpha}_2})^{\omega_2}>$$

If Eq. 7 holds for *n* = *k*, that is:

$$TIFWG_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_k)=$$
$$<(\prod_{j=1}^{k}a_j^{\omega_j},\prod_{j=1}^{k}b_j^{\omega_j},\prod_{j=1}^{k}c_j^{\omega_j});\prod_{j=1}^{k}u_{\alpha_j}^{\omega_j},1-\prod_{j=1}^{k}(1-v_{\tilde{\alpha}_j})^{\omega_j}>$$

According to Definition 4, we obtain:

$$TIFWG_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_k,\tilde{\alpha}_{k+1})=$$
$$<(\prod_{j=1}^{k+1}a_j^{\omega_j},\prod_{j=1}^{k+1}b_j^{\omega_j},\prod_{j=1}^{k+1}c_j^{\omega_j});\prod_{j=1}^{k+1}u_{\tilde{\alpha}_j}^{\omega_j},1-\prod_{j=1}^{k+1}(1-v_{\tilde{\alpha}_j})^{\omega_j}>$$

i.e., Eq. 7 holds for *n* = *k*+1. Hence, Eq. 7 holds for all *n* which completes the proof of theorem.

MCDM MODEL USING TIFNS

On the basis of the above analysis, we extend the classic TOPSIS method for solving the Multiple Criteria Decision Making (MCDM) problems with triangular intuitionistic fuzzy information.

Let A = {A_1, A_2, ... A_m} be the set of alternative, D = {d_1, d_2, ... d_t} be the set of decision maker, C = {c_1, c_2, ... c_n} be the set of criteria and ω (ω_1, ω_2, ...ω_n) be the weight vector of the criteria where $\omega_j \ge 0$ $\sum_{j=1}^{n}\omega_j=1$

Let:

$$\tilde{R}=(\tilde{r}_{ij})_{m\times n}=(<(a_{ij},b_{ij},c_{ij});u_{ij},v_{ij}>)_{m\times n}$$

be a triangular intuitionistic fuzzy decision matrix, where u_{ij} denotes the extent to which alternative A_i belongs to triangular intuitionistic fuzzy number $\tilde{r}_{ij}=<(a_{ij},b_{ij},c_{ij});u_{ij},v_{ij}>$ on the criteria c_j, v_{ij} denotes the extent to which alternative A_i does not belongs to triangular intuitionistic fuzzy number

$$\tilde{r}_{ij}=<(a_{ij},b_{ij},c_{ij});u_{ij},v_{ij}>$$

on the criteria c_j, with the conditions: $0\le u_{ij}\le 1$, $0\le v_{ij}\le 1$ and $u_{ij}+v_{ij}\le 1$.

The approach to resolve multiple criteria decision making problems with triangular intuitionistic fuzzy information mainly involves the following steps:

Step 1: Pool the decision maker's opinions to get appropriate TIFNs of alternative A_i∈ on criteria c_j∈C and construct the triangular intuitionistic fuzzy decision making matrix:

$$\tilde{R}^{(k)}=(\tilde{r}_{ij}^{(k)})_{m\times n},\ (k=1,2,\cdots,t)$$

The average rating value of the ith alternative A_i, with respect to the jth criteria, c_j, for t decision makers can be obtain as follows:

$$\tilde{R}'=(\tilde{r}_{ij}')_{m\times n}=(<(a_{ij}',b_{ij}',c_{ij}');u_{ij}',v_{ij}'>)_{m\times n}$$
$$=\frac{1}{t}(\tilde{r}_{ij}^{(1)}+\tilde{r}_{ij}^{(2)}+\cdots+\tilde{r}_{ij}^{(t)}) \quad (8)$$

Step 2: Standardize the triangular intuitionistic fuzzy decision making matrix, then the standardized methods are show as follows:

- For benefit type of criteria:

$$\tilde{r}_{ij}=<(a_{ij},b_{ij},c_{ij});u_{ij},v_{ij}>=<(\frac{a_{ij}'}{c_j^+},\frac{b_{ij}'}{c_j^+},\frac{c_{ij}'}{c_j^+});u_{ij},v_{ij}> \quad (9)$$

- For cost type of criteria:

$$\tilde{r}_{ij} = <(a_{ij}, b_{ij}, c_{ij}); u_{ij}, v_{ij}> = <(1-\frac{a'_{ij}}{c_j^+}, 1-\frac{b'_{ij}}{c_j^+}, 1-\frac{c'_{ij}}{c_j^+}); u_{ij}, v_{ij}> \quad (10)$$

Where:

$$c_j^+ = \max_i \{c_{ij} | i = 1, 2, \cdots, m\}, \quad (j = 1, 2, \cdots, n)$$

Step 3: Calculate the value of each alternative A_i using TIFWG operator:

$$\tilde{r}_i = <(a_i, b_i, c_i); u_i, v_i> = \prod_{j=1}^{n} \tilde{r}_{ij}^{\omega_j}$$
$$= <(\prod_{j=1}^{n} a_{ij}^{\omega_j}, \prod_{j=1}^{n} b_{ij}^{\omega_j}, \prod_{j=1}^{n} c_{ij}^{\omega_j}); \prod_{j=1}^{n} u_{ij}^{\omega_j}, 1-\prod_{j=1}^{n} (1-v_{ij})^{\omega_j}>$$

Step 4: Define the positive ideal solution and negative ideal solution

For a standardized triangular intuitionistic fuzzy decision making matrix, the triangular Intuitionistic Fuzzy Positive Ideal Solution (TIFPIS) $\tilde{r}^+$ and the Triangular Intuitionistic Fuzzy Negative Ideal Solution (TIFNIS) $\tilde{r}^-$ are defined as:

$$\tilde{r}^+ = (\tilde{r}_1^+, \tilde{r}_2^+, \cdots, \tilde{r}_n^+)$$

and:

$$\tilde{r}^- = (\tilde{r}_1^-, \tilde{r}_2^-, \cdots, \tilde{r}_n)$$

Where:

$$\tilde{r}_j^+ = <(\max_i\{a_i\}, \max_i\{b_i\}, \max_i\{c_i\}; \max_i\{\mu_i\}, \min_i\{v_i\}>$$

$$\tilde{r}_j^- = <(\min_i\{a_i\}, \min_i\{b_i\}, \min_i\{c_i\}; \min_i\{\mu_i\}, \max_i\{v_i\}>$$

According to the standardized triangular intuitionistic fuzzy decision making matrix, we knew that $\tilde{r}^+ = <(1,1,1); 1, 0>$, $\tilde{r}^- = <(0,0,0); 0, 1>$.

Step 5: Calculate the distance between the alternative and the TIFPIS ($\tilde{r}^+$) and the distance between the alternative and the TIFNIS ($\tilde{r}^-$) according to Hamming distance:

$$d_i^+ = \frac{1}{3}(|a_i - 1| + |b_i - 1| + |c_i - 1|) + \max(|u_i - 1|, v_i) \quad (11)$$

$$d_i^- = \frac{1}{3}(|a_i| + |b_i| + |c_i|) + \max(u_i, |v_i - 1|) \quad (12)$$

Step 6: Calculate the relative closeness (k_i) to the ideal solution:

$$k_i = \frac{d_i^-}{d_i^- + d_i^+} \quad (13)$$

A larger k_i implies a better alternative A_i, ordering the values k_i provides the rank of each alternative, the best alternative are those that have higher value k_i.

CONCLUSION

In this study, we propose a new approach for multiple criteria decision making based on triangular intuitionistic fuzzy information, where triangular intuitionistic fuzzy values are used to represent evaluation values of the decision makers with respect to alternatives. We first introduce the Hamming distance between two TIFNs, some operational laws of TIFNs and its properties. based on the proposed operational laws of TIFNs, we present the triangular intuitionistic fuzzy weighed geometric (TIFWG) operator. Then, a new approach for handling fuzzy multiple criteria decision making(MCDM) problems based on triangular intuitionistic fuzzy information is proposed. The proposed approach provides us with a useful way for dealing with DCDM problems under fuzzy environment.

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