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Necessary Conditions on Sampling Factors for Nonuniform Filter Banks Satisfying Perfect Reconstruction Property

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Abstract: Due to their flexibility in partitioning subbands, nonuniform filter banks (NUFBs) are widely used in signal processing applications and various methods have been proposed to achieve their Perfect-Reconstruction (PR) property. However, it should be noticed if the sampling factors cannot guarantee the NUFB to satisfy PR theoretically, the obtained NUFB still cannot achieve PR in spite of being designed by employing PR NUFB design methods. And thus in this study, we analyze and clarify the inter-relationships among those necessary conditions on sampling factors for NUFBs to achieve PR property. And then through the equivalent structure, the resulting conclusions can also be extended from integer sampling factors to rational cases. Based on these analyses, we can efficiently reject certain sets of sampling factors from being considered to build PR NUFBs. The work studied here can provide useful and practical hints for further exploring the necessary and sufficient conditions on sampling factors for PR NUFBs.

Key words: Nonuniform filter bank, perfect reconstruction, sampling factor, necessary condition

INTRODUCTION

Multirate filter banks find success in a variety of signal processing applications and the theory of Perfect Reconstruction (PR) filter banks with uniform frequency partitioning has been well established (Vaidyanathan, 1993). However, in some applications such as audio coding, nonuniform frequency spacing that matches the critical bands is much more preferred.

On the theory and design of nonuniform filter banks (NUFBs), many works have been done. Tree-structure is an easy way to construct PR NUFBs by cascading uniform ones. But the choice of sampling factors is very limited and the system delay is rather long. In the indirect structure (Xie *et al*., 2005; Li *et al*., 2009), certain subbands of a PR uniform bank are recombined by sets of transmultiplexers, producing a PR NUFB. The NUFB so obtained also has long system delay due to the two-stage architecture. In contrast, the direct structure with only one stage is much more attractive. The NUFBs are analyzed directly in time domain by Nayebi *et al*. (1993) to obtain their PR conditions. For the NUFBs with rational sampling factors, more detailed discussion on aliasing cancellation can be found in (Kovacevic and Vetterli, 1993). Also the PR condition and design procedures based on modulation technique were derived (Wada, 1995; Niamut and Heusdens, 2003) by merging the relevant filters of PR uniform modulated banks, inheriting good properties of uniform modulated banks.

For NUFBs with PR property, the above methods with either indirect or direct structures have to be employed according to the requirements of practical applications. However, if the sampling factors cannot make the NUFB satisfy PR theoretically, the resulting NUFB cannot be perfectly reconstructed in spite of being designed by PR NUFB design methods. Therefore before designing, the estimation for the sampling factors whether they can make the obtained NUFB achieve PR or not, is highly desired. This study focuses on the necessary conditions on sampling factors for the NUFBs satisfying PR property. On one hand, we summarize the known necessary conditions on sampling factors for the existence of PR NUFBs. Further on the other hand, based on the summarization, the interrelations among these necessary conditions are analyzed and extended, which helps to strengthen the clarity of known conditions. Based on these analyses, we can quickly exclude certain sampling factors from being considered to construct PR NUFBs. Although whether all conditions collectively can be necessary and sufficient ones on the sampling factors for PR NUFBs is still unknown, studies given here can provide useful and practical hints for further investigations.

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NECESSARY CONDITIONS ON INTEGER SAMPLING FACTORS AND THEIR INTERRELATIONS

Figure 1 shows the direct structure of *M*-channel NUFBs, where H_k (z) and F_k (z), 0<k<M-1, are respectively the analysis and synthesis filters. In this section, we assume all sampling factors n_k are integers and satisfy the critical sampling condition:

$$\sum_{k=0}^{M-1} 1/n_k = 1$$

The above system has PR property, if:

$$\hat{X}(z) = c_0 z^{-c_1} X(z)$$

(c_0 and c_1 are constants), that is, the output is a scaled and delayed version of the input. For the NUFBs, unlike the case in the uniform bank where the existence of realizable filters satisfying PR property is trivially assured, it is not always possible to achieve PR with realizable filters. Next we will summarize the known necessary conditions on integer sampling factors and further analyze the interrelations among them.

Condition 1: Compatible set: The necessary condition of maximally decimated PR NUFBs was firstly studied by Hoang and Vaidyanathan (1989), which shows for the complete aliasing cancellation, the set of integer sampling factors must be a compatible set.

Let S = {n_0, n_1,Y n_{M-1}} be an ordered set of integer sampling factors, n_0<n_1<Y n_{M-1}. Then S is a compatible set if it satisfies the following conditions:

- $\sum_{k=0}^{M-1} 1/n_k = 1$
- For every n_i, l_i (l_i< n_i-1), there exist n_j, l_j (l_j< n_j-1) with n_j … n_i, such that:

$$W_{n_i}^{l_i} = W_{n_j}^{l_j}$$

.

This compatible set condition is necessary for a PR system to cancel the aliasing distortion, since it guarantees the "pairing up" of all aliasing terms in the Alias-Component (AC) matrix. It has been pointed out that (Akkarakaran and Vaidyanathan, 2003), an equivalent and simpler restatement of the compatibility test is, each sampling factor must be a factor of some other sampling factors. In addition, it is important to note that the sampling factors generated by a tree-structure form a compatible set, but the converse is not necessarily true.

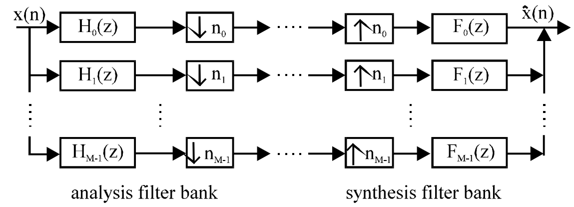

Fig. 1: M-channel NUFB with integer sampling factors

Condition 2: Pairwise noncoprimeness: Starting from the point that for a maximally decimated NUFB, the PR property is equivalent to biorthonormality, Djokovic and Vaidyanathan (1993) proposed another necessary condition "pairwise noncoprimeness". The biorthonormality of a PR NUFB can be expressed as:

$$\left(H_i(z)F_j(z)\right)\downarrow g_{ij} = \delta(i-j) \tag{1}$$

where g_{ij} is the greatest common divisor (gcd) of sampling factors n_i and n_j. If any n_i and n_j are relatively prime, then g_{ij} = 1 and biorthonormality implies H_i (z)F_j (z) = 0 which can only be achieved with ideal filters. Thus in order to achieve PR with realizable transfer functions, the sampling factors should be pairwise noncoprime.

This pairwise noncoprimeness condition has no connection with Condition 1. That is, the fact that one holds does not imply anything about whether the other holds or not. This can be proved by the sets {2, 6, 3} and {4, 6, 6 6, 6, 12}. The former has two coprime sampling factors (2 and 3) but can be verified not satisfying Condition 1. On the other hand, the latter does not have coprime sampling factors but still violates Condition 1 because the largest sampling factor 12 occurs only once.

Condition 3: Strong compatible set: Later, a strong compatibility condition (Djokovic and Vaidyanathan, 1994) is further developed by looking deeper into the details of aliasing cancellation, which includes the compatible set described in Condition 1 as a special case.

Notice that the sampling factors of an M-channel NUFB, n_0, n_1,Y n_{M-1}, may not be all distinct. Let us relabel them in terms of distinct integers v_j, with each v_j occurring N_i, times, where 0<j<K-1, N_0+ N_1+Y+N_{k-1} = M. Define the positive integers k_j = L/v_j with L = 1 cm {v_j}. Then in order to make the PR of corresponding NUFB possible, the sampling factors should satisfy the following strong compatible set condition:

$$m_j - 1 < N_j, 0 \le j \le K-1 \\ \text{where } m_j = \frac{\min_{i\ne j} \operatorname{lcm}(k_j,k_i)}{k_j} \tag{2}$$

The detailed proof can be found in (Djokovic and Vaidyanathan, 1994).

This strong compatibility condition is a generalization of Condition 1 which only said that any nonzero row of the AC matrix should have at least two nonzero aliasing terms. Its test is strictly stronger than the test for compatibility. For demonstration, let us take the NUFB with sampling factor set {3, 5, 6, 6, 15, 15} as an example. For this set, there are K = 4 distinct integers, L = 1cm $\{v_j\}$ = 30 and:

j	0	1	2	3
N_j	1	1	2	2
v_j	3	5	6	15
k_j	10	6	5	2
m_j	1	1	2	3.

Since, m_3 B1 = 2 = N_3, the PR of corresponding NUFB is not possible, although this set satisfies Condition 1.

Further we give a derivation for convenient test. Among the distinct v_j (0<j<k-1), if some v_j is a factor of some other v_i (i…j), that is $v_i = \alpha_j . v_j$, then there is no need to make the strong compatibility test for v_j. The following derivation shows that v_j can satisfy the strong compatibility automatically:

$$\left.\begin{array}{l} v_i = a_j v_j \\ v_j k_j = v_i k_i = L \end{array}\right\} \Rightarrow k_j = a_j k_i$$
$$\Rightarrow \left.\begin{array}{l} m_j = \dfrac{\min_{i\neq j} \operatorname{lcm}(a_j k_i,\, k_i)}{a_j k_i} = 1 \\ N_j \geq 1 \end{array}\right\} \Rightarrow m_j - 1 < N_j \tag{3}$$

It is evident for the NUFB with {3, 5, 6, 6, 15, 15}, there is no need to test sampling factors 3 and 5 for the strong compatibility, since they are all factors of 15. It should be noticed that the strong compatibility is not preserved by tree-structure as mentioned in (Li *et al*., 1997).

In 2003, Akkarakaran and Vaidyanathan (2003) presented another two necessary conditions that the sampling factors of PR NUFBs should satisfy. They are the pairwise gcd test and AC matrix test.

Condition 4: Pairwise GCD test: Among the sampling factor set S of a realizable NUFB, if there exists a subset S= containing g +1 (g = 1, 2, ,Y, M-1) sampling factors such that the gcd of any two elements from the subset is a factor of g, then the realizable NUFB cannot achieve PR property. In particular for the case of g = 1, this pairwise gcd test condition is simplified into Condition 2.

Condition 5: AC matrix test: In a given set of sampling factors, let v_0, v_1,Y, v_{k-1}, be the distinct values with v_j occurring N_j times. Let L be any common multiple of v_j (0<j<k-1) and define $k_j = L/v_j$. Then the algorithm for AC matrix test is as follows:

- Initialization. Create a matrix U with rows l numbered from 0 to L-1 and columns j from 0 to K-1, where the lj-th entry u_{lj} is 1 if l is a multiple of k_j and zero otherwise. Thus U is initialized to describe the positions of the zero and nonzero entries in the AC matrix. In particular, u_{0j} = 1 for all j
- Set U' = U. For all l, j such that u_{lj} is the only entry in the l-th row having value unity, set. u_{lj} = 2 This identifies sets of filters having the same sampling factor value v_j and satisfying an equation of the form:

$$\sum_i H_i\left(zW_L^l\right)F_i(z) = 0$$

- For each d = bk_j where integer b obeying 1<bk_j<[L/2], choose for s = 0, k_j, $2k_j$, Y., d-k_j. If u_{lj} = 2 for l/c_s^d(n) (mod L) for consecutive integers n, set u_{lj} = 2 for l / c_s^d(n) (mod L) for all integers n. Do this for each j = 0, 1,Y,K-1
- If u_{0j} = 2 for any j, the given set of sampling factors fails the AC matrix test. If U= = U, the set passes the test. If neither of these happens, go to step 2

As discussed by Akkarakaran and Vaidyanathan (2003), passing the above test is a necessary condition on the sampling factors of any realizable PR NUFB. It has also shown that the above AC matrix test implies Condition 3.

The five necessary conditions summarized above, Conditions 1-5, are all imposed on the sampling factor set, regardless of arrangement order of the elements in it. The last condition named as "feasible partitioning" concerns the arrangement order of sampling factors in a given set.

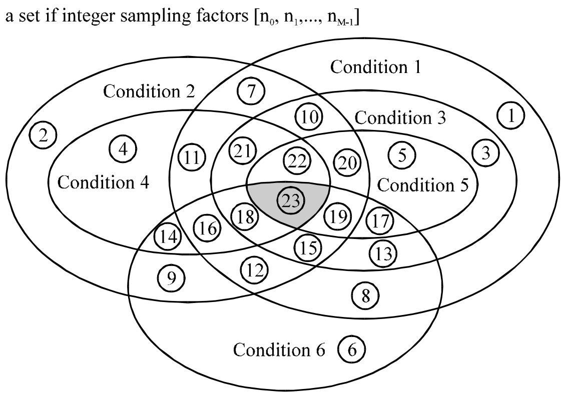

Fig. 2: Interrelation demonstration among Conditions 1-6

It has been mentioned (Li *et al.*, 1997; Absar and George, 2001) in different forms.

Condition 6: Feasible partitioning: Consider an M-channel NUFB with integer sampling factors n_0, n_1,.., n_{M-1}, In order to avoid large aliasing caused by the overlap of positive and negative frequency components, the sampling factors need to satisfy the feasible partitioning condition as:

$$\sum_{j=0}^{i-1} 1/n_j = l_i \times 1/n_i, 1 \le i \le M-1 \tag{4}$$

For some integer l_i with $1<l_i<n_i-1$. It has been shown that (Li *et al.*, 1997), for a feasible partitioned NUFB, it is possible to achieve PR if the analysis and synthesis filters are properly designed; while for a nonfeasible one, no matter how well the analysis/synthesis filters are designed, the large aliasing cannot be cancelled and thus PR cannot be obtained. Further it should be noticed that, for a sampling factor set which can form a compatible set, there does not always exist a combination of feasible partitioning, such as the sampling factor set {2, 3, 7, 84, 84}.

For clear description, we demonstrate the interrelations among Conditions 1-6 in the form of sets as shown in Fig. 2. It can be seen from Fig. 2 for a set of integer sampling factors [n_0, n_1,.., n_{M-1}], the different cases which satisfy some one or several necessary conditions are respectively denoted by notations □-23. The explanations of all cases are also illustrated in Table 1, together with the proof of examples. It should be noticed that, case 23 marked with gray shadow in Fig. 2 means that this set of integer sampling factors satisfies all above Conditions 1-6.

With the above analysis, we can quickly reject certain sampling factors from being considered to build PR NUFBs and further remain the decimators which have the possibility to construct NUFBs with PR property. It should be noticed that not all those remained decimators can realize PR of NUFBs, because the conditions collected here are all necessary for PR NUFBs, while the necessary and sufficient conditions on decimator set is still an open problem.

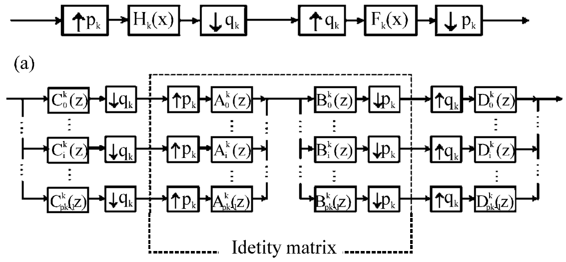

Fig. 3(a-b): (a) k-th branch of M-channel NUFB with rational sampling factors and (b) The equivalent structure of (a)

EXTENSIONS TO RATIONAL SAMPLING FACTORS

In this section, we further extend the necessary conditions on integer sampling factors to the rational case by employing the equivalent structure (Kovacevic and Vetterli, 1993).

For easy discussion, here we take the *k*-branch of *M*-channel NUFB with rational sampling factors p_k/q_k as an example shown in Fig. 3. The samplers p_k and q_k are coprime and satisfy the critical sampling condition . From Fig. 3 we can see that, the *k*-th branch with sampler p_k/q_k can be equivalent to p_k channels with integer sampler q_k. And thus the necessary conditions analyzed in the case of integer samplers can be further extended into the rational case. As a result: Those rational sampling factors which cannot make the corresponding NUFB achieve PR can also be excluded effectively.

CONCLUSION

This study explores the necessary conditions on sampling factors for the NUFBs satisfying PR property. We first analyze the known necessary conditions on integer samplers for the existence of PR NUFBs and then clarify the interrelations among them, which helps to strengthen the clarity of known conditions. Further by employing the equivalent structure, the resulting conclusions can also be applied to the rational case. Although, whether all conditions collectively can be necessary and sufficient ones on the sampling factors for PR NUFBs is still unknown, studies given here can provide useful and practical hints for further investigations.

Table 1: Case explanations and proof of examples on interrelation demonstration among Conditions 1-6

Case Explanations	Proof of Examples	Case Explanations	Proof of Examples
1 Conditions 1Y 2N 3N 4N 5N 6N	[2, 3, 7, 84, 84]	2 Conditions 1N 2Y 3N 4N 5N 6N	[2, 4, 6, 12]
3 Conditions 1Y 2N 3Y 4N 5N 6N	[2, 5, 10, 10, 10]	4 Conditions 1N 2Y 3N 4Y 5N 6N	[2, 4, 8, 12, 24]
5 Conditions 1Y 2N 3Y 4N 5Y 6N	[3, 4, 6, 12, 12, 12]	6 Conditions 1N 2N 3N 4N 5N 6Y	[2, 6, 3]
7 Conditions 1Y 2Y 3N 4N 5N 6N	[2, 4, 6, 24, 24]	8 Conditions 1Y 2N 3N 4N 5N 6Y	[3, 6, 6, 15, 15, 5]
8 Conditions 1N 2Y 3N 4N 5N 6Y	[4, 4, 4, 8, 24, 12]	9 Conditions 1Y 2Y 3Y 4N 5N 6N	[4, 6, 6, 6, 10, 20, 20, 20,]
10 Conditions 1Y 2Y 3N 4Y 5N 6N	[6, 6, 6, 6, 9, 9, 27, 27, 45, 270, 270, 270, 270]	?Conditions 1Y 2Y 3N 4N 5N 6Y	[2, 4, 48, 48, 48, 6]
? Conditions 1Y 2N 3Y 4N 5N 6Y	[2, 10, 5, 10, 10]	? Conditions 1N 2Y 3N 4Y 5N 6Y	[4, 12, 6, 6, 6, 6]
? Conditions 1Y 2Y 3Y 4N 5N 6Y	[4, 250, 10, 20, 20, 6, 6, 6]	? Conditions 1Y 2Y 3N 4Y 5N 6Y	[6, 6, 6, 6, 9, 9, 27, 27, 270, 270] 270, 270, 45]
? Conditions 1Y 2N 3Y 4N 5Y 6Y	$[3,15,5,\underbrace{15,\cdots,15}_{6}]$	? Conditions 1Y 2Y 3Y 4Y 5N 6Y	[6, 6, 6, 6, 9, 9, 72, 72, 24, 72, 72, 72,]
? Conditions 1Y 2Y 3Y 4N 5Y 6Y	[2, 4, 24, 48, 48, 6]	? Conditions 1Y 2Y 3Y 4N 5Y 6N	[2, 4, 6, 24, 48, 48]
11Conditions 1Y 2Y 3Y 4Y 5N 6N	[6, 6, 6, 6, 9, 9, 24, [6, 6, 6, 6, 9, 9, 24, 72, 72, 72, 72, 72] 72, 72, 72, 72, 72]	11Conditions 1Y 2Y 3Y 4Y 5Y 6N	[4, 2, 4]
11Conditions 1Y 2Y 3Y 4Y 5Y 6Y	$[6,30,10,30,15,\underbrace{30,30,\cdots,30}_{18}]$		

Y: Satisfying the corresponding condition, N: Not satisfying the corresponding condition

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