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Impulsive Exponential Consensus of Multi-agent Nonlinear Fuzzy Systems

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Abstract: In this study, we investigate the problem of impulsive exponential consensus of multi-agent systems, where each agent is represented by an identical T-S fuzzy model. Firstly, a fuzzy impulsive control protocol is designed for networks with fixed topology based on the local information of agents. Then sufficient conditions in terms of linear matrix equalities are given to guarantee the exponential consensus of the multi-agent fuzzy systems. Numerical simulations show the effectiveness of our theoretical results.

Key words: Multi-agent systems, fuzzy systems, T-S fuzzy model, consensus, impulsive control protocols, algebraic graph theory

INTRODUCTION

Since consensus is a kind of typical collective behaviors and basic motions in nature, the consensus problem of multi-agent systems have been intensively studied in various disciplines recently (Vicsek *et al*., 1995; Fax and Murray, 2004; Jadbabaie *et al*., 2003; Savkin, 2004; Tian and Liu, 2008). Olfati-Saber *et al*. (2007) investigated a systematical framework of consensus problem in networks of agents. Ren and Beard (2005) considered the problem of information consensus among multiple agents in the presence of limited and unreliable information exchange with dynamically changing interaction topologies. Ren (2008) proposed the consensus algorithms for double-integrator dynamics. The consensus problem for multi-agent systems was considered, in which all agents have an identical linear dynamic mode that can be of any order (Wang *et al*., 2008). Meanwhile, the consensus problem of multi-agent systems has also been considered by impulsive control method (Jiang and Bi, 2010; Guan *et al*., 2010; Wu *et al*., 2012; Zhang and Jiang, 2012; Jiang *et al*., 2011).

In the past few decades, the Takagi-Sugeno (T-S) fuzzy model has been used as an effective method to design and stability analysis of fuzzy control systems. More recently, stability analysis and synthesis of T-S fuzzy systems with impulse have gained considerable attention (Ho and Sun, 2007; Jiang *et al*., 2008; Zheng and Chen, 2009). Jiang *et al*. (2008) investigated the problem of robust fuzzy control for a class of nonlinear fuzzy impulsive systems with time-varying delay and proposed sufficient conditions for global exponential stability of systems.

Ho and Sun (2007) proposed some criteria for uniform stability and uniform asymptotic stability of T-S fuzzy time-delay systems with impulse were proposed. Zheng and Chen (2009) proposed a fuzzy impulsive controller for controlling chaotic dynamical systems by integrating T-S fuzzy model and impulsive control.

Motivated by the aforementioned discussions, in this study, we introduce fuzzy impulsive control protocols for multi-agent fuzzy systems. Firstly, a fuzzy impulsive protocol is designed for network with fixed topology based on the local information of agents. Then sufficient conditions in terms of linear matrix equalities are given to guarantee the exponential consensus of the multi-agent fuzzy systems by the theory of fuzzy systems and impulsive systems.

This study is organized as follows. In Section 2, we provide some results in matrix theory and algebraic graph theory. In Section 3, we formulate the consensus problem for multi-agent fuzzy systems and introduce an impulsive control protocol. The convergence analysis of the exponential consensus problem for network with fixed topology is discussed in Section 4. In Section 5, numerical simulations are included to show the effectiveness of our theoretical results. Some conclusions are drawn in Section 6.

Notation: Throughout this study, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; Let $\mathbb{R}_+ = [0, \infty)$, $\mathbb{N} = \{0, 1, 2,\ldots\}$, $\mathbb{N}_+ = \{1, 2,\ldots\}$; $\mathbb{R}_{n\times m}$ is the set of all n×m real matrices; For real symmetric matrices X and Y, the notation $X\geq Y$ (respectively, $X>Y$ means that the matrix X-Y is positive semi-definite (respectively, positive definite) In $\mathbb{R}^{n\times n}$ is an identity matrix; λ_{min} (P) (λ_{max}(P)) denotes the smallest (largest) eigenvalue of P. For a vector $x\in\mathbb{R}^n$, let $\|x\|$ denote the Euclidean vector norm, i.e., $\|x\|=\sqrt{x^T x}$. Let $1_N = (1, 1,\ldots, 1)^T\in\mathbb{R}^N$ and $e_i\in\mathbb{R}^n$, $e_i(i) = 1$, $e_i(j) = 0$, $j\neq 1$.

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PRELIMINARIES

In this section, we provide some results in matrix theory (Horn and Johnson, 1985, 1991) and algebraic graph theory (Godsil and Royle, 2001; Wang *et al*., 2008).

An undirected graph G of order N consists of a vertex set $V = \{1, 2, \ldots, N\}$ and an edge set $E = \{(i, j): i, j\in v\}\subset v\times v$. The set of neighbors of vertex i is denoted by $Ni = \{I\in V: (i, j)\in E, j\neq i\}$. A path between each distinct vertices i and j is meant a sequence of distinct edges of G of the form $(I, k_1), (k_1, k_2), \ldots, (k_l, j)$. If there is a path between any two vertices of a graph G, then G is connected, otherwise disconnected. A weighted adjacency matrix $A = [a_{ij}\in\mathbb{R}^{N\times N}]$, where $a_{ii} = 0$ and $a_{ji} = a_{ji}\geq 0$, $i\neq j$. $a_{ij}>0$ if and only if there is an edge between vertex i and vertex j. For an unweighted graph G, A is a 0-1 matrix. The out-degree of vertex i is defined as follows:

$$\deg_{out}(i) = \sum_{j=1}^{n} a_{ij}$$

Let D be the diagonal matrix with the out-degree of each vertex along the diagonal and call it the degree matrix of G. The Laplacian matrix of the weighted graph is defined as LG = D-A. For an unweighted graph G:

$$L_G = [l_{ij}]_{N\times N} \tag{1}$$

Where:

$$l_{ij} = \begin{cases} |N_i|, i = j, \\ -1, j\in N_i, \\ 0, \text{otherwise}. \end{cases} \tag{2}$$

By the definition, every row sum of L is zero.

Lemma 1: (Godsil and Royle, 2001; Wang *et al*., 2008) Let L be the Laplacian of an undirected graph G with N vertices, $\lambda_1\leq\lambda_2\leq\ldots\leq\lambda_N$ be the eigenvalues of L. Then:

- 0 is an eigenvalue of L and 1_N is the associated eigenvector, that is, $L1_N = 0$
- If G is connected, then $\lambda_1 = 0$ is the algebraically simple eigenvalue of L
- If 0 is the simple eigenvalue of L, then it is an n multiplicity eigenvalue of $L\otimes I_n$ and the corresponding eigenvectors are $1_N\otimes e_i$, $i = 1, 2, \ldots, n$

PROBLEM FORMULATION

Here, we consider a system consisting of N agents indexed by $i = 1, 2, \ldots, N$. The dynamics of each agent are represented by the following T-S fuzzy model.

Plant rule m:

$$\begin{aligned} &\text{IF } \theta_1(t) \text{ is } M_1^m \text{ and}, \cdots, \text{and } \theta_g(t) \text{ is } M_g^m \\ &\text{THEN } \dot{x}^i(t) = A_m x^i(t) + u^i(t), t\geq t_0\geq 0, \\ &i = 1, 2, \ldots, N, m = 1, 2, \cdots, q \end{aligned} \tag{3}$$

where, M_j^i is the fuzzy set, q is the number of rules, $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_g(t)]^T$ is the premise variable, $A_m\in\mathbb{R}^{n\times n}$ are constant matrices, $x^i(t) = (x_1^i(t), x_2^i(t), \ldots, x_n^i(t))^T\in\mathbb{R}^n$ and $u^i(t)\mathbb{R}^n$ are the state and the control input of agent i at time t, respectively.

Using the fuzzy inference method with singleton fuzzification, product inference and center average defuzzification, the overall fuzzy model of agent i has the following form:

$$\begin{aligned} &\dot{x}^i(t) = \sum_{m=1}^{q} h_m(\theta(t)) A_m x^i(t) + u^i(t), \\ &x^i(t)\in\mathbb{R}^n,\ i = 1, 2, \ldots, N, \end{aligned} \tag{4}$$

Where:

$$h_m(\theta(t)) = \frac{w_m(\theta(t))}{\sum_{m=1}^{q} w_m(\theta(t))}, w_m(\theta(t)) = \prod_{j=1}^{g} M_j^i(\theta_j(t))$$

We assume that $w_m(\theta(t))\geq 0$ and:

$$\sum_{m=1}^{q} w_m(\theta(t)) > 0$$

It is clear that:

$$h_m(\theta(t))\geq 0, \sum_{m=1}^{q} h_m(\theta(t)) = 1$$

The control input of agent i is designed as:

Plant rule m:

$$\begin{aligned} &\text{IF } \theta_1(t) \text{ is } M_1^m \text{ and}, \cdots, \text{and } \theta_g(t) \text{ is } M_g^m \\ &\text{THEN } u^i(t) = \sum_{k=1}^{+\infty} \delta(t-t_k) B_m^k \sum\nolimits_{j\in N_i} (x^j(t) - x^i(t)), \\ &k\in\mathbb{N}_+, i = 1, 2, \ldots, N, m = 1, 2, \cdots, q \end{aligned} \tag{5}$$

where, the discrete instants t_k satisfy $0\leq t_0<t_1<t_2<\ldots<t_{k-1}<t_k\ldots$ and:

$$\lim_{k\to\infty} t_k = +\infty$$

$\delta(t)$is the Dirac delta function. $B_k\in\mathbb{R}^{n\times n}$ are constant matrices to be designed later, N_i is the set of neighbors of agent i.

Using the same method as above, the overall control input of agent i has the following form:

$$u^i(t) = \sum_{k=1}^{+\infty} \delta(t - t_k) \sum_{m=1}^{q} h_m(\theta(t)) B_m^k \times \sum_{j \in N_i} (x^j(t) - x^i(t)), m = 1, 2, \dots, N, \tag{6}$$

Adopting a similar approach (Zheng and Chen, 2009; Guan and Zhang, 2008), from Eq. 4 and 5 we have:

$$x^i(t_k + \varepsilon) - x^i(t_k - \varepsilon) = \int_{t_k-\varepsilon}^{t_k+\varepsilon} (\sum_{m=1}^{q} h_m(\theta(t)) A_m x^i(s) + u^i(s)) ds,$$

where, $\varepsilon>0$ is sufficiently small. As $\varepsilon \to 0^+$, this becomes to:

$$\Delta x^i(t_k) = x^i(t_k^+) - x^i(t_k^-) = \sum_{m=1}^{q} h_m(\theta(t)) B_m^k \sum_{j \in N_i} (x^j(t_k^-) - x^i(t_k^-)),$$

Where:

$$x^i(t_k^+) = \lim_{t \to t_k^+} x^i(t)$$

and:

$$x^i(t_k^-) = \lim_{t \to t_k^-} x^i(t)$$

Without loss of generality, we assume that:

$$\lim_{t \to t_k^+} x^i(t) = x^i(t_k)$$

which means that the solution $x^i(t)$ is right continuous at time t_k. This implies that the agent i will suddenly update its state variable according to the state variables of itself and its neighbors at the instants t_k. Thus the control input $u^i(t)$ is called a fuzzy impulsive control protocol.

Under the fuzzy impulsive control protocol Eq. 5, the dynamics of agent i satisfy the following equations:

$$\begin{cases} \dot{x}^i(t) = \sum_{m=1}^{q} h_m(\theta(t)) A^m x^i(t), t \neq t_k, \\ \Delta x^i(t_k) = \sum_{m=1}^{q} h_m(\theta(t_k^-)) B_m^k \sum_{j \in N_i} (x^j(t_k^-) - x^i(t_k^-)), \\ i = 1, 2, \dots, N, k \in \mathbb{N}_+, \end{cases} \tag{7}$$

Definition1 For the system Eq. 4, the exponential consensus is said to be achieved under the fuzzy impulsive control protocol Eq. 5 if there exist M, $\gamma>0$ such that:

$$|e^{i,j}(t)| = \| x^i(t) - x^j(t) \| \leq M e^{-\gamma t} \to 0, \quad t \to \infty, i, j = 1, 2, \dots, N \tag{8}$$

Consider the following fuzzy impulsive system:

$$\begin{cases} \dot{z}(t) = \sum_{m=1}^{q} h_m(\theta(t)) C_m z(t), t \neq t_k, \\ \Delta z(t_k) = \sum_{m=1}^{q} h_m(\theta(t_k^-)) G_m^k z(t_k^-)), k \in \mathbb{N}_+, \end{cases} \tag{9}$$

Lemma 2: (Jiang *et al*., 2008) For a given positive constant α, if there exist there exist discrete instants t_k, impulsive matrices G_m^k, positive constants

$$0 < \eta_k \leq e^{\alpha(t_k - t_{k-1})}$$

and symmetric and positive definite matrix X, such that the following LMIs hold:

$$XC_m^T + C_m X + 2\alpha X < 0 \tag{10}$$

$$\begin{bmatrix} -\eta_k X & (X + G_m^k X)^T \\ X + G_m^k X & -X \end{bmatrix} < 0 \tag{11}$$

then the system Eq. 9 is global exponential stable. In this way, there exist M, $\gamma>0$ such that:

$$\| z(t) \| \leq M e^{-\gamma t} \to 0, t \to \infty \tag{12}$$

NETWORK WITH FIXED TOPOLOGY

In this section, we provide the convergence analysis of the consensus problem for network with fixed topology, i.e. G(t) = G for time t.

Let $x(t) = (x^1(t), x^2(t), \dots, x^N(t))^T$, then the system Eq. 7 can be described as:

$$\begin{cases} \dot{x}(t) = (I_N \otimes (\sum_{m=1}^{q} h_m(\theta(t)) A_m)) x(t), t \neq t_k, \\ \Delta x(t_k) = (I_N \otimes (\sum_{m=1}^{q} h_m(\theta(t_k^-)) B_m^k))(-L \otimes I_n) \\ \quad \times x(t_k^-), k \in \mathbb{N}_+ \end{cases} \tag{13}$$

Since L is symmetric, there is an orthogonal matrix $Y \in \mathbb{R}^{N \times N}$ such that:

$$YLY^{-1} = YLY^T = D = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N),$$

where $\{\lambda_1, \lambda_2, \dots \lambda_n\}$ is the spectrum of L.

Inspired by the method (Wang *et al*., 2008), let:

$$\tilde{x}(t) = (Y \otimes I_n) x(t) \tag{14}$$

Using the properties of Kronecker product, we have when $t\in[t_k, t_{k+1})$, $k\in\mathbb{N}_+$:

$$\begin{aligned}\dot{\tilde{x}}(t) &= (Y\otimes I_n)\dot{x}(t)\\ &= (Y\otimes I_n)(I_N\otimes(\sum_{m=1}^{q}h_m(\theta(t))A_m))x(t)\\ &= (Y\otimes I_n)(I_N\otimes(\sum_{m=1}^{q}h_m(\theta(t))A_m))(Y\otimes I_n)^{-1}\tilde{x}(t)\\ &= (I_N\otimes(\sum_{m=1}^{q}h_m(\theta(t))A_m))\tilde{x}(t)\end{aligned}$$

and:

$$\begin{aligned}\Delta\tilde{x}(t_k) &= (Y\otimes I_n)\Delta x(t_k)\\ &= (Y\otimes I_n)(I_N\otimes(\sum_{m=1}^{q}h_m(\theta(t_k^-))B_m^k))\\ &\quad\times(-L\otimes I_n)(Y\otimes I_n)^{-1}\tilde{x}(t_k^-)\\ &= (-D\otimes(\sum_{m=1}^{q}h_m(\theta(t_k^-))B_m^k))\tilde{x}(t_k^-)\end{aligned}$$

Thus Eq. 13 becomes to:

$$\begin{cases}\dot{\tilde{x}}(t) = (I_N\otimes(\sum_{m=1}^{q}h_m(\theta(t))A_m))\tilde{x}(t), t\neq t_k,\\ \Delta\tilde{x}(t_k) = (-D\otimes(\sum_{m=1}^{q}h_m(\theta(t_k^-))B_m^k))\tilde{x}(t_k^-)\end{cases}\tag{15}$$

Therefore:

$$\begin{cases}\dot{\tilde{x}}^i(t) = (\sum_{m=1}^{q}h_m(\theta(t))A_m)\tilde{x}^i(t), t\neq t_k,\\ \Delta\tilde{x}^i(t_k) = -\lambda_i(\sum_{m=1}^{q}h_m(\theta(t_k^-))B_m^k)\tilde{x}^i(t_k^-),\\ i=2,\ldots,N, k\in\mathbb{N}_+\end{cases}\tag{16}$$

Theorem 1: Consider the system Eq. 4. Assume that the graph G of the network is connected. For a given positive constant α, if there exist discrete instants t_k, impulsive matrices B_m^k, positive constants $0<\eta_k\leq e^{\alpha(t_k-t_{k-1})}$, symmetric and positive definite matrix X, such that the following LMIs hold for $i = 2,\ldots, N$, $k\in\mathbb{N}\mathbb{N}_+$

$$XA_m^T + A_mX + 2\alpha X < 0\tag{17}$$

$$\begin{bmatrix}-\eta_kX & (X+\lambda_iB_m^kX)^T\\ X+\lambda_iB_m^kX & -X\end{bmatrix}<0\tag{18}$$

then the exponential consensus is achieved under the fuzzy impulsive control protocol Eq. 5.

Proof: Since the graph G is connected, by Lemma 1, $\lambda_1 = 0$ is the algebraically simple eigenvalue of L. All the other eigenvalues of L are positive. Then we have:

$$0=\lambda_1<\lambda_2\leq\cdots\leq\lambda_N$$

It can be verified that:

$$\begin{aligned}&(L\otimes I_n)x(t)\\ &= (Y\otimes I_n)^{-1}(Y\otimes I_n)(L\otimes I_n)(Y^{-1}\otimes I_n)\tilde{x}(t)\\ &= (Y\otimes I_n)^{-1}(D\otimes I_n)\tilde{x}(t)\\ &= (Y\otimes I_n)^{-1}\begin{bmatrix}0 & \lambda_2\tilde{x}^2(t) & \cdots & \lambda_N\tilde{x}^N(t)\end{bmatrix}^T\end{aligned}$$

For a given positive constant α, if there exist discrete instants t_k, impulsive matrices B_k, constants $0<\eta_k\leq e^{\alpha(t_k-t_{k-1})}$, symmetric and positive definite matrix X, such that the following LMIs hold for $i = 2, \ldots, N$, $i = 2,\ldots n$ N, by Lemma 2 the system Eq. 16 is exponentially stable, i.e. $\|\tilde{x}^i(t)\|\leq M_ie^{-\gamma_it}\to 0$, $t\to+\infty$, $i = 2,\ldots, N$, then:

$$\begin{aligned}&|(L\otimes I_n)x(t)\|\leq\|(Y\otimes I_n)^{-1}\|\\ &\times\lambda_N\max\{M_2,\cdots,M_n\}e^{-\min\{\gamma_2,\cdots,\gamma_n\}t}\to 0, t\to\infty\end{aligned}$$

Since the graph G is connected, by Lemma 1, 0 is the eigenvalue of $L\otimes I_n$ with multiplicity n. The n linearly independent eigenvectors associated with the eigenvalue 0 of $L\otimes I_n$ are $1_N\otimes e_i$, $i = 1, 2,\ldots, n$. Therefore, $x\to 1_N\otimes s$, $t\to+\infty$, where:

$$|s=\sum_{i=1}^{n}\xi_ie_i\in\mathbb{R}^n$$

$\xi_i\in\mathbb{R}$, $i = 1, 2,\ldots, n$. Thus the system Eq. 4 achieves the exponential consensus under the fuzzy impulsive control protocol Eq. 5. This completes the proof.

SIMULATIONS

Consider the following multi-agent fuzzy systems:

Plant rule m:

$$\begin{aligned}&\text{IF } x_1^1(t) \text{ is } M_1^m \text{ and } x_1^2(t) \text{ is } M_2^m\\ &\text{THEN } \dot{x}^i(t) = A_mx^i(t)+u^i(t), t\geq t_0\geq 0,\\ &i=1,2, m=1,2,3,4,\end{aligned}$$

where, $x^i(t)=(x_1^i(t), x_2^i(t), x_3^i(t))^T$, a = 10, b = 28, c = 8/3, α = -20, β = 30, M_i^mi = 1, 2, m = 1, 2, 3, 4 are fuzzy sets which represent "Large" and "Small" in [-20, 30], respectively:

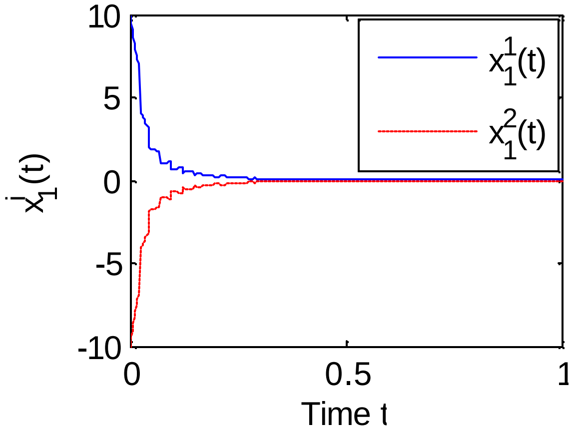

Fig. 1: Time histories of $x^1{}_1(t)$ and $x^2{}_1(t)$

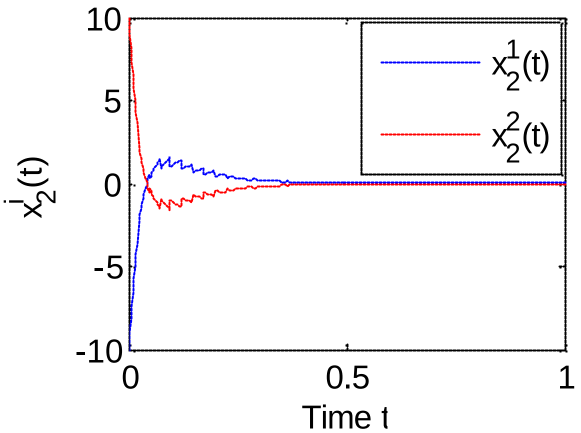

Fig. 2: Time histories of $x^1{}_2(t)$ and $x^2{}_1(t)$

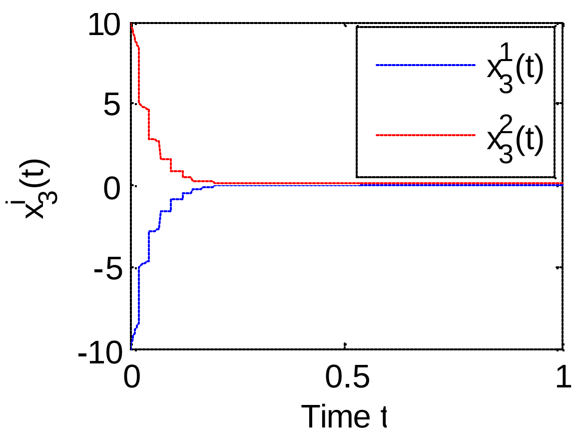

Fig. 3: Time histories of $x^1{}_3(t)$ and $x^2{}_3(t)$

$$A_1=\begin{bmatrix}-a & a & 0\\ b & -1 & -\alpha\\ 0 & \alpha & -c\end{bmatrix}, A_2=\begin{bmatrix}-a & a & 0\\ b & -1 & -\alpha\\ 0 & \beta & -c\end{bmatrix},$$
$$A_3=\begin{bmatrix}-a & a & 0\\ b & -1 & -\beta\\ 0 & \alpha & -c\end{bmatrix}, A_4=\begin{bmatrix}-a & a & 0\\ b & -1 & -\beta\\ 0 & \beta & -c\end{bmatrix}$$

The control input of agent i is designed as:

Plant rule m:

$$\text{IF } x_1^1(t) \text{ is } M_1^m \text{ and } x_1^2(t) \text{ is } M_2^m$$
$$\text{THEN } u^1(t)=\sum_{k=1}^{+\infty}\delta(t-t_k)B_m^k(x^2(t)-x^1(t)),$$
$$u^2(t)=\sum_{k=1}^{+\infty}\delta(t-t_k)B_m^k(x^1(t)-x^2(t)),$$
$$t\geq t_0\geq 0, k\in\mathbb{N}_+, m=1,2,3,4$$

Since the graph is a path on two vertices, we get $\lambda_2 = 2$. For simplicity, the impulsive matrices B_m^k, $k\in\mathbb{N}_+$, are chosen as p^*I_4, where, $0<p = 0.2$. Choose the equidistant impulsive interval $\Delta t_k = t_k\text{-}t_{k\text{-}1}, = \Delta = 0025$. By solving the linear matrix equalities, we get:

$$X=\begin{bmatrix}0.2540 & -0.3992 & 0.1283\\ -0.3992 & 0.4397 & 0.0541\\ 0.1283 & 0.0541 & 0.7695\end{bmatrix}$$

Thus the conditions of Theorem 1 are satisfied. Simulation results are shown in Fig. 1-3. The simulation results show that the fuzzy impulsive protocol is efficient to solve the exponential consensus problem.

CONCLUSION

In this study, we have introduced fuzzy impulsive control protocols for multi-agent fuzzy systems. Convergence analysis of the fuzzy impulsive control protocol is presented.

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REFERENCES

Fax, J.A. and R.M. Murray, 2004. Information flow and cooperative control of vehicle formations. IEEE Trans. Automat. Control, 49: 1465-1476.

Ho, D.W.C. and J.T. Sun, 2007. Stability of Takagi-Sugeno fuzzy delay systems with impulse. IEEE Trans. Fuzzy Syst., 5: 784-790.

Horn, R.A. and C.R. Johnson, 1985. Matrix Analysis. 1st Edn., Cambridge University Press, London.

Horn, R.A. and C.R. Johnson, 1991. Topics in Matrix Analysis. Cambridge University Press, Cambridge, UK.

Jadbabaie, A., L. Jie and A.S. Morse, 2003. Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans. Automat. Control, 48: 988-1001.

Jiang, H.B., J.J. Yu and C.G. Zhou, 2008. Robust fuzzy control of nonlinear fuzzy impulsive systems with time-varying delay. IET Control Theory Appl., 2: 654-661.

Jiang, H.B. and Q.S. Bi, 2010. Impulsive synchronization of networked nonlinear dynamical systems. Phys. Lett. A, 374: 2723-2729.

Jiang, H.B., J.J. Yu and C.G. Zhou, 2011. Consensus of multi-agent linear dynamic systems via impulsive control protocols. Int. J. Syst. Sci., 42: 967-976.

Godsil, C. and G. Royle, 2001. Algebraic Graph Theory. Springer-Verlag, New York.

Guan, Z.H. and H. Zhang, 2008. Stabilization of complex network with hybrid impulsive and switching control. Chaos Solitons Fractals, 37: 1372-1382.

Guan, Z.H., Z.W. Liu, G. Feng and Y.W. Wang, 2010. Synchronization of complex dynamical networks with time-varying delays via impulsive distributed control. IEEE Trans. Circuits Syst.-I, 57: 2182-2195.

Olfati-Saber, R., J.A. Fax and R.M. Murray, 2007. Consensus and cooperation in networked multi-agent systems. Proc. IEEE, 95: 215-233.

Ren, W. and R.W. Beard, 2005. Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans. Automat. Control, 50: 655-661.

Ren, W., 2008. On consensus algorithms for double-integrator dynamics. IEEE Trans. Automat. Control, 53: 1503-1509.

Savkin, A.V., 2004. Coordinated collective motion of groups of autonomous mobile robots: Analysis of Vicseks model. IEEE Trans. Automat. Control, 49: 981-982.

Tian, Y.P. and C.L. Liu, 2008. Consensus of multi-agent systems with diverse input and communication delays. IEEE Trans. Automat. Control, 53: 2122-2128.

Vicsek, T., A. Czirok, E. Ben-Jacob, I. Cohen and O. Sochet, 1995. Novel type of phase transition in a system of self-driven particle. Phys. Rev. Lett., 75: 1226-1229.

Wang, J.H., D.Z. Cheng and X.M. Hu, 2008. Consensus of multi-agent linear dynamic systems. Asian J. Control, 10: 144-155.

Wu, Q.J., J. Zhou and L. Xiang, 2012. Impulsive consensus seeking in directed networks of multi-agent systems with communication time delays. Int. J. Syst. Sci., 43: 1479-1491.

Zhang, L.P. and H.B. Jiang, 2012. Impulsive cluster anticonsensus of discrete multiagent linear dynamic systems. Discrete Dyn. Nat. Soc., 10.1155/2012/857561

Zheng, Y.A. and G. Chen, 2009. Fuzzy impulsive control of chaotic systems based on TS fuzzy model. Chaos Solitons Fractals, 39: 2002-2011.