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# Internal Force Evaluation of Portal Frame with Non-uniform Beam under the Combination of Vertical and Horizontal Loads

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**Abstract:** To obtain the accurate analytical solution of internal force in portal frame being composed of non-uniform members subjected to the combination of vertical and horizontal loads, the force method was used to solve internal force of every key section of the non-uniform beam in portal frame with single-span. Gauss numerical integration method was used to simplify the complex integration when the deformation energy was calculated. The analytical solutions were verified by finite element method. Under the precondition of satisfied accuracy, the explicit expressions of bending moment, shear force and axial force of every key section of beam were obtained. The research work can be used as reference for the evaluation of portal frame members and for the programming of portal frame design codes.

**Key words:** Portal frame, non-uniform beam, internal force evaluation, force method, numerical integration

## INTRODUCTION

The bending moment at different section of beam differs significantly in lightweight steel portal frames. To realize economical design, the beam is often designed as a non-uniform member. The portal frame being composed of non-uniform members are usually analyzed by finite element method. The theoretically deduced elements in the stiffness matrix of non-uniform members are expressed as integral form. The analytical solutions of these integrations are difficult to obtain. In practical application, non-uniform members are often approximately simulated by several uniform segments (Dong and Qian, 2000). Therefore, the accuracy of computation depends on the number of element section types and the number of elements. With the number of defined sections and elements increasing, the workload of modeling and computation will increase significantly. In order to obtain the accurate controlling forces of members in portal frame conveniently and directly, based on the linear elastic theory, force method (Long and Bao, 1994) was used to solve internal forces of every key section of beam. Gauss numerical integration method (Gao, 1987) was used to simplify the evaluation when meeting the complex integration expressions. Under the precondition of satisfied accuracy, the analytical expressions of bending moment, shear force and axial force of every key section of beam were proposed.

## ANALYSIS MODEL

The single-span double-pitch portal frame with both hinged column bases was analyzed. The beam is subjected to vertical uniformly distributed pressure q. The columns are subjected to horizontal uniformly distributed loads $q_1$ and $q_2$. The model diagram and displacement coordinate system are shown in Fig. 1. The constant cross section is designed for column. The non-uniform I-shaped cross section with varying web height is designed for

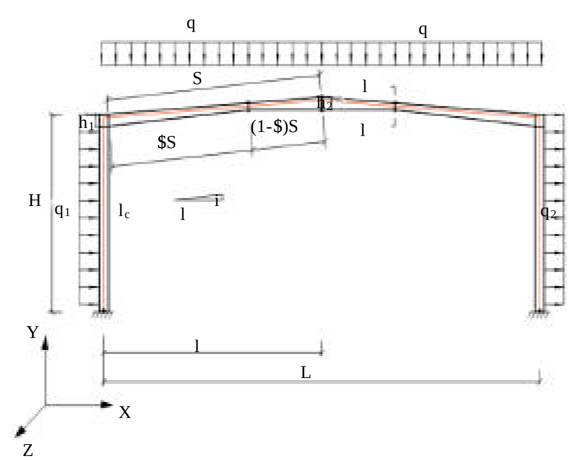


Fig. 1: Model of the portal frame

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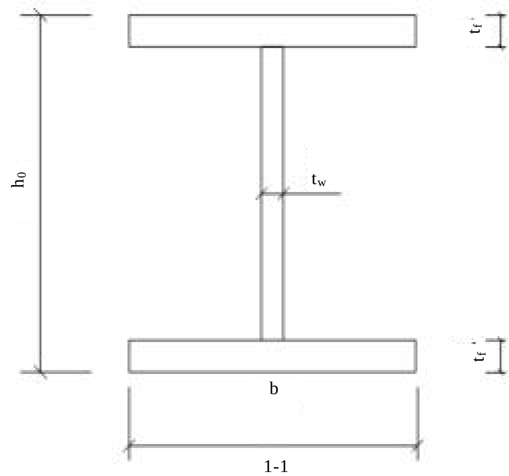


Fig. 2: Minimum section of beam

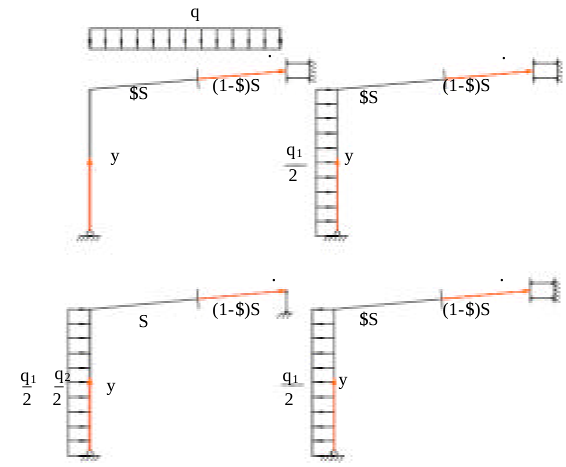


Fig. 3: Basic evaluation diagram of portal frame

beam. Beam span is L and its half-span length is S. The height of column is H. The slope ratio of beam is i. The inertia moment of column is $I_c$. The axial distance between the end of tapering beam and its minimum section is βS. The axial distance between the mid-span section and the minimum section is (1-β)S. The height of the end section of beams is $h_1$ and the height of the mid-span section of beams is $h_2$. The minimum section of beam with the height $h_0$ is shown in Fig. 2. The width and thickness of the flange are b and $t_f$, respectively. The thickness of web is $t_w$. The tapering ratio of the end section to the minimum section is $\gamma_1 = h_1/h_0-1$, and that of the mid-span section to the minimum section is $\gamma_2 = h_2/h_0-1$.

## SOLUTION OF THE INTERNAL FORCE FOR BEAM

For the convenience of solution, the analysis of entire portal frame can be simplified as that of half structure in consideration of symmetry. The basic force method calculation diagram is shown in Fig. 3. The end section of beam is subjected to the bending moment $M_e$, shear force $V_e$ and axial force $N_e$ while the mid-span section is subjected to the bending moment $M_m$, shear force $V_m$, and axial force $N_m$.The forementioned key internal forces are all shown in Fig. 4. The bending moment is taken as positive when it makes the lower fiber of beam be pulled. The shear force is taken as positive when it makes the member rotate clockwise. The axial force is taken as positive when it is tension. For engineering design, the evaluation of internal force is often based on the linear elastic theory; consequently, elastic material and linear small deformation is taken into account. The elastic modulus of the material is E.

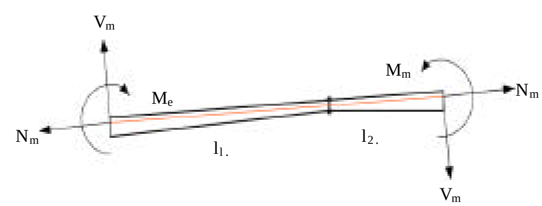


Fig. 4: Internal force of beam

Force method is used for the evaluation of internal force. The basic equation of force method is listed in Eq.1:

$$\delta_{11} \times M_m + \Delta_{1p} = 0 \qquad (1)$$

where, $\delta_{11}$ is the rotation displacement of the mid-span section for beam in the basic structure subjected to the unit bending moment at mid-span. $\Delta_{1p}$ is the rotation displacement of mid-span section for beam under all the design loads. When the deformation energy is calculated, the portal frame shown in Fig. 3 is divided into three parts for integral according to their own local coordinate systems. The inertia moment of different parts of beam is respectively defined $I_{1æ}$ and $I_{2æ}$ which are cubic function about æ. The equations are listed in Eq. 2 and 3:

$$I_{1\zeta} = \frac{bt_f^3}{6} + 2bt_f\left[\frac{(\frac{-\gamma_1\zeta}{\beta S}+1)h_0 - t_f}{2}\right]^2 + \frac{t_w\left[(\frac{-\gamma_1\zeta}{\beta S}+1)h_0 - 2t_f\right]^3}{12} \qquad (2)$$

$$I_{2\zeta}=\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(\frac{\gamma_2\zeta}{\beta S}+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(\frac{\gamma_2\zeta}{\beta S}+1)h_0-2t_f\right]^3}{12} \tag{3}$$

The integral expressions of $\delta_{11}$ and $\Delta_{1p}$ are listed as following:

$$\delta_{11}=\int_0^H \frac{\overline{M}_1^2}{EI_c}dy+\int_{-\beta S}^0 \frac{\overline{M}_1^2}{EI_{1\zeta}}d\zeta+\int_0^{(1-\beta)S}\frac{\overline{M}_1^2}{EI_{2\zeta}}d\zeta \tag{4}$$

$$\Delta_{1p}=\int_0^H \frac{\overline{M}_1M_p}{EI_c}dy+\int_{-\beta S}^0 \frac{\overline{M}_1M_p}{EI_{1\zeta}}d\zeta+\int_0^{(1-\beta)S}\frac{\overline{M}_1M_p}{EI_{2\zeta}}d\zeta \tag{5}$$

For Eq. 2 and 3, it is difficult to obtain analytical solutions explicitly. Thus, Gauss numerical integration method is introduced into the integration solutions. The simplified expressions of integration are obtained. Based on the superposition principle, the bending moment of the mid-span section can be calculated by Eq. 6:

$$M_m=-\frac{\Delta_{1p}^1+\sum_{k=2}^{3}\sum_{j=1}^{3}\Delta_{1p}^{kj}}{\delta_{11}^1+\sum_{k=2}^{3}\sum_{j=1}^{3}\delta_{11}^{kj}}\times q-\frac{\Delta_{1p}^4+\sum_{v=5}^{6}\sum_{j=1}^{3}\Delta_{1p}^{vj}}{\delta_{11}^1+\sum_{k=2}^{3}\sum_{j=1}^{3}\delta_{11}^{kj}}\times q_1+\frac{\Delta_{1p}^4+\sum_{v=5}^{6}\sum_{j=1}^{3}\Delta_{1p}^{vj}}{\delta_{11}^1+\sum_{k=2}^{3}\sum_{j=1}^{3}\delta_{11}^{kj}}\times q_2 \tag{6}$$

Parameters in Eq. 6 are listed as following:

$$\Delta_{1p}^1=-\frac{l^2H^3}{6EI_c(H+il)^2} \tag{7}$$

$$\Delta_{1p}^{21}=\frac{\beta S}{2E}\frac{\left\{0.556\times\frac{l^2}{2}(1-0.887\beta)\times(1.11\beta-\frac{H}{H+il})\times\left[1-\frac{il}{H+il}(1-0.887\beta)\right]\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.112\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.112\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{8}$$

$$\Delta_{1p}^{22}=\frac{\beta S}{2E}\frac{\left\{0.889\times\frac{l^2}{2}(1-0.5\beta)(1.5\beta-\frac{H}{H+il})\times\left[1-\frac{il}{H+il}(1-0.5\beta)\right]\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.5\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.5\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{9}$$

$$\Delta_{1p}^{23}=\frac{\beta S}{2E}\frac{\left\{0.556\times\frac{l^2}{2}(1-0.112\beta)(1.89\beta-\frac{H}{H+il})\left[1-\frac{il}{H+il}(1-0.112\beta)\right]\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.887\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.887\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{10}$$

$$\Delta_{1p}^{31}=\frac{(1\text{-}\beta)S}{2E}\frac{\left\{0.556\times\frac{l^2}{2}\times\left[1-\frac{il}{H+il}\times0.887(1-\beta)\right]\times0.887(1-\beta)\left[\frac{1+\beta}{2}-0.387(1-\beta)-\frac{H}{H+il}\right]\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.112\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.112\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{11}$$

$$\Delta_{1p}^{32}=\frac{(1\text{-}\beta)S}{2E}\frac{\left\{0.889\times\frac{l^2}{2}\times\left[1-\frac{il}{H+il}\times0.5(1-\beta)\right]\times0.5(1-\beta)\left[\frac{1+\beta}{2}-\frac{H}{H+il}\right]\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.5\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.5\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{12}$$

$$\Delta_{1p}^{33}=\frac{(1\text{-}\beta)S}{2E}\frac{\left\{0.556\times\frac{l^2}{2}\times\left[1-\frac{il}{H+il}\times0.112(1-\beta)\right]\times0.112(1-\beta)\left[\frac{1+\beta}{2}+0.387(1-\beta)-\frac{H}{H+il}\right]\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.887\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.887\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{13}$$

$$\delta_{11}^1=\frac{H^3}{3EI_c(H+il)^2} \tag{14}$$

$$\delta_{11}^{21}=\frac{\beta S}{2E}\frac{0.556\times\frac{l^2}{2}\left[1-\frac{il}{H+il}(1-0.887\beta)\right]^2}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.112\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.112\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{15}$$

$$\delta_{11}^{22}=\frac{\beta S}{2E}\frac{0.889\times\frac{l^2}{2}\left[1-\frac{il}{H+il}(1-0.5\beta)\right]^2}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.5\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.5\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{16}$$

$$\delta_{11}^{23}=\frac{\beta S}{2E}\frac{0.556\times\frac{l^2}{2}\left[1-\frac{il}{H+il}(1-0.112\beta)\right]^2}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.887\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.887\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{17}$$

$$\delta_{11}^{31}=\frac{(1\text{-}\beta)S}{2E}\frac{0.556\times\left[1-\frac{il}{H+il}\times0.887(1-\beta)\right]^2}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.112\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.112\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{18}$$

$$\delta_{11}^{32}=\frac{(1\text{-}\beta)S}{2E}\frac{0.889\times\left[1-\frac{il}{H+il}\times0.5(1-\beta)\right]^2}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.5\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.5\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{19}$$

$$\delta_{11}^{33}=\frac{(1\text{-}\beta)S}{2E}\frac{0.556\times\left[1-\frac{il}{H+il}\times0.112(1-\beta)\right]^2}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.887\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.887\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{20}$$

$$\Delta_{1p}^{4}=\frac{5ilH^4-3H^5}{48EI_c(H+il)^2} \tag{21}$$

$$\Delta_{1p}^{51}=\frac{\beta S}{2E}\frac{\left\{0.556\times\left[1-\frac{il}{H+il}(1-0.887\beta)\right]\times(1-0.887\beta)\frac{H(il)^2}{2(il+H)}\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.112\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.112\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{22}$$

$$\Delta_{1p}^{52}=\frac{\beta S}{2E}\frac{\left\{0.889\times\left[1-\frac{il}{H+il}(1-0.5\beta)\right]\times(1-0.5\beta)\frac{H(il)^2}{2(il+H)}\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.5\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.5\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{23}$$

$$\Delta_{1p}^{52}=\frac{\beta S}{2E}\frac{\left\{0.556\times\left[1-\frac{il}{H+il}(1-0.112\beta)\right]\times(1-0.112\beta)\frac{H(il)^2}{2(il+H)}\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.887\gamma_1+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.887\gamma_1+1)h_0-2t_f\right]^3}{12}\right\}} \tag{24}$$

$$\Delta_{1p}^{61}=\frac{(1-\beta)S}{2E}\frac{\left\{0.556\times\left[1-\frac{il}{H+il}\times0.887\times(1-\beta)\right]\times0.887\times(1-\beta)\frac{H(il)^2}{2(il+H)}\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.112\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.112\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{25}$$

$$\Delta_{1p}^{62}=\frac{(1-\beta)S}{2E}\frac{\left\{0.889\times\left[1-\frac{il}{H+il}\times0.5\times(1-\beta)\right]\times0.5\times(1-\beta)\frac{H(il)^2}{2(il+H)}\right\}}{\left\{\frac{bt_f^{\,3}}{6}+2bt_f\left[\frac{(0.5\gamma_2+1)h_0-t_f}{2}\right]^2+\frac{t_w\left[(0.5\gamma_2+1)h_0-2t_f\right]^3}{12}\right\}} \tag{26}$$

$$\Delta_{1p}^{63}=\frac{(1-\beta)S}{2E}\frac{\left\{\begin{array}{l}0.556\times\left[1-\frac{il}{H+il}\times0.112\times(1-\beta)\right]\\ \times0.112\times(1-\beta)\frac{H(il)^2}{2(il+H)}\end{array}\right\}}{\left\{\begin{array}{l}\frac{bt_f^{\ 3}}{6}+2bt_f\left[\frac{(0.887\gamma_2+1)h_0-t_f}{2}\right]^2\\ +\frac{t_w\left[(0.887\gamma_2+1)h_0-2t_f\right]^3}{12}\end{array}\right\}} \quad (27)$$

Based on the bending moment at mid-span of beam $M_m$, every item of internal force for the end section and the mid-span section can be obtained as following:

$$V_m=\left[\frac{1}{H+il},M_m-\frac{ql^2}{2})-\frac{H^2q_1}{4l}-\frac{H^2q_2}{4l}\right]\frac{1}{\sqrt{1+i^2}} \quad (28)$$

$$N_m=\frac{M_m-\frac{ql^2}{2}}{H+il}\frac{i}{\sqrt{1+i^2}} \quad (29)$$

$$M_e=\frac{H}{H+il}(M_m-\frac{ql^2}{2}+\frac{q_1ilH}{4}-\frac{q_2ilH}{4})+\frac{(q_1+q_2)H^2}{4} \quad (30)$$

$$V_e=\frac{M_m-\frac{ql^2}{2}}{H+il}\frac{i}{\sqrt{1+i^2}}+\frac{ql}{\sqrt{1+i^2}}-\frac{(q_1+q_2)H^2}{4l\sqrt{1+i^2}} \quad (31)$$

$$N_e=\frac{M_m-\frac{ql^2}{2}}{H+il}\frac{1}{\sqrt{1+i^2}}-\frac{i}{\sqrt{1+i^2}}ql+\frac{(q_1+q_2)iH^2}{4l\sqrt{1+i^2}} \quad (32)$$

The internal forces of column can be obtained conveniently by the equilibrium relationship after the internal forces of beam being solved:

## VERIFICATION FOR ANALYTICAL SOLUTIONS

To verify the correctness of the derivation process and the accuracy of the simplified evaluation method by using Gauss numerical integration, the finite element code package ANSYS was used. Five project models were built based on the typical geometries and loads of practical portal frames. For the finite element models, portal frames were simulated by the element of SHELL181. Finite element results of every internal force at the end and mid-span of portal frame beam were compared with the results from the forementioned analytical derivation. The difference is less than 9%. The reason possibly lies in it that the deformation of bending, shearing and axial deformation are all taken into account when finite element method is adopted to evaluate the internal forces. However, only the influence of bending deformation is taken into account when the force method is used to solve the internal forces. Consequently, the proposed solutions herein can be considered to be accurate and reliable.

## CONCLUSION

Based on linear elastic theory, force method was used to solve the explicit expressions of internal forces of key sections in portal frame with non-uniform beam. The solutions were verified by finite element methods. For the engineering design of portal frame being composed of non-uniform members, the accurate solution of every controlling internal force item is proposed. The research lay the foundations for the further programming work of design and optimization code for portal frame.

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