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## Research on Hydraulic Characteristic of Drop-pits

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**Abstract:** The hydraulic factors affected the cavity's back flow quantity, cavity's length, jet's impacting angle, back flow's momentum and cavity's pressure of drop-pit are analyzed and also the relationships between the hydraulic factors and cavity's back flow quantity, cavity's length, jet's impacting angle, back flow's momentum and cavity's pressure of drop-pit are analyzed by using the jet flow theory in this paper. Verification results show that the analyzed results in this paper agree well with the model experimental conclusions.

**Key words:** Jet flow, backflow, cavity, impacting angle, cavity's pressure

### INTRODUCTION

In many cases, there are much small bubbles in high velocity flows, which cannot be observed by naked eyes (Wei and DeFazio, 1982). The small bubbles develop and grow in low pressure area. If they flow to high pressure area, they may rupture and generate strong explosion, which can damage buildings, such as dam, discharge tunnel etc (Yuan, 1980). The above phenomenon is called cavitations damage by many experts. Many methods have been explored to prevent cavitations damage by hydraulic experts in the past years. Aeration in high velocity flows is an effective way to decrease cavitations damage (Rutschmann, 1990). Aeration in high velocity flows must rely on certain device (Pinto *et al.*, 1982). Drop-pit is the most commonly used device to aerate (Rutschmann, 1990). Flow's movement track above drop-pit is shown in Fig. 1. Oval FEC in Figure 1 is a cavity, where air is aerated.  $X_c$  is the length of cavity,  $2b$  is the depth of flows, AB is flow's center line, B is the node between flow's center line and the floor line of drop-pit,  $s_r$  is arc length from A to B,  $\theta$  is crossing angle between flow's center line and the floor line of drop-pit, D is the height of drop-pits.

Wang *et al.* (2003) regarded that if there are much backflows in cavity, the air will be submerged by water, aeration effects will become poor, so it is important to decrease the backflows (Wang *et al.*, 2003). The length of cavity can affect negative pressure in cavity, the greater the negative pressure is in cavity, the greater the ventilation quantity in cavity. Zhang and Xu (2006) think that the backflows increase with the increase of the inflow flux. Zhang and Xu (2006) also regarded the greater the crossing angle  $\theta$  is, the greater the backflows are. Xu *et al.* (2004) regarded that the length of cavity  $X_c$  has relationship with the height of drop-pit D,  $X_c$  increases

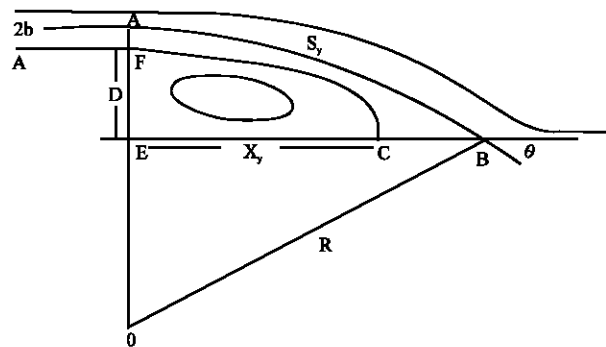


Fig. 1: Flows in drop-pits

with the increase of D. Although a lot of experts have researched the cavity of drop-pit and some useful conclusions also are obtained, their research method mainly confined in simulation and model test. Very few people use theory to analysis the hydraulic characteristics of drop-pit. Simulation and model test need to consume large amounts of money and manpower. Besides, about the hydraulic characteristics of drop-pit, there are many issues to discuss, such as the momentum of backflows (Jin and Guo, 1987), the factors affecting crossing angle  $\theta$  and so on (Zhang *et al.*, 2004). So it is necessary to use theory method to comprehensive research the hydraulic characteristics of drop-pit. The purpose of this paper is to study the factors affecting cavity backflows, the factors affecting cavity length and momentum of backflows by using jet flow theory.

### HYDRULIC CHARACTERISTICS OF CAVITY

**Factors affecting backflows:** According to jet flow theory and refer to Fig. 1, the following equation can be obtained:

$$Q_1 = \left[ \frac{3J_0 (s_r + s_0)}{4\rho\sigma} \right]^2 (1 - K_r) \quad (1)$$

Where,  $Q_1$  is backflow's flux;  $J_0$  is jet flows' original momentum;  $R$  is the radius of jet flows movement;  $\sigma$  is jet flows' diffusion coefficient;  $\rho$  is flows' density;  $s_0$  is the distance from the jet flows' imaginary origin point to jet flows' exit section;  $s_r$  is arc length of jet flows' center line.  $K_r$  and  $s_r$  can be expressed respectively as the following:

$$K_r = \tanh \frac{\sigma y_r}{s_r + s_0} = 2 \cos \frac{\theta + \pi}{3} \quad (2)$$

$$s_r = \frac{2b\sigma}{3} \left( \frac{1}{K_r^2} - 1 \right) \quad (3)$$

The following equations are derived from the geometrical relationship in Fig. 1:

$$R = \frac{s_r}{\theta} \quad (4)$$

$$D = R \times (1 - \cos \theta) - b \quad (5)$$

$$X_r = R \sin \theta + (D + b) \sin \theta - \frac{y_r}{\sin \theta} \quad (6)$$

The coefficient  $y_r$  is defined as the following:

$$y_r = \frac{2b}{3K_r^2} \times \frac{1}{2} \ln \frac{1 + K_r}{1 - K_r} = \frac{b}{3K_r^2} \ln \frac{1 + K_r}{1 - K_r} \quad (7)$$

Equation 3-5 can replace the coefficient  $R$  in Eq. 6, so Eq. 6 can be changed into:

$$\frac{D}{b} + 1 = \frac{2\sigma}{3\theta} \left[ \frac{1}{4 \cos^2 \frac{\theta + \pi}{3}} - 1 \right] (1 - \cos \theta) = f(\theta) \quad (8)$$

By using mathematics knowledge, it can be learned that:

$$\cos \theta \approx 1 - \frac{1}{2}\theta^2 + \frac{1}{24}\theta^4 \quad (9)$$

$$\cos \frac{\theta + \pi}{3} \approx 1 - \frac{1}{2} \left( \frac{\theta + \pi}{3} \right)^2 + \frac{1}{24} \left( \frac{\theta + \pi}{3} \right)^4 \quad (10)$$

By using Eq. 8-10 can be changed into:

Table 1: Relationship between  $\theta$  and  $D/b$

$\theta$ (°)	20.00	25.00	30.00	35.00	40.00	45.00	50.00	55.00	60.00
$D/b$	0.23	1.15	2.49	4.42	7.21	11.22	17.18	26.26	40.77

$$\frac{D}{b} + 1 = \frac{2\sigma}{3} \left[ \frac{1}{4 \left[ 1 - \frac{1}{2} \left( \frac{\theta + \pi}{3} \right)^2 + \frac{1}{24} \left( \frac{\theta + \pi}{3} \right)^4 \right]} - 1 \right] \left( \frac{1}{2} - \frac{1}{24}\theta^2 \right) = f(\theta) \quad (11)$$

Equation 8 shows that  $\theta$  is closely related with  $D/b$  and  $\theta$  is the function of  $D/b$ . The results data in Table 1 can be commutated By using Eq. 11. The data in Table 1 illuminates that  $\theta$  increases with the increasing of  $D/b$ .

The following equation can be obtained from jet flow theory:

$$s_0 = \frac{2\sigma b}{3} \quad (12)$$

If combining Eq. 3 and 12, Eq. 1 can be expressed:

$$Q_1 = \left[ \frac{bJ_0}{2\rho} \right]^2 \times \frac{1 - K_r}{K_r} = bu \times \frac{1 - K_r}{K_r} = q \frac{1 - K_r}{K_r} \quad (13)$$

where,  $u$  is flow's velocity,  $q$  is flow's original discharge per unit width.  $(1 - K_r)/K_r$  decreases with the increasing of  $K_r$  and Eq. 2 shows that  $K_r$  is only related with  $\theta$  and it decreases with the increase of  $\theta$ , so  $(1 - K_r)/K_r$  increases with the increase of  $\theta$ . From Eq. 2, 11 and 13, the following conclusions can be obtained: backflow's flux  $Q_1$  is the function of  $D$ ,  $b$ ,  $q$ ; when  $D$  and  $b$  are constant, the relationship between  $Q_1$  and  $u$  is Linear relationship; when  $u$  and  $b$  are constant,  $Q_1$  increases with the increase of  $D$ ; when  $u$  and  $D$  are constant,  $Q_1$  decreases with the increase of  $b$ ;  $Q_1$  increases with the increase of  $D/b$ .

**Factors affecting cavity's length:** By using Eq. 7 and 2, 6 can be changed:

$$X_r = \frac{2b\sigma}{3\theta} \left( \frac{1}{4 \cos^2 \frac{\theta + \pi}{3}} - 1 \right) \sin \theta + (D + b) \sin \theta - \frac{b}{12 \cos^2 \frac{\theta + \pi}{3} \sin \theta} \left[ \ln \left( 1 + 2 \cos \frac{\theta + \pi}{3} \right) - \ln \left( 1 - 2 \cos \frac{\theta + \pi}{3} \right) \right] \quad (14)$$

Equation 14, illuminates that the length of cavity  $X_r$  is the function of  $\theta$ ,  $D$  and  $b$ . because  $\theta$  is only the function of  $D$  and  $b$ ,  $X_r$  is only the function of  $D$  and  $b$ . The data in Table 2 is calculated by using Eq. 14. Combining Eq. 8 and the results data in Table 2, the following conclusions can be obtained: when  $b$  does not vary,  $\theta$  increases with the increase of  $D$ , so  $X_r$  increases with the increase of  $D$ .

Table 2: The relationship between  $\theta$  and  $X_r$

$\theta$ ( $^\circ$ )	20	25	30	35	40	45	50	55	60
$X_r$	3.98b+0.34D	7.38b+0.42D	11.16b+0.51D	15.61b+0.57D	21.12b+0.64D	28.21b+0.71D	37.71b+0.77D	51.05b+0.82D	70.91b+0.87D

Table 3: The relationships among  $X_r$ ,  $\theta$  and b (D=0.5m)

b (m)	0.012	0.019	0.029	0.044	0.069	0.11	0.20	0.43	2.17
$\theta$ ( $^\circ$ )	60.000	55.000	50.000	45.000	40.000	35.00	30.00	25.00	20.00
$X_r$ (m)	1.310	1.390	1.480	1.610	1.780	2.06	2.48	3.41	8.81

Table 4:  $J_1$  and  $J_2$  (D = 0.5m)

b (m)	0.012	0.0190	0.0290	0.0440	0.069	0.110	0.2000	0.4300	2.1700
$\theta$ ( $^\circ$ )	60.000	55.0000	50.0000	45.0000	40.000	35.000	30.0000	25.0000	20.0000
$J_1$ ( $\rho u^2$ )	0.018	0.0299	0.0477	0.0752	0.122	0.200	0.3736	0.8200	4.2160
$J_2$ ( $\rho u^2$ )	0.006	0.0081	0.0104	0.0130	0.016	0.0198	0.0266	0.0396	0.1268

When D does not vary, such as D = 0.5 m, Table 2 can be changed into Table 3. The data in Table 3 shows that when D is constant,  $X_r$  increases with then increase of b,  $\theta$  decreases with the increase of b.

**Factors affecting backflow’s momentum:** According to jet flow theory, downstream export flows’ momentum and backflows’ momentum in cavity can expressed as the following, respectively:

$$\begin{cases} J_1 = \frac{3}{4}J_0\left(\frac{2}{3} + K_r - \frac{1}{3}K_r^3\right) = \frac{3}{2}b\rho u^2\left(\frac{2}{3} + K_r - \frac{1}{3}K_r^3\right) \\ J_2 = \frac{3}{4}J_0\left(\frac{2}{3} - K_r + \frac{1}{3}K_r^3\right) = \frac{3}{2}b\rho u^2\left(\frac{2}{3} - K_r + \frac{1}{3}K_r^3\right) \end{cases} \quad (15)$$

where,  $J_1$  is downstream export flows’ momentum;  $J_2$  is backflows’ momentum in cavity. According to the above analysis, it can be learned that  $K_r$  is the function of  $\theta$  and  $\theta$  is the function of D/b. So  $J_1$  and  $J_2$  are only the function of b, D and u. Equation 15 illuminates that  $J_1$  and  $J_2$  increase with the increase of u. The data in Table 4 are calculated by using Eq. 15, 8 and 2 when D is 0.5 m. The data in Table 4 shows that when D is constant,  $J_1$  and  $J_2$  increase with the increase of b. Equation 15 indicates that  $J_1$  increases with the increase of  $K_r$ ,  $J_2$  decreases with the increase of  $K_r$ . When D increases,  $\theta$  increases and  $K_r$  decreases,  $J_1$  decreases and  $J_2$  increases.

**Factors affecting cavity’s negative pressure:** Because the pressure upper jet flow is atmospheric pressure, the bottom pressure is cavity pressure. The cavity’s pressure is the function of the difference between atmospheric pressure and cavity pressure. But the difference between atmospheric pressure and cavity pressure is related with  $\theta$ . If the difference between atmospheric pressure and cavity pressure becomes bigger; the crossing angle  $\theta$  also becomes bigger. The above theoretical analysis shows that  $\theta$  is the function of D/b, when b is constant and D increases,  $\theta$  also increases, the difference between atmospheric pressure and cavity pressure increases. Thus cavity’s pressure becomes lower. Similarly, if D does not

vary and b increases,  $\theta$  becomes smaller, the difference between atmospheric pressure and cavity pressure becomes smaller and the cavity’s pressure becomes bigger.

## VERIFICATION

**Verification for cavity’s backflows:** Wang *et al.* (2003) got much data about backflow in cavity of drop-pits by model experiment (6). Figure 2 is derived from Wang’s model experiment data. Figure 2 shows the relationships between q and flow’s height H in cavity when cavity’s geometry is continual drop-pits. The curves in Fig. 2 illuminate that when other factors are constant, the backflow’s flux increases with the increase of q, which are consistent with the results in this study.

Zhang and Xu (2006) got more conclusions about hydraulic characteristics of drop-pits by model experimental. Figure 3 is derived from his model experiment data, where H is flow’s height in cavity. He regarded that backflow’s flux is related with crossing angle  $\theta$ , the larger the  $\theta$  is, the more are the backflow’s flux. His above view point on backflow’s flux can verify the relationship between cavity’s backflow’s flux and  $\theta$  in this study.

**Verification for cavity’s bottom length:** The data in Table 5 are derived from (Zhang and Xu, 2006) model experiment. The data in Table 3 show that cavity’s bottom length increases with original flow’s depth b and also, cavity’s bottom length increases with the increase of drop-pits height D. Zhang’s model experiment conclusions are consistent with the results about cavity’s bottom length in this study.

**Verification for cavity’s pressure:** The data in Table 6 are also derived from (Zhang and Xu, 2006) model experiment. From Table 6, it can be learned that if b is constant, cavity’s pressure increases with the increase of D; if D is

Table 5: Cavity's bottom length at different b and D

q (m <sup>3</sup> /m.s)	u (m/s)	2b (m)	D (m)	Cavity's bottom length
211.18	41	5.2795	1.29	38.28
220.00	41	5.2380	1.29	24.80
169.00	35	4.8280	1.93	38.66
169.00	35	4.8280	0.97	21.13

Table 6: Cavity's pressure at different b and D

q(m <sup>3</sup> /m.s)	u (m/s)	2b (m)	D (m)	Cavity's pressure
61.75	25	2.470	0.99	-0.67
141.4	35	4.040	0.99	-0.59
169.0	35	4.828	1.93	-1.30
169.0	35	4.828	0.97	-1.27

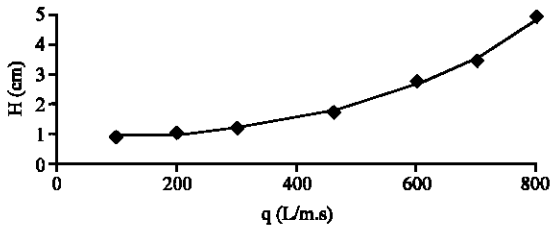


Fig. 2: Relationships between q and H

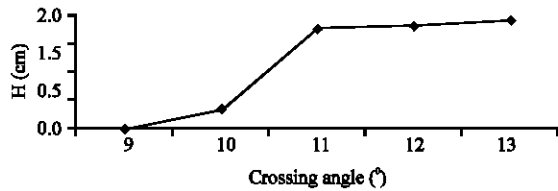


Fig. 3: Relationship between  $\theta$  and H

constant, cavity's pressure decreases with the increase of b. Zhang's model experiment conclusions about cavity's pressure can well verify the results about cavity's pressure in this study.

### CONCLUSIONS

According to jet flow theory, the hydraulic characteristics of drop-pits are analyzed in this paper. The analyzed results show that when other factors are not varied, the backflow's flux in cavity increases with the increase of flow's original discharge per unit width q; and also when other factors are not varied, the backflow's flux in cavity increases with the increase of crossing angle between flow's center line and the floor line of drop-pits  $\theta$ ;  $\theta$  is related with D/b,  $\theta$  increases with the increase of

D/b; Cavity's bottom length is related with D and b, cavity's bottom length increases with the increase of b when D is constant, cavity's bottom length increases with the increase of D when b is constant; downstream export flows' momentum  $J_1$  and backflows' momentum in cavity  $J_2$  all increases with the increase of b;  $J_1$  decreases and  $J_2$  increases when b is constant and D increases; cavity's pressure is closely related with  $\theta$ ; the larger is the D/b, the larger is the cavity's pressure. All theory analyzed results in this paper are verified by model experiment data.

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