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Study on Some Nonlinear Dynamics Problems of Rotor-sliding Bearing System with Impact-rubbing

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Abstract: An rotor-sliding bearing dynamics model supported on sliding bearing was established_DThe dynamic model of the nonlinear rigidity-rotor system with rubbing fault was set up by the physics nonlinear factors. The nonlinear dynamic behaviors of the system caused by rubbing fault were studied by using the numerical value integral and Poincaré mapping methods. The bifurcation diagrams of the response were given following the changing of rotating speed_Aeccentricity and stiffness of the stator. Some typical Poincaré maps, phase plane portraits, trajectory of journal centers and amplitude spectra, et al, were also given. There are doubling-periods, approximate-periods and chaos behaviors in the rotor system. Which may provide theory references to fault diagnoses, vibration control, safety operating and enable early prediction of the fault in rotating machinery.

Key words: Rotor, nonlinear rigidity, fault, rubbing, bifurcation, chaos

INTRODUCTION

Rotating machinery is widely used in industry. In order to improve the performance of rotating machinery, the clearance between the rotor-stator is designed to smaller and this also raise the possibility of rubbing. The rubbing can cause blade fracture, rotor instability and even cause serious incidents.

Rotor-sliding bearing system is a typical nonlinear system and the rub-impact fault exhibit strong nonlinear characteristics, so the vibration spectrum is very complex. (Zhang, 1990) Establishing a reasonable model of the friction force and rotor is the key to solve the problem. The model of short bearings theory and the model of oil film proposed by Zhang Wen of the analytical expression are the same and they are the simplified form of finite Explicit Model Proposed by Yang Jinfu (Liu *et al*., 2008). In this study, used the model of short bearings theory. This study used the smooth friction force model with piecewise-linearity.

Rotor-sliding bearing system with nonlinear support stiffness is a complex nonlinear system. With impact stiffness coefficient increased, the phenomenon of frequency division and chaos are induced more frequently. This study established a rotor-sliding bearing system with nonlinear support stiffness and oil-film force. The numerical integration method of Runge-Kutta is used to obtain nonlinear dynamic responses of this system and using time-domain waveform diagrams, the orbit diagrams of axle center, the frequency spectrogram, the Poincare maps, the bifurcation plots etc., (He and Li, 2013; Tian *et al*., 2013). The sliding-bearings clearance, impact stiffness coefficient on system responses are studied and the faults characteristics of rotor misalignment and rubbing between rotor and stator are also studied.

A DYNAMICS MODEL OF ROTOR-SLIDING BEARING WITH NONLINEAR OIL FILM FORCE

As the complexity of nonlinear rub-impact model, the system is simplified in this study. The rotor is simplified as elastic rotation shaft with concentrated weight. The system is the model with nonlinear oil film force and has rigid foundation and support (Xu and Zhang, 2000) Fig. 1 shows the model.

The rotor is supported at both ends by sliding bearing. O_1 is the geometrical center of bearing bush. O_2 is the geometrical center of the rotor. O_3 is the weight center of the rotor. k_c is the stiffness of stator. k is the linear stiffness of elastic rotation shaft. c_1 is the damping coefficient of the rotor on the sliding bearing. c_2 is the damping coefficient of disk rotor. δ_0 is the gap between the stator and disk rotor. e is the eccentric weight of disk rotor. m_1 is the concentrated weight of the rotor on the sliding bearing. m_2 is the concentrated weight of the rotor on the disc. It is the elastic shaft without weight between the sliding bearing and the disk. R is the radius of the sliding bearing. L is the length of the sliding bearing.

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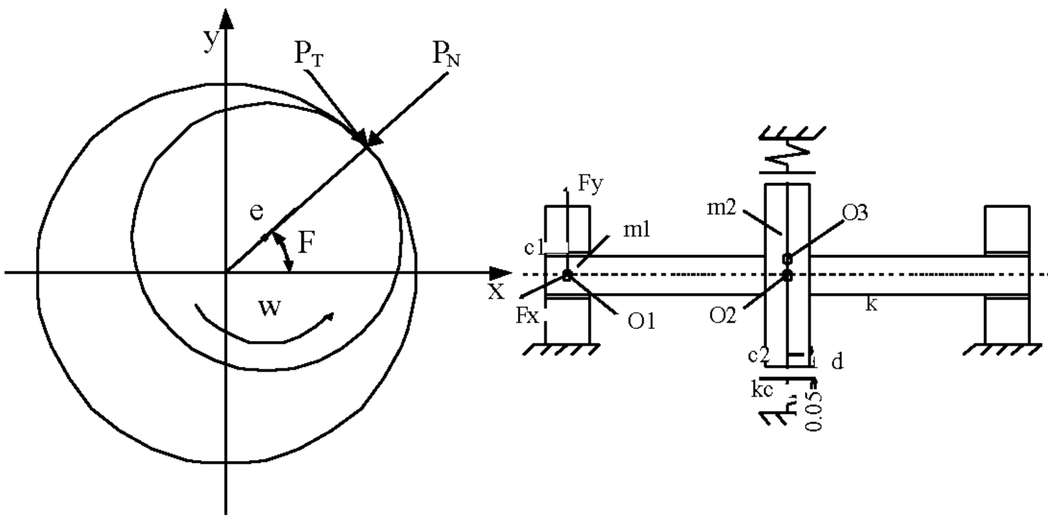

Fig. 1: Model of the rotor-bearing system

In the complex plane, $z = x+iy$. The nonlinear dynamic equation of the system is expressed as follows:

$$M\ddot{z} + D\dot{z} + Kz = G + F_z + F_{rub} \tag{1}$$

In this equation, M is the weight matrix, D is the damping matrix, K is the stiffness matrix, G is the gravity vector, F_z is the unbalanced force vector, F_{rub} is the rubbing force.

Kinetic equation is expressed as follows:

$$\begin{cases} m_1x_1'' + c_1(x_1' - x_2') + k(x_1 - x_2) + k_r[(x_1 - x_2)^2 + (y_1 - y_2)^2](x_1 - x_2) = f_x \\ m_1y_1'' + c_1(y_1' - y_2') + k(y_1 - y_2) + k_r[(x_1 - x_2)^2 + (y_1 - y_2)^2](y_1 - y_2) = f_y - m_1g \\ m_2x_2'' + c_2(x_2' - x_1') + k(x_2 - x_1) + k_r[(x_1 - x_2)^2 + (y_1 - y_2)^2](x_2 - x_1) = p_x + m_2rw^2\cos(wt) \\ m_2y_2'' + c_2(y_2' - y_1') + k(y_2 - y_1) + k_r[(x_1 - x_2)^2 + (y_1 - y_2)^2](y_2 - y_1) = p_y + m_2rw^2\sin(wt) - m_2g \end{cases} \tag{1}$$

Nonlinear rubbing force: The rubbing force is decomposed as P_x and P_y by coulomb law of friction. P_x is the radial rubbing force, P_x is the tangential rubbing force.

c is the average oil film thickness. If $\hat{x} = x/c$, $\hat{y} = y/c$, In the x-y Coordinate System, the rubbing force after non-dimensionalization could be show as follows:

$$\begin{cases} \begin{Bmatrix} P_{\hat{x}} \\ P_{\hat{y}} \end{Bmatrix} = -c(1-\delta_0/r)k_c \begin{bmatrix} 1 & -f \\ f & 1 \end{bmatrix} \begin{Bmatrix} \hat{x} \\ \hat{y} \end{Bmatrix}, & r \geq \delta_0 \\ \begin{Bmatrix} P_{\hat{x}} \\ P_{\hat{y}} \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \hat{x} \\ \hat{y} \end{Bmatrix}, & r < \delta_0 \end{cases} \tag{2}$$

In this equation, δ_0 is the gap between the stator and disk rotor, k_c is the radial stiffness of the stator, f is the friction coefficient. r is the radial displacement of disc rotor, shows as follows:

$$r = \sqrt{x^2 + y^2}$$

Dimensionless model of nonlinear oil-film force: In the rotor-sliding bearing system, because the perturbations of axle journal, lubricant in the bearing will be rotated and formed the oil film, then the bracing force hold up the rotor. All the model of oil force is based on Reynolds equation and the pressure on the boundary film is zero. Capone proposed a model about nonlinear oil-film force on the infinitely short journal bearing in 1991, the calculating results shows better accuracy and convergence (Chen and Zhang, 2011).

The force after non-dimensionalization shows as follows:

$$\left[\frac{r_0}{L}\right]^2 \frac{\partial}{\partial z}\left[h^3 \frac{\partial p}{\partial z}\right] = x\sin\varphi - y\cos\varphi - 2(x'\cos\varphi + y'\sin\varphi) \tag{3}$$

We can get the equation as follows by decomposed to radial direction and axis direction:

$$\begin{Bmatrix} f_{\hat{x}} \\ f_{\hat{y}} \end{Bmatrix} = \frac{\sqrt{(\hat{x} - 2\hat{y}')^2 + (\hat{y} + 2\hat{x}')^2}}{1 - \hat{x}^2 - \hat{y}^2} \begin{Bmatrix} 3\hat{x}\cdot V(\hat{x},\hat{y},\alpha) - \sin\alpha\cdot G(\hat{x},\hat{y},\alpha) - 2\cos\alpha\cdot S(\hat{x},\hat{y},\alpha) \\ 3\hat{y}\cdot V(\hat{x},\hat{y},\alpha) + \cos\alpha\cdot G(\hat{x},\hat{y},\alpha) - 2\sin\alpha\cdot S(\hat{x},\hat{y},\alpha) \end{Bmatrix} \tag{4}$$

In the equation:

$$V(\hat{x},\hat{y},\alpha) = \frac{2 + (\hat{y}\cos\alpha - \hat{x}\sin\alpha)\cdot G(\hat{x},\hat{y},\alpha)}{1 - \hat{x}^2 - \hat{y}^2}$$

$$S(\hat{x},\hat{y},\alpha) = \frac{\hat{y}\sin\alpha + \hat{x}\cos\alpha}{1 - (\hat{y}\sin\alpha + \hat{x}\cos\alpha)^2}$$

$$G(\hat{x},\hat{y},\alpha)=\frac{2}{1-\hat{x}^2-\hat{y}^2}[\frac{\pi}{2}+\arctan\frac{\hat{y}\cos\alpha-\hat{x}\sin\alpha}{\sqrt{1-\hat{x}^2-\hat{y}^2}}]$$

Dynamics system: Take Eq. 2 and 3 into Eq. 1, we can get the dynamics equation shows as follows:

$$\begin{cases}\hat{x}_1''+\frac{c_1}{\omega m_1}(\hat{x}_1'-\hat{x}_2')+\frac{k}{\omega^2 m_1}(\hat{x}_1-\hat{x}_2)+\frac{k_r c^2}{\omega^2 m_1} \\ [(\hat{x}_1-\hat{x}_2)^2+(\hat{y}_1-\hat{y}_2)^2](\hat{x}_1-\hat{x}_2)=\frac{sM}{\omega^2 m_1\sigma}f_x(\hat{x}_1,\hat{y}_1,\hat{x}_1',\hat{y}_1') \\ \hat{y}_1''+\frac{c_1}{\omega m_1}(\hat{y}_1'-\hat{y}_2')+\frac{k}{\omega^2 m_1}(\hat{y}_1-\hat{y}_2)+\frac{k_r c^2}{\omega^2 m_1} \\ [(\hat{x}_1-\hat{x}_2)^2+(\hat{y}_1-\hat{y}_2)^2](\hat{y}_1-\hat{y}_2)=\frac{sM}{\omega^2 m_1\sigma}f_y(\hat{x}_1,\hat{y}_1,\hat{x}_1',\hat{y}_1')-G \\ \hat{x}_2''+\frac{c_2}{\omega m_2}(\hat{x}_2'-\hat{x}_1')+\frac{k}{\omega^2 m_2}(\hat{x}_2-\hat{x}_1)+\frac{k_r c^2}{\omega^2 m_2} \\ [(\hat{x}_1-\hat{x}_2)^2+(\hat{y}_1-\hat{y}_2)^2](\hat{x}_2-\hat{x}_1)=\frac{P_x(\hat{x}_2,\hat{y}_2)}{\omega^2 m_2\sigma}+b\cdot\cos(\tau) \\ \hat{y}_2''+\frac{c_2}{\omega m_2}(\hat{y}_2'-\hat{y}_1')+\frac{k}{\omega^2 m_2}(\hat{y}_2-\hat{y}_1)+\frac{k_r c^2}{\omega^2 m_2} \\ [(\hat{x}_1-\hat{x}_2)^2+(\hat{y}_1-\hat{y}_2)^2](\hat{y}_2-\hat{y}_1)=\frac{P_y(\hat{x}_2,\hat{y}_2)}{\omega^2 m_2\sigma}+b\cdot\sin(\tau)-G\end{cases}$$

In the equation, $\tau = \omega t$, ω is angular speed of rotor, u is the viscosity of lubrication, M is half of the weight of rotor disc, $G = g/(w^2c)$ is the gravitation after non-dimensionalization.

$$b=r/c,\ \hat{x}_1=x_1/c,\ \hat{y}_1=y_1/c,\ \hat{x}_1'=x_1'/\omega c,\ \hat{y}_1'=y_1'/\omega c,$$
$$\hat{x}_1''=x_1''/\omega^2c,\ \hat{y}_1''=y_1''/\omega^2c,\ \hat{x}_2=x_2/c,\ \hat{y}_2=y_2/c,$$
$$\hat{x}_2'=x_2'/\omega c,\ \hat{y}_2'=y_2'/\omega c,\ \hat{x}_2''=x_2''/\omega^2c,\ \hat{y}_2''=y_2''/\omega^2c$$

S is the Sommerfeld-correction coefficient, the expression shows as follows:

$$s=\frac{u\omega RL}{M}(\frac{R}{c})^2(\frac{L}{2R})^2$$

PERFORMANCE ANALYSIS AND SIMULATIONS

The initial parameters in the system can pick and choose as follows:

$m_1 = 4$ kg, $m_2 = 32.1$ kg, R = 25 mm, L = 12 mm, c = 0.11 mm, u = 0.018 Pa.s, $c_1 = 1050$ N.s/m, $c_2 = 2100$ N.s/m, kc $=0.5\times10^6$ N/m, $5 = 2.5\times10^7$ N/m^3, $k_r = 2.5X12$ N/m^3, e = 0.5 mm, $\delta_0 = 0.05$ mm

f_0 is the characteristic frequency of the system, it shows as follows:

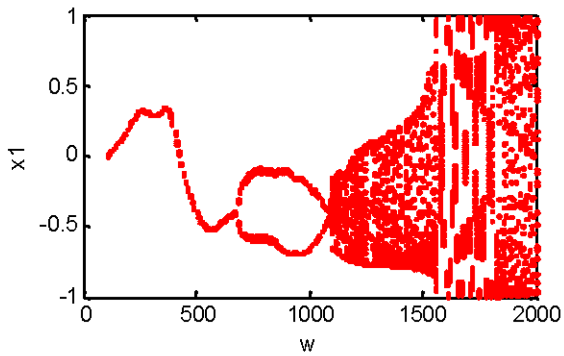

Fig. 2: Bifurcation diagram of ω as a control parameter with bending vibration

$$f_0=\sqrt{k/m_2}=882.5056 \text{ Hz}$$

The numerical integration method of Runge-Kutta is used to obtain nonlinear dynamic responses of this system by Matlab. Making the time dimensionless and integral step selection is 1/512.

Influences about bearing-sliding rotor response system by the variety of ω: The bifurcation diagram of ω as a control parameter with bending vibration is shown as Fig. 2, when ω changes from 0 to 2000 rad/s. Form Fig. 7, we can see that system has different motion states, Chaos movement, periodic motion and quasi-periodic motion.

When ω changes from 0 to 670 rad/s, the system is in P-1 motion state. When ω changes from 670 to 1090 rad/s, the system is in P-2 motion state. When ω changes from 1090 to 2000/rad/s, the system was excited into chaos state. At the same time, the reduction of half-frequency further shows oil film whirl case appeared alleviative.

As ω = 1160 rad/s, Fig. 3 shows the vibration waveform of rotor vibration, the axis orbit of the rotor, the axis orbit of the rotor disc, amplitude spectra of the vibration waveform and the poincare maps. Spectrogram of the vibration waveform has half-frequency component, basic frequency component and three-second frequency component. Three-second frequency component is caused by the nonlinear stiffness of rotor. And the value of half-frequency increased, caused the oil film whirl seriously. And for speed increases, the rubbing fault is more seriously. The occurrence of the rubbing has multiperiodicity. The frequency spectrum of the rubbing force becomes more complex. Even some frequency can do some work to the rotor's vibration, it makes the rotor's vibration more complex. Poincare maps show that the system is in chaos state.

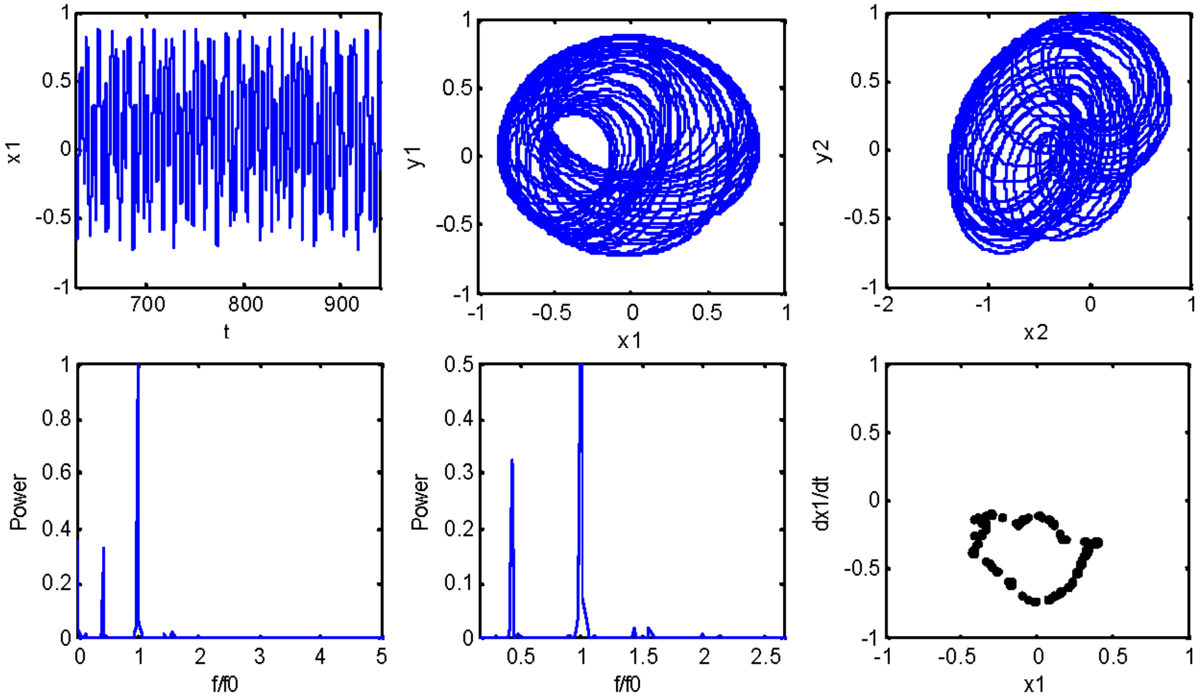

Fig. 3: Vibration waveform of rotor vibration, axis orbit of the rotor, axis orbit of the rotor disc, amplitude spectra of the vibration waveform, Poincare maps. (ω = 1160 rad/s)

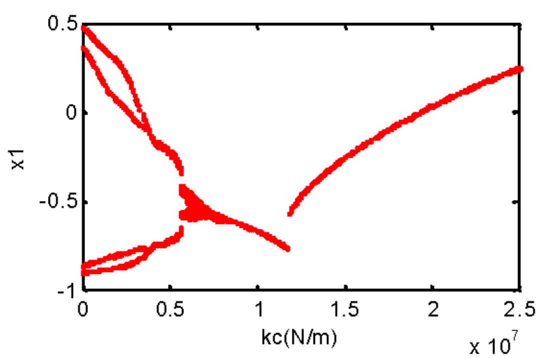

Fig. 4: Bifurcation diagram of k_c as a control parameter with bending vibration

Influences about bearing-sliding rotor system by the variety of k_c: The bifurcation diagram of k_c as a control parameter with bending vibration is shown as Fig. 4, when ω = 800 rad/s. Form Fig. 4, we can see that system has different motion states, chaos state, periodic motion state and quasi-periodic motion state. When the value of k_c is small, nonlinear oil-film force and the nonlinear stiffness of rotor affect the system greatly and the system is in P-4 motion state. With the increment of k_c, the system's response is in P-2 motion state, finally into the P-1 motion state. Make k_c increasing can take the oil film whirl began to recede. When k_c changes from 0 to 3.6×10^6 N/m, the system is in P-4 motion state and the oil film whirl is seriously. For the increase of k_c, the system is in P-2 motion state shortly. When k_c increased from 4.1 to 6.1×10^6 N/m, the response of the system shows more nonlinear characteristics and excited into chaotic state. At the same time, half-frequency component reduce the condition of oil film whirl further. For continues to increase k_c, the system is quickly into P-1 motion state. When k_c changes from 0 to 3.6×10^6 N/m, the system is in P-4 motion state. For the increase of k_c, the system is in P-2 motion state shortly. When k_c increased from 4.1 to 6.1×10^6 N/m, the response of the system shows more nonlinear characteristics and excited into chaotic state. At the same time, half-frequency component reduce the condition of oil film whirl further. For continues to increase k_c, the system is quickly into P-1 motion state.

Influences about bearing-sliding rotor response system by the variety of e: The shaft and the impeller always have the residual offset more or less, during the processing and installation. The eccentric weight of disk rotor could cause vibration during the rotation of rotor. In this study, pick out the system parameters as follows:

$k_c = 3.5\times10^6$ N/m, $k = 2.5\times10^7$ N/m,
$k_r = 2.5\times10^{12}$ N/m^3, e = 0.05 mm,
c = 0.11 mm, ω = 900 rad/s,
c1 = 1050 N.s/m, e = $[0\sim1.0\times10^{-4}]$m

The bifurcation diagram of ω as a control parameter with bending vibration is shown as Fig. 5. Form Fig. 5, we can see that system has different motion states, Chaos movement_Aperiodic motion and quasi-periodic motion.

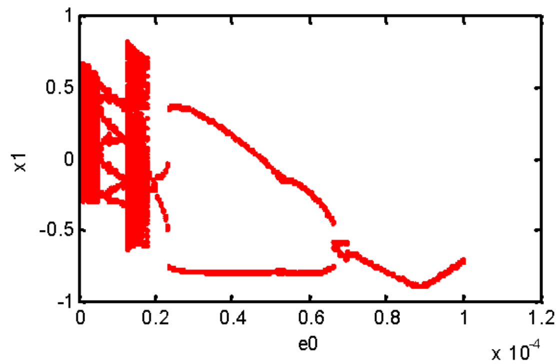

Fig. 5: Bifurcation diagram of e as a control parameter with bending vibration

When e changes from 0 to 0.55×10^{-5}m, the system is in choas state. There is a clear bifurcation of the responses of the system in Fig. 5, when e changes from 0.55 to 1.25×10^{-5}m.

Figure 6 shows the vibration waveform of rotor vibration, the axis orbit of the rotor, the axis orbit of the rotor disc, amplitude spectra of the vibration waveform and the Poincare maps as e = 0.4×10^{-5} m. Spectrogram of the vibration waveform has half-frequency component. The value of half-frequency component shows the oil film whirl is very seriously. The shape in poincare maps is a closed curve, it shows that the system is in choas state.

When e changes from 1.25 to 1.75×10^{-5} m, the system is in chaos state again. There is a clear bifurcation of the responses of the system in the Fig. 11, as e changes from 1.75 to 7.05×10^{-5} m. it shows the system is in P-2 motion state.

As e increases, the system is more stable and come in P-1 motion state.

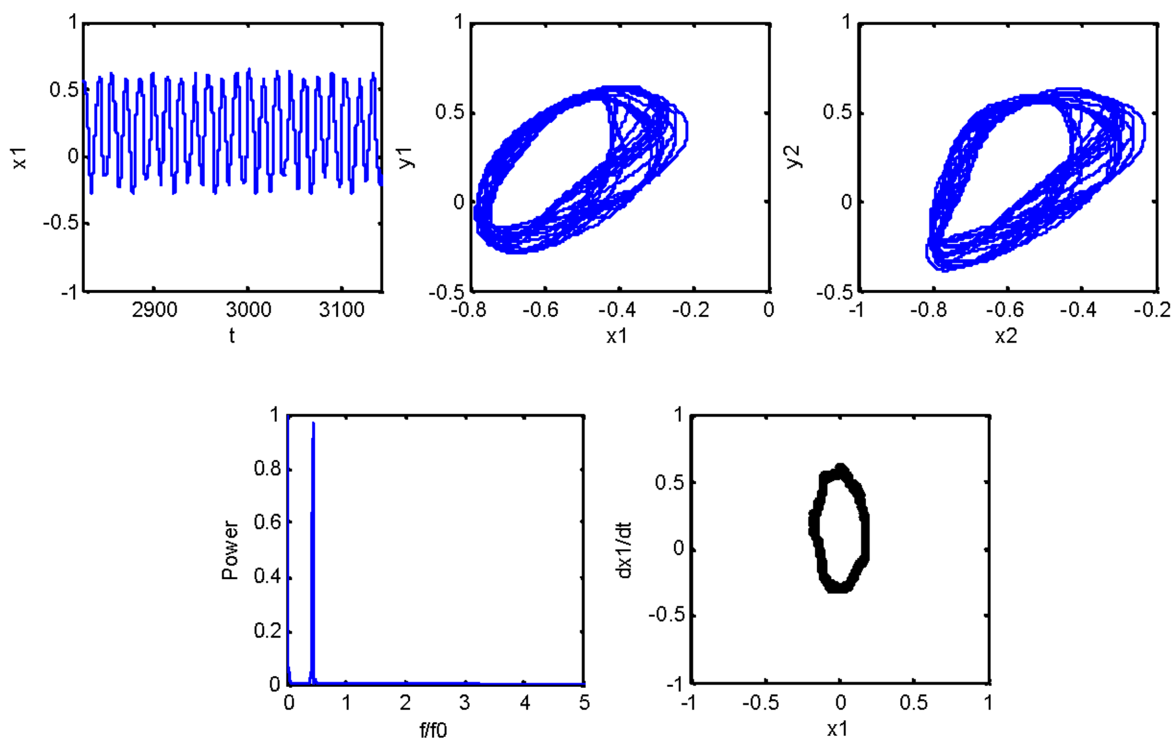

Fig. 6: Vibration waveform of rotor vibration, axis orbit of the rotor, axis orbit of the rotor disc, amplitude spectra of the vibration waveform, poincare maps. (e = 0.4×10^{-5} m)

CONCLUSION

This study studied a rub-impact dynamical behavior of a nonlinear rigid rotor-bearing system with nonlinear oil film force. The bifurcation and the chaos character of the rotor-bearing with operating speed changing are analyzed. The effect of the system parameter on the dynamical character is studied. The motion character with impect-rubbing in the system is obtained. There are doubling-periods, approximate-periods and chaos behaviors in the rotor system.

Through calculation and analysis, we can obtain the following conclusions.

As the rotation speed increases, the steady-state response of the system became more complex, from periodic motion to p-K periodic motion, eventually into a chaotic state. As the radial stiffness of the stator increases, the steady-state response of the system became more stability. As the eccentric weight of disk rotor increases, the steady-state response of the system became more stability. The response of the system always has chaos behaviors. As the eccentric weight of disk rotor increases, system gradually evolved into the quasi-periodic motion and periodic motion.

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