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Optimal Inventory Replenishment Strategy for Perishable Products in a Multi-period Environment

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Abstract: In this study, we investigate a multi-period deteriorating inventory model in which the demand is determined by inventory level and decreases with period. Considering the falling demand phenomenon for the perishable items which are out of fashion, we prove that the net profit has a maximum value about the length of waiting for inventory replenishment.

Key words: Perishable items, multi-period, inventory

INTRODUCTION

Because of the rapid development of technology and changes in customer preferences, the previous products will lose their competitiveness and out of fashion which are also reflected in a decline in the utility or demand. In addition, the demand of items is heavily dependent on the inventory level and is generally declined in this study. The products will be replenished to the maximal content of the stock after the same interval. At the end of the last period, the worth of the excess items will be treated as a constant salvage value which can be sold at a lower price.

In the researches of the demand in single-period, Urban and Baker (1997) investigated a single-period model in which the demand of the product is a deterministic, multivariate function of price, time and the level of inventory. Chun (2003) divided the period into several portions and each portion has its respective price. Abad (2003) developed a inventory model in which the demand of the product is sensitive to the price charged by the retail to the consumer. Backlogging is considered in Abad's model in order to avoid costs due to deterioration, though it will result in the loss of demand. However, we establish a multi-period inventory model in which the assumption of demand is gradually decreased during the periods and is dependent on the inventory level.

In the aspect of multi-period aspect, Wee (1995) assumed a inventory model of deterioration items with backlogging phenomenon appeared at the end of each period and the demand is gradually decreasing. On the contrary, Dye *et al.* (2006) supposed the gradually increasing demand and the shortages occurred in the beginning of each period. You and Chen (2007) segmented the customers into two types, namely spot purchase customers and the forward purchase customers. Tsao and Sheen (2008) adopted a price and time-dependent demand function to model the finite time horizon inventory for the perishable products. Wang and Lin (2012) considered a two-echelon supply chain with the demand which is exponentially decreasing and price-dependent. In this study, however, the demand in the model is dependent on stock and gradually decreased with periods.

The rest of this study is organized as follows. Section 2 presents the related assumption, notation and formulates the proposed model. Section 3 gives the solution procedure of the model and find out a optimal replenishment policy for the schedule.

NOTATIONS AND ASSUMPTIONS

Notations:

- $I_i(t)$ the inventory level at time t during the ith period
- $D_i(t)$ the demand rate at time t in the ith period
- θ the items deterioration rate (constant), $0<\theta<1$
- μ the demand decrease rate
- p the retail price per unit
- c the purchasing cost per unit
- W the maximum inventory level per period, i.e., the initial inventory level
- k the replenishment cost per order
- h the holding cost per unit item per unit time
- c_b salvage value per unit after n periods, $c_b<c$

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- T the length of each period
- π_n the total profits after n periods
- AP_n the profit per unit time during n periods

Assumptions:

- Replenishment rate is infinite and lead time is zero
- Shortage of stock is not allowed
- The distribution of time to the deterioration of the items follows exponential distribution with parameter θ (i.e., constant rate of deterioration)
- The demand rate function in the ith period $D_i(t)$, is deterministic and is a function of instantaneous stock level during the ith period $I_i(t)$; the functional $D_i(t)$ is given by: $D_i(t) = [\alpha+\beta I(t)]\mu^{i-1}$, where a and b are non-negative constants, μ is the demand decrease rate and $0<\mu<1$ and i = 1, 2, 3,..., n
- Considering that the benefit received from a unit of inventory is greater than or equal to its cost, we assume that:

$$\beta\mu^{i-1}p \ge \beta\mu^{i-1}c + c\theta + h,\ i = 1, 2, 3, \cdots, n$$

MATHEMATICAL FORMULATION

According to above assumptions, the inventory starts with full stock at zero time. Because of the demand and the deterioration, the stock decreases gradually from the top. T is the time when the stock is replenished to maximum scale. After n periods, the unsold product will be processed at salvage value c_b.

During $t\in[0, T]$, $I_i(t)$ is governed by:

$$\frac{dI_i(t)}{dt} = -D_i(t) - \theta I_i(t),\ I_i(0) = W \tag{1}$$

Combined with the demand rate function $D_i(t)$, the solution of Eq. 1 is:

$$I_i(t) = We^{-(\beta+\theta)t} + \frac{\alpha}{\beta+\theta}\left[e^{-(\beta+\theta)t} - 1\right] \tag{2}$$

And the inventory level at time T in the ith period is:

$$I_i(T) = We^{-\left(\beta\mu^{i-1}+\theta\right)T} + \frac{\alpha\mu^{i-1}}{\beta\mu^{i-1}+\theta}\left[e^{-\left(\beta\mu^{i-1}+\theta\right)T} - 1\right] > 0 \tag{3}$$

With the function of $D_i(t)$, we also can depict the income of the ith period:

$$P_i = p\int_0^T D_i(t)dt = p \left\{\frac{\mu^{i-1}\alpha\theta}{\beta\mu^{i-1}+\theta}T - \frac{W\beta\mu^{i-1}\left(\beta\mu^{i-1}+\theta\right) + \alpha\beta\mu^{2i-2}}{\left(\beta\mu^{i-1}+\theta\right)^2}\left[e^{-\left(\beta\mu^{i-1}+\theta\right)T} - 1\right]\right\} \tag{4}$$

the ordering quantity after the ith period as:

$$Q_i = W - I_i(T) = W\left[1 - e^{-\left(\beta\mu^{i-1}+\theta\right)T}\right] - \frac{\alpha\mu^{i-1}}{\beta\mu^{i-1}+\theta}\left[e^{-\left(\beta\mu^{i-1}+\theta\right)T} - 1\right] \tag{5}$$

the holding cost in the ith period as:

$$H_i = h\int_0^T I_i(t)\,dt = -\frac{\alpha hT\mu^{i-1}}{\beta\mu^{i-1}+\theta}\frac{h}{\beta\mu^{i-1}+\theta}\left(W + \frac{\alpha\mu^{i-1}}{\beta\mu^{i-1}+\theta}\right)\left[1 - e^{-\left(\beta\mu^{i-1}+\theta\right)T}\right] \tag{6}$$

Consequently, the total profits after n periods is:

$$\begin{aligned}
\pi_n &= \sum_{i=1}^{n} p\int_0^T D_i(t)dt + c_b I_n(T) - c\left(W + \sum_{i=1}^{n-1} Q_i\right) - \sum_{i=1}^{n} h\int_0^T I_i(t)\,dt - (n-1)k \\
&= \sum_{i=1}^{n} p\left\{\begin{array}{l}\dfrac{\mu^{i-1}\alpha\theta}{\beta\mu^{i-1}+\theta}T - \\ \dfrac{W\beta\mu^{i-1}\left(\beta\mu^{i-1}+\theta\right) + \alpha\beta\mu^{2i-2}}{\left(\beta\mu^{i-1}+\theta\right)^2}\left[e^{-\left(\beta\mu^{i-1}+\theta\right)T} - 1\right]\end{array}\right\} \\
&+ c_b\left\{We^{-\left(\beta\mu^{n-1}+\theta\right)T} + \frac{\alpha\mu^{n-1}}{\beta\mu^{n-1}+\theta}\left[e^{-\left(\beta\mu^{n-1}+\theta\right)T} - 1\right]\right\} \\
&- c\left\{W + \sum_{i=1}^{n-1}\left\{\begin{array}{l}W\left[1 - e^{-\left(\beta\mu^{i-1}+\theta\right)T}\right] \\ -\dfrac{\alpha\mu^{i-1}}{\beta\mu^{i-1}+\theta}\left[e^{-\left(\beta\mu^{i-1}+\theta\right)T} - 1\right]\end{array}\right\}\right\} \\
&- \sum_{i=1}^{n}\left\{\begin{array}{l}\dfrac{h}{\beta\mu^{i-1}+\theta}\left(W + \dfrac{\alpha\mu^{i-1}}{\beta\mu^{i-1}+\theta}\right)\left[1 - e^{-\left(\beta\mu^{i-1}+\theta\right)T}\right] \\ -\dfrac{\alpha hT\mu^{i-1}}{\beta\mu^{i-1}+\theta}\end{array}\right\} - (n-1)k
\end{aligned} \tag{7}$$

Therefore, we can further obtain the profit per unit time during n periods as the following:

$$AP_n = \frac{\pi_n(T)}{nT} \tag{8}$$

We will solve this equation and find optimal policies in the rest of the study.

The total profits π_n and the profit per unit time AP_n. We first present the key results.

Proposition 1: The second derivative of $\pi_n(T)$ is less than zero.

Proof: First note that from Eq. 7, the first derivative of $\pi_n(T)$ is:

$$\begin{aligned}
\pi_n{}'(T) &= \sum_{i=1}^{n} p\left\{\frac{\mu^{i-1}\alpha\theta}{\beta\mu^{i-1}+\theta} + \frac{W\beta\mu^{i-1}\left(\beta\mu^{i-1}+\theta\right) + \alpha\beta\mu^{2i-2}}{\beta\mu^{i-1}+\theta}e^{-\left(\beta\mu^{i-1}+\theta\right)T}\right\} \\
&- c_b\left[W\left(\beta\mu^{n-1}+\theta\right) + \alpha\mu^{n-1}\right]e^{-\left(\beta\mu^{n-1}+\theta\right)T} - \sum_{i=1}^{n-1} c\left[W\left(\beta\mu^{i-1}+\theta\right) + \alpha\mu^{i-1}\right] \\
&e^{-\left(\beta\mu^{i-1}+\theta\right)T} + \sum_{i=1}^{n} h\left[\frac{\alpha\mu^{i-1}}{\beta\mu^{i-1}+\theta} - \left(W + \frac{\alpha\mu^{i-1}}{\beta\mu^{i-1}+\theta}\right)e^{-\left(\beta\mu^{i-1}+\theta\right)T}\right]
\end{aligned} \tag{9}$$

Hence, we can further write the second derivative of $\pi_n(T)$ as the following:

$$\pi_n''(T)=\sum_{i=1}^{n}\left(h-p\mu^{i-1}\beta\right)\left[W\left(\beta\mu^{i-1}+\theta\right)+\alpha\mu^{i-1}\right]e^{-\left(\beta\mu^{i-1}+\theta\right)T}$$
$$+c_b\left(\beta\mu^{n-1}+\theta\right)\left[W\left(\beta\mu^{n-1}+\theta\right)+\alpha\mu^{n-1}\right]e^{-\left(\beta\mu^{n-1}+\theta\right)T} \quad (10)$$
$$+\sum_{i=1}^{n-1}c\left(\beta\mu^{i-1}+\theta\right)\left[W\left(\beta\mu^{i-1}+\theta\right)+\alpha\mu^{i-1}\right]e^{-\left(\beta\mu^{i-1}+\theta\right)T}$$

Follow the Assumptions, we have $\beta\mu^{i-1}p\geq\beta\mu^{i-1}c+c\theta+h$, $(i = 1, 2, 3,\ldots, n)$ and combined with $c_b<c$, $\pi_n''(T)$ can be derived as:

$$\pi_n''(T)<\sum_{i=1}^{n}\left(h-p\mu^{i-1}\beta\right)\left[W\left(\beta\mu^{i-1}+\theta\right)+\alpha\mu^{i-1}\right]e^{-\left(\beta\mu^{i-1}+\theta\right)T}$$
$$+c\left(\beta\mu^{n-1}+\theta\right)\left[W\left(\beta\mu^{n-1}+\theta\right)+\alpha\mu^{n-1}\right]e^{-\left(\beta\mu^{n-1}+\theta\right)T}$$
$$+\sum_{i=1}^{n-1}c\left(\beta\mu^{i-1}+\theta\right)\left[W\left(\beta\mu^{i-1}+\theta\right)+\alpha\mu^{i-1}\right]e^{-\left(\beta\mu^{i-1}+\theta\right)T} \quad (11)$$
$$=\sum_{i=1}^{n}\left[c\theta+h-\beta\mu^{i-1}(p-c)\right]\left[W\left(\beta\mu^{i-1}+\theta\right)+\alpha\mu^{i-1}\right]e^{-\left(\beta\mu^{i-1}+\theta\right)T}<0$$

This completes the proof of proposition 1.

Theorem 1: The profit per unit time AP_n is concave in T.

Proof: According to Eq. 9, the first derivative of $AP_n(T)$ is:

$$\frac{dAP_n}{dT}=\frac{\pi_n'(T)}{nT}-\frac{\pi_n(T)}{nT^2} \quad (12)$$

Hence, the second derivative of $AP_n(T)$ is:

$$\frac{d^2AP_n}{dT^2}=\frac{\pi_n''(T)}{nT}-\frac{2\pi_n'(T)}{nT^2}+\frac{2\pi_n(T)}{nT^3} \quad (13)$$

Define T = T0 as the condition of:

$$\frac{dAP_n}{dT}=\frac{\pi_n'(T)}{nT}-\frac{\pi_n(T)}{nT^2}=0$$

using proposition 1, we have:

$$\left.\frac{d^2AP_n}{dT^2}\right|_{T=T^0}=\frac{\pi_n''(T^0)}{nT^0}<\frac{1}{nT^0}\sum_{i=1}^{n}\left[c\theta+h-\beta\mu^{i-1}(p-c)\right] \quad (14)$$
$$\left[W\left(\beta\mu^{i-1}+\theta\right)+\alpha\mu^{i-1}\right]e^{-\left(\beta\mu^{i-1}+\theta\right)T^0}<0$$

In theorem 1, we finally find a unique global maximum for the profit per unit time.

CONCLUSION

In this study, we mostly develop a multi-period deterioration inventory model. And the demand is gradually decrease with time and stock-sensitivity. We propose some useful procedures for finding the optimal length of the period which can maximize the unit profit of retailer. It is also interesting to study the dynamic pricing and replenishment strategy for deterioration products with other assumption of demand in the multi-period model.

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