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Mean-Square Exponential Stable Control for Fuzzy Networked Systems with Stochastic Delays

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Abstract: The problem of mean-square exponential stable control for fuzzy networked systems with stochastic delays is considered in this study. A new T-S fuzzy model of nonlinear networked control systems is derived. The networked induced delays are assumed to satisfy Bernoulli distribution characteristics. By constructing a new Lyapunov function, the mean-square exponential stabilization condition is obtained in terms of linear matrix inequality, then, the state feedback fuzzy controller is designed. Finally, a numerical example is given to demonstrate the validity of the results.

Key words: Networked control systems, mean-square exponential stable, stochastic delay

INTRODUCTION

Networked control systems (NCSs) (Fridman *et al.*, 2004) are systems where the feedback loop is closed via a communication network in which information, from various components such as sensors, controllers and actuators, is exchanged with limited bandwidth. NCSs have received increasing attentions in recent years (Mondie and Kharitonov, 2005; Gao *et al.*, 2008).

However, the network itself is dynamic system that exhibits characteristics such as network-induced delays. Those delays come from the time sharing of the communication medium (Walsh *et al.*, 2001). As is known, network-induced delays can degrade a system's performance and even cause system instability. Many researchers have studied stability analysis and controller design for NCSs (Xiong *et al.*, 2003). It is quite common in practice that the time delays occur in a random way, rather than a deterministic way. Both network-induced delay and packet dropout in network transmission have very large influence for stability of the whole system. A model of NCSs was provided under consideration of the network-induced delay and the packet dropout in transmission (Yue *et al.*, 2004). A scheduling method for network-based control systems is given to design the state feedback controller (Park *et al.*, 2002). Liu obtained a new stability criterion of networked predictive control systems with random network delay in the feedback channel (Liu and Xia, 2007). Zhang gave the guaranteed cost networked control for T-S fuzzy systems with time delays (Zhang *et al.*, 2007a). How to analyze stability of

nonlinear NCSs is a challenging and interesting topic. Some results about the stability of non-linear NCS were obtained. A new T-S model of NCSs with uncertainties is obtained in (Zhang *et al.*, 2007b). Based on the model, Zhang gave the approach to design the robust H_∞ controller which makes the NCSs exponentially stable. Jiang and Han designed fuzzy controllers for a class of non-linear networked control systems (Jiang and Han, 2008).

PROBLEM FORMULATION

Consider the following nonlinear control system with delay Rule i:

IF $z_1(t)$ is $M_1^i, \dots$ and $z_n(t)$ is M_n^i . Then:

$$\begin{aligned} \dot{x}(t) = & (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t-d) \\ & + (B_i + \Delta B_i(t))u(t)x(t) = \phi(t) \quad t \in [-d, 0] \end{aligned} \quad (1)$$

where, $z(t) = [z_1(t) \ z_2(t) \ \dots \ z_n(t)]^T$ is the premise variable, $x(t) \in \mathbb{R}^m$ is the systems state vector, $u(t) \in \mathbb{R}^m$ is the controlled input vector, M_k^i are fuzzy sets. $A, A_{di} \in \mathbb{R}^{n \times n}$ are known constant matrices, $B_i \in \mathbb{R}^{n \times m}$ is input matrix, $\phi(t) \in \mathbb{R}^n$ is the given initial state on $[-d, 0]$, d is state delay, q is the number of IF-THEN rules. $\Delta A_i(t), \Delta A_{di}(t) \in \mathbb{R}^{n \times n}$ representing the uncertainties and satisfying $[\Delta A_i(t) \ \Delta A_{di}(t) \ \Delta B_i(t)] = DF(t)[E_{i1} \ E_{i2} \ E_{i3}]$ where $D, E_{i1}, E_{i2}, E_{i3}$ are constant matrices with appropriate dimensions, $F(t)$ is a matrix with appropriate dimensions satisfying $F^T(t)F(t) \leq I$. By using a center average defuzzifier, product inference and a singleton fuzzifier, the

global dynamics of the T-S fuzzy systems are described by:

$$\begin{aligned} \dot{x}(t) = & \sum_i \mu_i(z(t))[(A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t-d) + \\ & (B_i + \Delta B_i(t))u(t)]x(t) = \phi(t) \quad t \in [-d, 0] \end{aligned} \quad (2)$$

Where

$$\omega_i(z(t)) = \prod_{k=1}^n M_k^i(z_k(t)), \quad \mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^q \omega_i(z(t))}$$

where $\omega_i(z(t))$ satisfying:

$$\omega_i(z(t)) \geq 0, \quad \sum_{i=1}^q \omega_i(z(t)) > 0, \quad i = 1, 2, \dots, q$$

Throughout this note, we suppose that all the system's states are available for a state feedback control. In the presence of the control network, data transfers between the controller and the remote system, e.g., sensors and actuators in a distributed control system will induce network delay in addition to the controller proceeding delay. We introduce stochastic delay $\tau(t)$ to denote the network-induced delay. In this note we make the following assumptions:

Assumption 1: Sensor and controller are clock-driven

Assumption 2: Actuator is event-driven.

We will design the state feedback fuzzy controller:

$$u(t) = \sum_i \mu_i(z(t))K_i x(t - \tau(t)) \quad (3)$$

where, $\tau(t)$ is the stochastic network-induced delay satisfying $\tau(t) \in [0, \tau]$.

Inserting the controller (3) into system (2), we obtain the closed system:

$$\begin{aligned} \dot{x}(t) = & \sum_i \sum_j \mu_i(z(t))\mu_j(z(t))[(A_i + \Delta A_i(t))x(t) \\ & + (A_{di} + \Delta A_{di}(t))x(t-d) + (B_i + \Delta B_i(t))K_j x(t - \tau(t))] \\ x(t) = & \psi(t) \quad t \in [-\bar{d}, 0] \end{aligned} \quad (4)$$

The initial condition of the state is supplemented as $x(t) = \psi(t)$, where $\psi(t)$ is a smooth function on $[-\bar{d}, 0]$, $\bar{d} = \max\{\tau, d\}$. Therefore, there exists a positive constant $\bar{\Psi}$ satisfying:

$$\|\psi(t)\| \leq \bar{\Psi} \quad t \in [-\bar{d}, 0]$$

It is assumed that there exists a constant $\tau_1 \in [0, \tau]$ such that the probability of $\tau(t)$ taking values on $[0, \tau_1)$ and $[\tau_1, \tau]$ can be observed. In order to employ the information of probability distribution of the delay in the system model, the following sets are proposed firstly:

$$\Omega_1 = \{t : \tau(t) \in [0, \tau_1)\}, \quad \Omega_2 = \{t : \tau(t) \in [\tau_1, \tau]\}$$

Obviously $\Omega_1 \cap \Omega_2 = \Phi$

Now we define two functions:

$$h_1(t) = \begin{cases} \tau(t) & t \in \Omega_1 \\ 0 & t \notin \Omega_1 \end{cases}, \quad h_2(t) = \begin{cases} \tau(t) & t \in \Omega_2 \\ \tau_1 & t \notin \Omega_2 \end{cases} \quad (5)$$

and a stochastic variable:

$$\beta(t) = \begin{cases} 1 & t \in \Omega_1 \\ 0 & t \in \Omega_2 \end{cases} \quad (6)$$

By using the new functions $h_1(t), h_2(t)$ and stochastic variable $\beta(t)$, the systems (3) can be equivalently written as:

$$\begin{aligned} \dot{x}(t) = & \sum_i \sum_j \mu_i(z(t))\mu_j(z(t))\bar{A}_{ij}\xi(t) \\ x(t) = & \phi(t) \quad t \in [-\bar{d}, 0] \end{aligned} \quad (7)$$

Where:

$$\begin{aligned} \bar{A}_{ij} = & [\bar{A}_i \quad \bar{A}_{di} \quad \beta(t)\bar{B}_i K_j \quad (1-\beta(t))\bar{B}_i K_j] \\ \xi^T(t) = & [x^T(t), x^T(t-d), x^T(t-h_1(t)), x^T(t-h_2(t))] \\ \bar{A}_i = & A_i + \Delta A_i(t) \quad \bar{A}_{di} = A_{di} + \Delta A_{di}(t) \\ \bar{B}_i = & B_i + \Delta B_i(t) \end{aligned}$$

MAIN RESULTS

Definition 1: Yue *et al.* (2005) For the systems (7), if there exists constants $\alpha > 0$ and $\gamma \geq 1$ such that:

$$E\{\|x(t)\|\} \leq \gamma \sup_{-\bar{d} \leq s \leq 0} E\{\|\psi(s)\|\} e^{-\alpha t}, \quad t \geq 0$$

Then the systems (7) is mean-square exponentially stable. Lemma 1 (Xia *et al.*, 2008) For matrices $X_i, Y_i (1 \leq i \leq r)$ and matrix $S > 0$ with appropriate dimensions, the following inequality is hold:

$$\begin{aligned}
 & 2 \sum_{i=1}^r \sum_{j=1}^r \sum_{p=1}^r \sum_{l=1}^r \mu_i \mu_j \mu_p \mu_l X_{ij}^T S Y_{pl} \\
 & \leq \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j (X_{ij}^T S X_{ij} + Y_{ij}^T S Y_{ij})
 \end{aligned}$$

where, $\mu_i (1 \leq i \leq r)$ denotes $\mu_i(z(t)) \geq 0$ and:

$$\sum_{i=1}^r \mu_i(z(t)) = 1$$

Lemma 2: Xia *et al.* (2009) For any vectors a , b and matrices N , X , Y , Z with appropriate dimensions, if the following matrix inequality holds:

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$$

then we have:

$$-2a^T N b \leq \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Lemma 3: Zheng *et al.* (2006) The LMI:

$$\begin{bmatrix} Y(x) & W(x) \\ * & R(x) \end{bmatrix} > 0$$

is equivalent to:

$$R(x) > 0, Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

where, $Y(x) = Y^T(x)$, $R(x) = R^T(x)$ depend on x .

Lemma 4: Zhou and Li (2005) For known constant $\varepsilon > 0$ and matrices D , E , F which satisfying $F^T F \leq I$, then the following matrix inequality is hold:

$$DEF + E^T F^T D^T \leq \varepsilon DD^T + \varepsilon^{-1} E^T E$$

Theorem 1: For the given constants $\alpha > 0, 1 \geq \beta \geq 0$ and $i, j = 1, 2, \dots, q$, if there exist positive-definite matrices $P, Q, R, T \in \mathbb{R}^{n \times n}$ and matrices $K_j \in \mathbb{R}^{m \times n}$ and X_{1ij}, X_{2ij}, Y_{ij} with appropriate dimensions, such that the following matrix inequalities hold:

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & \Theta_{23} & \Theta_{24} \\ * & * & \Theta_{33} & \Theta_{34} \\ * & * & * & \Theta_{44} \end{bmatrix} \quad (8)$$

Where:

$$\begin{aligned}
 \Theta_{11} &= P\bar{A}_i + \bar{A}_i^T P + Q + 2\alpha P + \tau_1 X_{111} + \tau X_{211} + Y_{11} \\
 &\quad + Y_{11}^T + Y_{21} + Y_{21}^T + \tau_1 \bar{A}_i^T R \bar{A}_i + \tau \bar{A}_i^T T \bar{A}_i \\
 \Theta_{12} &= P\bar{A}_d + \tau_1 X_{112} + \tau X_{212} + Y_{12}^T + Y_{22}^T + \tau_1 \bar{A}_i^T R \bar{A}_d + \tau \bar{A}_i^T T \bar{A}_d \\
 \Theta_{13} &= P\beta \bar{B}_i K_j + \tau_1 X_{113} + \tau X_{213} - Y_{11} + Y_{13}^T + Y_{23}^T \\
 &\quad + \tau_1 \bar{A}_i^T R \beta \bar{B}_i K_j + \tau \bar{A}_i^T T \beta \bar{B}_i K_j \\
 \Theta_{14} &= P(1-\beta) \bar{B}_i K_j + \tau_1 X_{114} + \tau X_{214} + Y_{14}^T - Y_{21} + Y_{24}^T \\
 &\quad + \tau_1 \bar{A}_i^T R (1-\beta) \bar{B}_i K_j + \tau \bar{A}_i^T T (1-\beta) \bar{B}_i K_j \\
 \Theta_{22} &= -e^{-2\alpha d} Q + \tau_1 X_{122} + \tau X_{222} + \tau_1 \bar{A}_d^T R \bar{A}_d + \tau \bar{A}_d^T T \bar{A}_d \\
 \Theta_{23} &= \tau_1 X_{123} + \tau X_{223} - Y_{12} + \tau_1 \bar{A}_d^T R \beta \bar{B}_i K_j + \tau \bar{A}_d^T T \beta \bar{B}_i K_j \\
 \Theta_{24} &= \tau_1 X_{124} + \tau X_{224} - Y_{22} + \tau_1 \bar{A}_d^T R (1-\beta) \bar{B}_i K_j \\
 &\quad + \tau \bar{A}_d^T T (1-\beta) \bar{B}_i K_j \\
 \Theta_{33} &= \tau_1 X_{133} + \tau X_{233} - Y_{13} - Y_{13}^T + \tau_1 K_j^T \bar{B}_i^T R \beta \bar{B}_i K_j \\
 &\quad + \tau K_j^T \bar{B}_i^T T \beta \bar{B}_i K_j \\
 \Theta_{34} &= \tau_1 X_{134} + \tau X_{234} - Y_{14}^T - Y_{23} \\
 \Theta_{44} &= \tau_1 X_{144} + \tau X_{244} - Y_{24} - Y_{24}^T + \tau_1 K_j^T \bar{B}_i^T R (1-\beta) \bar{B}_i K_j \\
 &\quad + \tau K_j^T \bar{B}_i^T T (1-\beta) \bar{B}_i K_j
 \end{aligned}$$

with the fuzzy controller (3), the network control systems (7) is mean-square exponentially stable.

Proof: Choose a Lyapunov functional candidate for the system (7) as follows:

$$\begin{aligned}
 V(t) &= x^T(t) P x(t) + \int_{t-d}^t x^T(s) Q e^{2\alpha(s-t)} x(s) ds \\
 &\quad + \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s) R e^{2\alpha(s-t)} \dot{x}(s) ds d\theta \\
 &\quad + \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s) T e^{2\alpha(s-t)} \dot{x}(s) ds d\theta
 \end{aligned}$$

where, P, Q, R, T positive-definite matrices in theorem 1. Then, along the solution of system (7), we have:

$$\begin{aligned}
 & \dot{V}(t) + 2\alpha V(t) \\
 &= 2x^T(t) P \dot{x}(t) + x^T(t) Q x(t) \\
 &\quad - x^T(t-d) Q e^{-2\alpha d} x(t-d) + \tau_1 \dot{x}^T(t) R \dot{x}(t) \\
 &\quad + \tau \dot{x}^T(t) T \dot{x}(t) + 2\alpha x^T(t) P x(t) \\
 &\quad - \int_{t-\tau_1}^t \dot{x}^T(s) R e^{2\alpha(s-t)} \dot{x}(s) ds - \int_{t-\tau}^t \dot{x}^T(s) T e^{2\alpha(s-t)} \dot{x}(s) ds
 \end{aligned} \quad (9)$$

With

$$x(t) - x(t-h_1(t)) - \int_{t-h_1(t)}^t \dot{x}(s) ds = 0$$

and

$$x(t) - x(t - h_2(t)) - \int_{t-h_2(t)}^t \dot{x}(s) ds = 0$$

For any $4n \times n$ matrices:

$$N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}, \quad M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

and

we know:

$$0 = \xi^T(t) N [x(t) - x(t - h_1(t)) - \int_{t-h_1(t)}^t \dot{x}(s) ds] \quad (10)$$

$$0 = \xi^T(t) M [x(t) - x(t - h_2(t)) - \int_{t-h_2(t)}^t \dot{x}(s) ds] \quad (11)$$

With lemma 2 and (10-11), we obtain:

$$0 \leq 2\xi^T(t) Y_1 [x(t) - x(t - h_1(t))] + \tau_1 \xi^T(t) X_1 \xi(t) + \int_{t-\tau_1}^t \dot{x}^T(s) R e^{2\alpha(s-t)} \dot{x}(s) ds \quad (12)$$

$$0 \leq 2\xi^T(t) Y_2 [x(t) - x(t - h_2(t))] + \tau_2 \xi^T(t) X_2 \xi(t) + \int_{t-\tau_2}^t \dot{x}^T(s) T e^{2\alpha(s-t)} \dot{x}(s) ds \quad (13)$$

Inserting (12-13) into (9), we obtain:

$$\begin{aligned} & \dot{V}(t) + 2\alpha V(t) \\ & \leq \sum_i \sum_j h_i(z(t)) h_j(z(t)) \{x^T(t) [P \bar{A}_i + \bar{A}_i^T P + Q \\ & \quad + 2\alpha P] x(t) + 2x^T(t) P \bar{A}_{\bar{a}} x(t - d) \\ & \quad + 2x^T(t) P \beta(t) \bar{B}_i K_j x(t - h_1(t)) \\ & \quad + 2x^T(t) P (1 - \beta(t)) \bar{B}_i K_j x(t - h_2(t)) \\ & \quad - x^T(t - d) Q e^{-2\alpha d} x(t - d) \\ & \quad + 2\xi^T(t) Y_i [x(t) - x(t - h_1(t))] \\ & \quad + \tau_1 \xi^T(t) X_1 \xi(t) + 2\xi^T(t) Y_2 [x(t) - x(t - h_2(t))] \\ & \quad + \tau_2 \xi^T(t) X_2 \xi(t) \} + \tau_1 \dot{x}^T(t) R x(t) + \tau_2 \dot{x}^T(t) T x(t) \end{aligned} \quad (14)$$

With Lemma 1, we have:

$$\begin{aligned} \tau_1 \dot{x}^T(t) R x(t) & \leq \tau_1 \sum_i \sum_j \mu_i(z(t)) \mu_j(z(t)) \xi^T(t) \bar{A}_{\bar{a}}^T R \bar{A}_{\bar{a}} \xi(t) \\ & = \tau_1 \sum_i \sum_j \mu_i(z(t)) \mu_j(z(t)) \xi^T(t) \end{aligned}$$

$$\bullet \begin{bmatrix} \bar{A}_i^T R \bar{A}_i & \bar{A}_i^T R \bar{A}_{\bar{a}} & \bar{A}_i^T R \beta(t) \bar{B}_i K_j \\ * & \bar{A}_{\bar{a}}^T R \bar{A}_{\bar{a}} & \bar{A}_{\bar{a}}^T R \beta(t) \bar{B}_i K_j \\ * & * & \beta^2(t) K_j^T \bar{B}_i^T R \bar{B}_i K_j \\ * & * & * \end{bmatrix} \xi(t)$$

$$\begin{bmatrix} \bar{A}_i^T R (1 - \beta(t)) \bar{B}_i K_j \\ \bar{A}_{\bar{a}}^T R (1 - \beta(t)) \bar{B}_i K_j \\ \beta(t) (1 - \beta(t)) K_j^T \bar{B}_i^T R \bar{B}_i K_j \\ (1 - \beta(t))^2 K_j^T \bar{B}_i^T R \bar{B}_i K_j \end{bmatrix} \xi(t) \quad (15)$$

$$\begin{aligned} \tau \dot{x}^T(t) T x(t) & \leq \tau \sum_i \sum_j \mu_i(z(t)) \mu_j(z(t)) \xi^T(t) \bar{A}_{\bar{a}}^T T \bar{A}_{\bar{a}} \xi(t) \\ & = \tau \sum_i \sum_j \mu_i(z(t)) \mu_j(z(t)) \xi^T(t) \\ & \bullet \begin{bmatrix} \bar{A}_i^T T \bar{A}_i & \bar{A}_i^T T \bar{A}_{\bar{a}} & \bar{A}_i^T T \beta(t) \bar{B}_i K_j \\ * & \bar{A}_{\bar{a}}^T T \bar{A}_{\bar{a}} & \bar{A}_{\bar{a}}^T T \beta(t) \bar{B}_i K_j \\ * & * & \beta^2(t) K_j^T \bar{B}_i^T T \bar{B}_i K_j \\ * & * & * \end{bmatrix} \xi(t) \end{aligned} \quad (16)$$

It is easy to know that:

$$\begin{aligned} & 2\xi^T(t) Y_1 [x(t) - x(t - h_1(t))] \\ & = \xi^T(t) \begin{bmatrix} Y_{11} + Y_{11}^T & Y_{12}^T & -Y_{11} + Y_{13}^T & Y_{14}^T \\ * & 0 & -Y_{12} & 0 \\ * & * & -Y_{13} - Y_{13}^T & -Y_{14}^T \\ * & * & * & 0 \end{bmatrix} \xi(t) \end{aligned} \quad (17)$$

$$\begin{aligned} & 2\xi^T(t) Y_2 [x(t) - x(t - h_2(t))] \\ & = \xi^T(t) \begin{bmatrix} Y_{21} + Y_{21}^T & Y_{22}^T & Y_{23}^T & -Y_{21} + Y_{24}^T \\ * & 0 & 0 & -Y_{22} \\ * & * & 0 & -Y_{23} \\ * & * & * & -Y_{24} - Y_{24}^T \end{bmatrix} \xi(t) \end{aligned} \quad (18)$$

Inserting (15-18) into (14), we have:

$$\begin{aligned} & E\{\dot{V}(t) + 2\alpha V(t)\} \\ & \leq \sum_i \sum_j \mu_i(z(t)) \mu_j(z(t)) \xi^T(t) \Theta \xi(t) \end{aligned}$$

With matrix inequality (5), we know:

$$E\{\dot{V}(t)\} < -2\alpha E\{V(t)\}$$

Therefore:

$$\begin{aligned} E\{V\} &< E\{V(0)\}e^{-2\alpha t} \leq [\lambda_{\max}(P) + d\lambda_{\max}(Q) \\ &+ \tau_1\lambda_{\max}(R)\bar{\Psi}^2 + \tau\lambda_{\max}(T)\bar{\Psi}^2]E\{\|\psi(t)\|^2\}e^{-2\alpha t} \end{aligned} \quad (19)$$

Obviously:

$$E\{V(t)\} \geq \lambda_{\min}(P)E\{\|x(t)\|^2\} \quad (20)$$

From (19-20), we obtain:

$$\begin{aligned} E\{\|x(t)\|\} &< \sqrt{\frac{\lambda_{\max}(P) + d\lambda_{\max}(Q) + \tau_1\lambda_{\max}(R)\bar{\Psi}^2 + \tau\lambda_{\max}(T)\bar{\Psi}^2}{\lambda_{\min}(P)}} E\{\|\psi(t)\|\}e^{-\alpha t} \end{aligned}$$

With the Lyapunov stability theorem and the above inequality, we know that the system (7) is exponentially stable.

Theorem 2: For the given constants $\alpha > 0, 1 \geq \beta \geq 0$ and $i, j = 1, 2, \dots, q$, if there exist positive-definite matrices $\bar{P}, \bar{Q}, \bar{R}, \bar{T} \in \mathbb{R}^{n \times n}$ and matrices $\bar{K}_j \in \mathbb{R}^{m \times n}, \bar{X}_{1ij}, \bar{X}_{2ij}, \bar{Y}_{ij}$, with appropriate dimensions, such that the following linear matrix inequalities hold:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} < 0 \quad (21)$$

Where:

$$\begin{aligned} \Xi_{11} = & \begin{bmatrix} A_i\bar{P} + \bar{P}A_i^T + \bar{Q} + 2\alpha\bar{P} & A_d\bar{P} + \tau_1\bar{X}_{112} \\ +\tau_1\bar{X}_{111} + \tau\bar{X}_{211} + \bar{Y}_{11} & +\tau\bar{X}_{212} + \bar{Y}_{12} \\ +\bar{Y}_{11}^T + \bar{Y}_{21} + \bar{Y}_{21}^T + \epsilon_1DD^T & +\bar{Y}_{22}^T \\ * & -e^{-2\alpha d}\bar{Q} + \tau_1\bar{X}_{122} + \tau\bar{X}_{222} \\ * & * \\ * & * \\ \beta B_i\bar{K}_j + \tau_1\bar{X}_{113} + \tau\bar{X}_{213} & (1-\beta)B_i\bar{K}_j + \tau_1\bar{X}_{114} \\ -\bar{Y}_{11} + \bar{Y}_{13}^T + \bar{Y}_{23}^T & +\tau\bar{X}_{214} + \bar{Y}_{14}^T - \bar{Y}_{21} + \bar{Y}_{24}^T \\ \tau_1\bar{X}_{123} + \tau\bar{X}_{223} - \bar{Y}_{12} & \tau_1\bar{X}_{124} + \tau\bar{X}_{224} - \bar{Y}_{22} \\ \tau_1\bar{X}_{123} + \tau\bar{X}_{223} - \bar{Y}_{13} - \bar{Y}_{13}^T & \tau_1\bar{X}_{134} + \tau\bar{X}_{234} - \bar{Y}_{14}^T - \bar{Y}_{23} \\ * & \tau_1\bar{X}_{144} + \tau\bar{X}_{244} - \bar{Y}_{24} - \bar{Y}_{24}^T \end{bmatrix} \\ \Xi_{12} = & \begin{bmatrix} \tau_1\beta\bar{P}A_i^T & \tau_1(1-\beta)\bar{P}A_i^T & \tau\beta\bar{P}A_i^T & \tau(1-\beta)\bar{P}A_i^T \\ \tau_1\beta\bar{P}A_d^T & \tau_1(1-\beta)\bar{P}A_d^T & \tau\beta\bar{P}A_d^T & \tau(1-\beta)\bar{P}A_d^T \\ \tau_1\beta\bar{K}_j^TB_i^T & 0 & \tau\beta\bar{K}_j^TB_i^T & 0 \\ 0 & \tau_1(1-\beta)\bar{K}_j^TB_i^T & 0 & \tau(1-\beta)\bar{K}_j^TB_i^T \\ \bar{P}E_{i1}^T & \tau_1\beta\bar{P}E_{i1}^T & \tau_1(1-\beta)\bar{P}E_{i1}^T & \tau\beta\bar{P}E_{i1}^T & \tau(1-\beta)\bar{P}E_{i1}^T \\ \bar{P}E_{i2}^T & \tau_1\beta\bar{P}E_{i2}^T & \tau_1(1-\beta)\bar{P}E_{i2}^T & \tau\beta\bar{P}E_{i2}^T & \tau(1-\beta)\bar{P}E_{i2}^T \\ \beta\bar{K}_j^TE_{i3}^T & \tau_1\beta\bar{K}_j^TE_{i3}^T & 0 & \tau\beta\bar{K}_j^TE_{i3}^T & 0 \\ (1-\beta)\bar{K}_j^TE_{i3}^T & 0 & \tau_1(1-\beta)\bar{K}_j^TE_{i3}^T & 0 & \tau(1-\beta)\bar{K}_j^TE_{i3}^T \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \Xi_{22} = & \text{diag}\{-\tau_1\beta\bar{R} + \epsilon_2DD^T, -\tau_1(1-\beta)\bar{R} + \epsilon_3DD^T, -\tau\beta\bar{T} \\ & + \epsilon_4DD^T, -\tau(1-\beta)\bar{T} + \epsilon_5DD^T, -\epsilon_1I, -\epsilon_2I, -\epsilon_3I, -\epsilon_4I, -\epsilon_5I\} \end{aligned}$$

with $K_1 = \bar{K}_1\bar{P}^{-1}, K_2 = \bar{K}_2\bar{P}^{-1}$, the systems (7) is mean-square exponentially stable.

Proof: With lemma 3 and lemma4, by giving some transformations:

$$\begin{aligned} \bar{P} &= P^{-1}, \quad \bar{Q} = P^{-1}QP^{-1}, \quad \bar{K}_j = K_jP^{-1}, \quad \bar{X}_{ijk} = P^{-1}X_{ijk}P^{-1}, \\ \bar{Y}_{ij} &= P^{-1}Y_{ij}P^{-1}, \quad \bar{R} = R^{-1}, \quad \bar{T} = T^{-1} \end{aligned}$$

we know that inequality $\Theta < 0$ is equivalent to (21). Therefore, the linear matrix inequality (21) is equivalent to (8). With Theorem1, the systems (7) is mean-square exponentially stable.

CONCLUSION

This study considers the exponential stability control problem for a class of nonlinear networked control systems (NCSs) with stochastic network-induced delay. A T-S fuzzy model is employed to represent the nonlinear controlled plant in the NCSs. With the Lyapunov stability theory, the exponential stability condition and the state feedback fuzzy controller design method are given in term of LMI.

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