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On the Performance of Dual-hop Amplify-and-forward Transmission in Asymmetric Multipath/shadowing Fading Channels

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Abstract: In this study, we present the end-to-end performance of a dual-hop amplify-and-forward relaying system over asymmetric multipath/shadowing fading environment, where the composite Nakagami-lognormal distribution is approximated by using mixture gamma distribution. Based on the cumulative distribution function of the end-to-end signal-to-noise ratio, novel closed-form expressions of the outage probability and symbol error rate for the dual-hop system are derived. Finally, numerical and simulation results are shown to verify the accuracy of the analytical results.

Key words: Dual-hop relaying, asymmetric fading channels, mixture gamma distribution, performance analysis

INTRODUCTION

Multihop cooperative transmission has emerged as a promising technique for extending coverage, enhancing connectivity and saving transmitter power in wireless networks. In a cooperative network, a source communicates with the destination via one or several intermediate terminals called relays. In the past decade, the performance of cooperative networks has been widely studied in term of Outage Probability (OP) and average bit/symbol error rate (ABER/ASER) for various systems (dual-hop/multi-hop/two-way) and channel models (Rayleigh, Nakagami-m, Rician, Weibull, Lognormal and Generalized-K (GK)).

In a practical scenario, Relaying nodes (R) are usually located in different geographical locations and at different distances with respect to the source node (S) and the Destination node (D). The signal in one link may be in Line Of Sight (LOS) situation and other links may be in NLOS situation, even in shadowing or composite multipath/shadowing situation. In the published literature, such scenario has been regarded as asymmetric or mixed fading models. On the contrary, the channel situation is regarded as symmetric fading models in which all the links experience the same fading conditions. So far, most previous works have considered the latter, only a few works have evaluated the former.

Recently, there is an increasing research interest on the former. Suraweera *et al.* (2009) studied the end-to-end performance of dual-hop Amplify-and-Forward (AF) relaying over mixed Rayleigh and Rician fading channels. After that, more cooperative models are studied in mixed

Rayleigh and Rician fading channels, for example, the dual-hop Decode-and-Forward (DF) cooperative model (Yang *et al.*, 2011), the dual-hop AF cooperative model (Ouyang *et al.*, 2012), the two-hop AF model with beamforming (Chen *et al.*, 2010) and so on. Xu *et al.* (2010) analyzed the performance of dual-hop AF relaying in mixed Nakagami and Rician fading channels. Nuri *et al.* (2013) studied the performance of an AF cooperative system under mixed Rayleigh and generalized Gamma fading channels.

Despite these recent studies related to the analysis of AF or DF relaying over mixed fading channels, their fading condition is limited to the mixture of various multipath fading, i.e., Rayleigh/Ricean, Nakagami-m/Ricean and Rayleigh/Gamma fading. The performance of the cooperative networks has not as yet been studied under mixed multipath/shadowing fading channels. Recently, Atapattu *et al.* (2010) developed an alternative approach to approximate the Nakagami-lognormal (NL) distribution by using the Mixture Gamma (MG) distribution. To the best of our knowledge, there are few studies in performance analysis of cooperative system over mixed fading channels by using MG fading model.

In this study, we consider an asymmetric scenario of a dual-hop AF relaying system. The S-R (first-hop) and the R-D (second-hop) links experience Nakagami-m or NL fading, where the NL fading model is approximated by using MG model. The main contributions of this study are as follows: Firstly, some exact closed-form expressions of OP and SER for the dual-hop system over asymmetric fading channels are derived. And then, the numerical and simulation results are given to show the accuracy of the theoretical analysis.

SYSTEM AND CHANNEL MODELS

We consider a wireless dual-hop AF relaying system consisting of S, D and R. The whole transmission is divided into two phase. In the first phase, S only transmits its signals to R and in the second phase, R amplifies the received signal by a gain factor and then forwards their amplified versions to D. Thus, the instantaneous end-to-end signal-to-noise (SNR), γ_{SRD} , at the D can be expressed as (Laneman *et al.*, 2004; Hasna and Alouini, 2003):

$$\gamma_{SRD} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + c} \tag{1}$$

where, $\gamma_i = \rho |h_i|^2$ is the instantaneous SNR of the *i*th-hop link, $i \in \{1, 2\}$, $\rho = P/N_0$ denotes the un-faded SNR, $|h_i|$ is the fading amplitude of the *i*th-hop link, P is the transmitted power of S or R, N_0 is the power of the additive white Gaussian noise component. In addition, exact γ_{SRD} is given by substituting $c = 1$ when the gain factor is $1/(P|h_1|^2 + N_0)$ and well approximated at medium and high SNR by substituting $c = 0$ when N_0 in the gain factor is ignored.

Note that due to the symmetry of γ_{SRD} in (1) with respect to γ_1 and γ_2 , the statistics of γ_{SRD} will be identical despite that the *i*th-hop link is subject to Nakagami-*m* or NL fading. If the *i*th-hop link experiences Nakagami-*m* fading, γ_i is a Gamma distributed variable with the Probability Density Function (PDF) given by Simon and Alouini (2005):

$$f_{\gamma_i}(\gamma) = \frac{m_i^{m_i} \gamma^{m_i-1}}{\Gamma(m_i) \bar{\gamma}_i^{m_i}} \exp\left[-\frac{m_i \gamma}{\bar{\gamma}_i}\right] \tag{2}$$

where, $\bar{\gamma}_i = \rho E[|h_i|^2] = \rho \Omega_i$ is the average SNR of the *i*th-hop link, m_i is Nakagami-*m* fading parameter, $E(*)$ denotes the statistical expectation, $\Gamma(*)$ is the standard Gamma function. Due to capture the path-loss effect, we use the local mean power $\Omega_i = (d_0/d_i)^\epsilon$, d_0 denotes the distance between S and D, d_i is the distance of the *i*th-hop link and ϵ is the path-loss exponent. The Cumulative Distribution Function (CDF) of γ_i can be obtained as Simon and Alouini (2005):

$$F_{\gamma_i}(\gamma) = 1 - \frac{\Gamma(m_i, m_i \gamma / \bar{\gamma}_i)}{\Gamma(m_i)} \tag{3}$$

where, $\Gamma(*, *)$ is the incomplete gamma function.

When the *i*th-hop link experiences NL fading, γ_i is a composite Gamma-lognormal distribution variable with the PDF given by Simon and Alouini (2005):

$$f_{\gamma_i}(\gamma) = \int_0^\infty \frac{m_i^{m_i} \gamma^{m_i-1} \exp(-m_i \gamma / \rho y)}{\Gamma(m_i) (\rho y)^{m_i}} \frac{1}{\sqrt{2\pi} \lambda_i y} \exp\left[-\frac{(\ln y - \mu_i)^2}{2\lambda_i^2}\right] dy \tag{4}$$

where, μ_i and γ_i are the mean and the standard deviation of lognormal shadowing, respectively. $\mu_i = \ln \Omega_i$, $\gamma_i = (\ln 10/10) \sigma$, σ is the standard deviation in dB.

Since a closed-form expression of (4) is not available in the published literature, the performance metrics of digital systems over composite NL distribution are intractable. Some approximations of (4) have been given great attention recently, such as, KG distribution and MG distribution. Due to that KG distribution includes modified Bessel functions, some expressions of the performance metrics still keep mathematical complications and further approximations have to be adopted. In order to avoid the above problems, we use MG distribution to approximate the composite NL distribution in this study. Thus, the PDF of γ_i can be expressed as (Atapattu *et al.*, 2010):

$$f_{\gamma_i}(\gamma) = \sum_{j=1}^N \frac{C a_j}{2 \rho^{m_j}} \gamma^{m_j-1} \exp\left(-\frac{b_j \gamma}{\rho}\right) \tag{5}$$

Where:

$$a_j = 2 m_j^{m_j} w_j \exp[-m_j(\sqrt{2} \lambda_{t_j} + \mu_i)] / \sqrt{\pi} \Gamma(m_j), \quad b_j = m_j \exp[-(\sqrt{2} \lambda_{t_j} + \mu_i)]$$

C is the normalization factor, defined as:

$$C = \sqrt{\pi} / \sum_{j=1}^N w_j$$

w_j and t_j are abscissas and weight factors for Gaussian- Hermite integration. w_j and t_j for different *N* values are available (Abramowitz and Stegun, 1965). The CDF of γ_i over MG fading can be obtained as:

$$F_{\gamma_i}(\gamma) = 1 - \sum_{j=1}^N \frac{C a_j}{2 b_j^{m_j}} \Gamma(m_j, b_j \gamma / \rho) \tag{6}$$

PERFORMANCE ANALYSIS

Outage probability: The OP is an important performance metric that is commonly used to characterize a wireless communication system. It is defined as the probability that the instantaneous SNR (γ) falls below a given threshold (γ_{th}), this is:

$$P_{out} = F_{\gamma}(\gamma_{th}) = \Pr(\gamma \leq \gamma_{th}) = \int_0^{\gamma_{th}} f_{\gamma}(\gamma) d\gamma \tag{7}$$

where $F_{\gamma}(\gamma)$ and $f_{\gamma}(\gamma)$ is the CDF and PDF of γ , respectively.

For the dual-hop system, assuming that the first-hop link is subject to Nakagami-*m* fading and the second-hop link is subject to NL fading, by using (1) and (7), the OP can be expressed as:

$$P_{out} = \Pr(\gamma_{SRD} \leq \gamma_{th}) = \Pr\left[\frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + c} \leq \gamma_{th}\right] \quad (8)$$

After applying some manipulations, (8) can be rewritten as:

$$P_{out} = 1 - \int_0^\infty \bar{F}_1(\gamma_{th} + \frac{\gamma_{th}^2 + c\gamma_{th}}{x}) f_1(\gamma_{th} + x) dx \quad (9)$$

where, $\bar{F}_\gamma(*)$ is the complementary CDF of γ_1 , which is defined as $\bar{F}_\gamma(*) = 1 - F_\gamma(*)$. According to Nakagami-m fading of the first-hop link $\bar{F}_\gamma(*)$, can be expressed by using (3) as:

$$\bar{F}_\gamma(\gamma_{th}) = \frac{1}{\Gamma(m_1)} \Gamma(m_1, \frac{m_1}{\rho\Omega_1} (\gamma_{th} + \frac{\gamma_{th}^2 + c\gamma_{th}}{x})) \quad (10)$$

By using (5) and (10) and substituting them into (11), then using the series expression of $\Gamma(*,*)$ and the binomial expansion when m_1 is an integer, we can obtain the OP as:

$$P_{out} = 1 - \sum_{i=1}^N \sum_{k=0}^{m_1-1} \sum_{j=0}^{m_1-1-k} \Xi(i,k,r,j) \exp[-\Phi(i)\gamma_{th}/\rho] K_{m_1+r_j}[2\Theta(i)] \quad (11)$$

Where:

$$\Xi(i,k,r,j) = \frac{C_{m_1-1}^i C_k^j C_a C_b m_1^{k+\frac{m_1-1}{2}} \gamma_{th}^{k+\frac{m_1-1}{2}} (\gamma_{th}+c)^{\frac{m_1-1}{2}}}{b_1^{m_1-1} \rho^{m_1-j} \Omega_1^{k+\frac{m_1-1}{2}} k!},$$

$$\Phi(i) = m_1/\Omega_1 + b_1, \Theta(i) = \sqrt{(\gamma_{th}^2 + c\gamma_{th}) b_1 m_1 / \Omega_1 \rho^2},$$

$$C_j^i = j! / [(j-i)! i!]$$

is the binomial coefficients, $K\alpha(*)$ is the second kind modified Bessel function of order α .

Average symbol error rate: The average SER is a useful measurement for investigating the performance of wireless systems. The ASER of M-ary modulations can be obtained by using Moment-Generation Function (MGF) approach. MGF is defined as Laplace transform of PDF. However, MGF can also be obtained by using CDF. By using the integration property of Laplace transform, the MGF of γ_{SRD} can be obtained as:

$$M(s) = \int_0^\infty \exp(-sx) F_{SRD}(x) dx \quad (12)$$

where, $F_{SRD}(x)$ can be obtained by substituting $\gamma_{th} = x$ into (11) with $c = 0$ which is analytically more tractable. Therefore, by using Eq. (6.621.3) (Gradshteyn *et al.*, 2000), the analytical expression of (12) over mixed fading channels can be obtained as:

$$M_{\gamma_{SRD}}(s) = 1 - \sum_{i=1}^N \sum_{k=0}^{m_1-1} \sum_{r=0}^{m_2-1-k} \sum_{j=0}^k \Upsilon(i,k,r,j) {}_2F_1(u+v, v+1/2; u+1/2; \frac{\Phi(i)\zeta(i)+\rho s}{\Phi(i)\zeta(i)+\rho s}) \quad (13)$$

where:

$$u = m_2 + k + 1, v = m_2 - r - j, \zeta(i) = 2\sqrt{m_1 b_1 / \Omega_1},$$

$$\Upsilon(i,k,r,j) = \frac{4^{m_2-r-j} s \sqrt{\pi} C_{m_2-1}^r C_k^j C_a C_b \Gamma(u+v) \Gamma(u-v)}{k! (\Omega_1 / m_1)^{k+r} [\Phi(i) + \zeta(i) + \rho s]^{u+v} \Gamma(u+0.5)}$$

${}_2F_1(a,b,c;z)$ is the hypergeometric function.

By using the MGF method, the ASER of M-ary phase-shift keying signals (MPSK) can be given by Simon and Alouini (2005):

$$P_{e-MPSK} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\gamma_{SRD}}(\frac{g_M}{\sin^2 \theta}) d\theta \quad (14)$$

where, $g_M = \sin^2(\pi/M)$. Thus, the ASER of dual-hop system can be numerically evaluated by substituting (13) into (14).

NUMERICAL AND SIMULATION RESULTS

In this section, we present some numerical and simulation results to evaluate the system performance in asymmetric fading channels, where the S-R link is subject to the Nakagami-m fading, R-D link is subject to the MG fading, vice versa.

Figure 1 illustrates the ASER of BPSK and 16PSK of the dual-hop system. In this case, a symmetric network geometry is assumed, where $d_0 = 1, d_1 = d_2 = 0.5, \epsilon = 4$. Each hop has the same multipath parameters ($m_1 = m_2 = 2$), $N = 10$ for MG model. It can be seen from Fig. 1 that the analytical results of (14) coincide perfectly with the

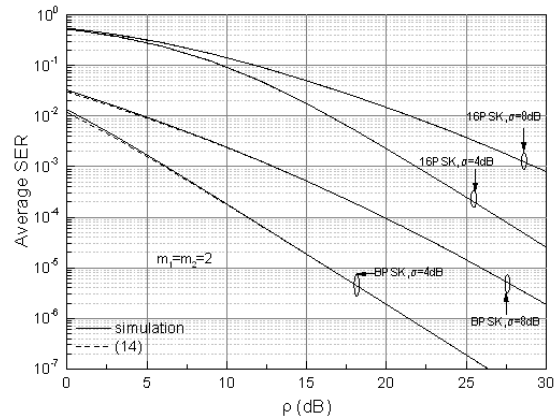


Fig. 1: ASER of BPSK and 16PSK for the dual-hop system versus ρ

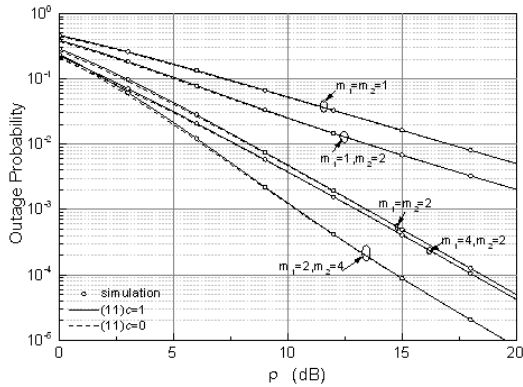


Fig. 2: OP for the dual-hop system versus ρ

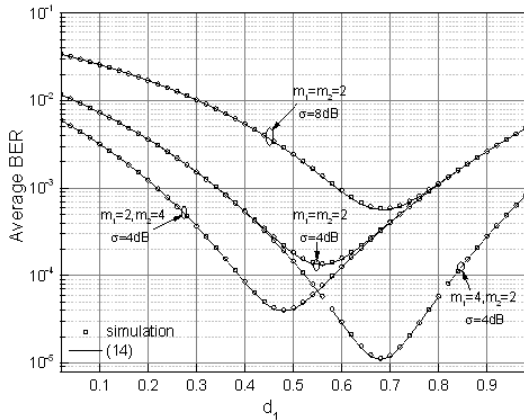


Fig. 3: ABER of BPSK for the dual-hop system versus d_1

simulation results ($c = 1$), only a small gap in low SNR region for BPSK. As expected, the ASER of BPSK and 16PSK is degraded when the shadow deviation increases ($\sigma = 4\text{dB} \rightarrow 8\text{dB}$) and the performance of BPSK outperforms that of 16PSK at the same channel conditions.

In Figure 2, the analysis and simulation results of OP are shown, where each hop has different multipath parameters, $d_1 = d_2 = 0.5$, $\sigma = 4\text{dB}$ and $N = 10$ for MG model. It can be seen from Figure 2 that the analytical results of (11) with $c = 1$ coincide perfectly with the simulation results. When $c = 0$, these results are similar as that in Fig. 1 and have only a small gap in low SNR region. Moreover, it can be seen the diversity performance of the dual-hop is determined by the hop with the minimum value between m_1 and m_2 .

In Fig. 3, we show the effect of the relay location on the ABER of BPSK for the dual-hop system. In this section, one asymmetric network geometry is examined where R is moved on a straight line between S and D, d_1 denotes the distance between S and R. Each hop has different fading parameters, $\rho = 10\text{dB}$, $N = 10$ for MG model. It can be seen from Fig. 3 that the optimum

performance is nearby the middle of the line when the fading parameters of each hop are same values, however, the optimum performance moves towards the side of the weaker-hop link when they are different. When R is closer to S (D), the performance is determined by the fading conditions of the second (first) hop. It shows that the system performance is determined by the hop with long transmitted distance. This is due to the fact that the path loss shows more important effect on the performance of dual-hop than the multipath and shadowing parameters. These results are helpful to the selection of relaying nodes in cooperative networks. Moreover, the simulation results match well with (14).

CONCLUSION

In this study, we investigated the end-to-end performance of a dual-hop AF relaying system over asymmetric Nakagami/ MG fading environment. Based on the CDF of the end-to-end SNR, novel closed-form expressions of the OP and ASER for the dual-hop AF system are derived. Finally, we showed numerical and simulation results to verify the accuracy of the analytical results and discussed the effect of the location of relaying node on the performance of the dual-hop system.

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