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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (18): 4385-4389, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.4385.4389

A Simulation Fast Algorithm for Parameters Estimation of the Multicomponent LFM Signals

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Abstract: A simulation algorithm of parameters estimation based on Radon-Wigner transform is presented by analyzing WVD and Radon transform in detail. The algorithm accomplished the Radon-Wigner transform of LFM signal by Radon transform of an expanded matrix which rows and lines of the two dimension matrix obtained by WVD are expanded with a certain proportion. The algorithm enhances the performance in terms of speed and accuracy. By simulation analysis of multicomponent LFM signals, the results revealed the speed of simulation has been improved more significantly than which obtained by dechiping.

Key words: Radon-wigner transformation, rows and lines extension, dechirping, parameter estimation, simulation algorithm

INTRODUCTION

In the applications of science and engineering, all of the received signals have continuous instantaneous phase. According to Weierstrass theorem, the instantaneous phase can be well approximated by a finite-order polynomial in time within a finite-duration interval (Wu *et al*., 2008). As a second-order polynomial signal, linear frequency modulation signal (LFM: Linear Frequency Modulation) is a non-stationary signal whose frequency linearly changes with time, while it is very important for LFM signal to estimate its parameters. (meanwhile, the estimation of parameters has significant impacts on LFM).

The maximum likelihood estimator offers one solution to estimate the parameters of LFM signals (Boashash, 1992). However, the algorithm needs plenty of computations and is not suitable to estimate the parameters of LFM signals (Zhou *et al*., 2006), which proves to be impracticable. In the early 1990s, Djuric and Kay (1990) put forward an algorithm to estimate the parameters of LFM signals, which adopted the linear least squares recursive approach to estimate the single component of LFM signal and needs to unwrap the phase of the signal when the signal-to-noise ratio is sufficiently high. Then, Peleg and Friedlander (1995) come up with the discrete polynomial-phase transform, which can estimate polynomial signal without unwrapping the phase of the signal, including LFM signal, but mainly aims to estimate parameters of the single component of polynomial signal. For multicomponent polynomial signal, there are also two main problems. First, the interaction between components, often called cross-terms, can interfere with the detection and estimation of parameters. Second, the principle of demodulation of polynomial signal with single component no longer works with polynomial signal with Multicomponent (Pham and Zoubir, 2007).

Any algorithm is used as an analytical tool for non-stationary signal; we wish it will have good time-frequency local property. However, the Wigner-Ville distribution of LFM signal has ideal time-frequency aggregative performance and is of great benefit to study single component LFM signal. Nevertheless, when it comes to multi-component LFM signal, the Wigner-Ville distribution inevitably causes serious cross-term interference in terms of time-frequency distribution. Based on above-mentioned, we put forward the Radon-Wigner transform, which is a projection transform of linear integral and can effectively solve the cross-term interference.

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WIGNER-VILLE DISTRIBUTION (WVD) ANALYSIS

Analysis of the time aggregative performance of the WVD: Let z (t) be the analytic signal of signal s (t), the WVD expression of signal s(t) is as follows:

$$W_z(t,f)=\int_{-\infty}^{\infty} z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{j2\pi\tau f}d\tau \tag{1}$$

To simplify the notation, we denote the amplitude of single-component LFM signal is 1.Hence, z (t) can be expressed as:

$$z(t)=e^{j2\pi(f_0t+\frac{1}{2}mt^2)} \tag{2}$$

Then multiplied signal can be described as:

$$\begin{aligned} &z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2}) \\ &=e^{j2\pi[f_0(t+\frac{\tau}{2})+\frac{1}{2}m(t+\frac{\tau}{2})^2]}e^{-j2\pi[f_0(t-\frac{\tau}{2})+\frac{1}{2}m(t-\frac{\tau}{2})^2]} \\ &=e^{j2\pi(f_0+mt)\tau} \end{aligned} \tag{3}$$

The WVD of the LFM signal can be obtained by substituting (3) into (1) .Thus:

$$\begin{aligned} W_{LFM}(t,f)&=\int_{-\infty}^{\infty} z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau \\ &=\int_{-\infty}^{\infty} e^{j2\pi\tau(f_0+mt)}e^{-j2\pi\tau f}d\tau \\ &=\delta[f-(f_0+mt)] \end{aligned}$$

From function (4), we note that the WVD of the single component LFM signal is the shock linear spectrum along $f = f_0 + mt$, so the WVD has ideal time frequency aggregative performance.

Analysis of cross-terms of WVD: For multi-component LFM signal, if:

$$\begin{aligned} z(t)&=z_1(t)+z_2(t) \\ &=e^{j2\pi(\frac{1}{2}m_1t^2+f_1t)}+e^{j2\pi(\frac{1}{2}m_2t^2+f_2t)} \end{aligned} \tag{5}$$

the WVD is expressed as:

$$\begin{aligned} W_{z_1(t)+z_2(t)}(t,f)&=\int_{-\infty}^{\infty} z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau \\ &=W_{z_1}(t,f)+W_{z_2}(t,f)+W_{z_1z_2}(t,f)+W_{z_2z_1}(t,f) \\ &=\delta[f-(f_1+m_1t)]+\delta[f-(f_2+m_2t)] \\ &\quad+W_{z_1z_2}(t,f)+W_{z_2z_1}(t,f) \end{aligned} \tag{6}$$

$$\begin{aligned} W_{z_1z_2}(t,f)&=\int_{-\infty}^{\infty} z_1(t+\frac{\tau}{2})z_2^*(t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau \\ W_{z_2z_1}(t,f)&=\int_{-\infty}^{\infty} z_2(t+\frac{\tau}{2})z_1^*(t-\frac{\tau}{2})e^{-j2\pi\tau f}d\tau \end{aligned} \tag{7}$$

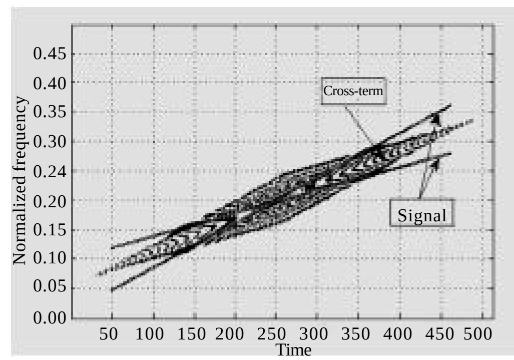

Fig. 1: Time-frequency plane of WVD for LFM signal of two components

the W_{z1z2} (t, f) and W_{z2z1} (t, f) are the cross-terms in (7), which are also the distracters for the parameter estimation of LFM signal. Equation (7) shows that, even through one signal component is far enough from another in time-frequency plane, the cross-terms of WVD of them also occur. Set the initial and termination frequency of the two signal as (0.01, 0.04) and (0.1, 0.3), respectively, the time-frequency plane is shown in Fig. 1.

As illustrated in Fig. 1, although the two signal components are sufficiently far apart in the time frequency plane, the cross-terms are still relatively large in their WVD.

RADON-WIGNER TRANSFORM

Definition of the RWT: According to the definition of Radon transform in reference (Wang *et al*., 1998), RW transform of 2D function g (x, y) can be written as:

$$g(\rho,\theta)=\iint g(x,y)\delta(\rho-x\cos\theta-y\sin\theta)dxdy \tag{8}$$

however, the RWT express W_z (t, f) of the WVD of g (x, y) in (7) and in reference (Wood and Barry, 1994), the RWT can be expressed as:

$$\begin{aligned} D_f(r,\phi)&=R_\omega[f(t)]=R[W_f(t,\omega)] \\ &=\int W_f(r\cos(\phi)-s\sin(\phi),r\sin(\phi)+s\cos(\phi))ds \end{aligned} \tag{8}$$

where, r and s, respectively express t and ω axes that are rotated counter-clockwise by the angle ϕ, W_f represents the Wigner transform of signal f(t), R [] represents the Radon transform, D_f (r, ϕ) represents the RWT of signal f (t), because z (t) is the analytic signal of signal f (t), (8) can be represented as follows:

$$\begin{aligned} D_z(r,\phi)&=R_\omega[z(t)]=R[W_z(t,\omega)] \\ &=\int W_z(r\cos(\phi)-s\sin(\phi),r\sin(\phi) \\ &\qquad+s\cos(\phi))ds \end{aligned} \tag{9}$$

For the transformation relationship of two-dimensional coordinates:

$$\begin{cases} t = r\cos\phi - s\sin\phi \\ \omega = r\sin\phi + s\cos\phi \end{cases} \tag{10}$$

The parameter (r, ϕ) can represent the Radon transform in (8), this is because the Radon transform is equivalent to a projection integration (for r integral projection). RWT is equivalent to line integration of W_f (t, ω). While the amplitude r and dip angle ϕ can express the line equation of (t, ω) plane, namely:

$$r = t\cos\phi + \omega\sin\phi \tag{11}$$

Due to the RWT is the line integration and the WVD has ideal time-frequency aggregation performance, that is, angular frequency ω is linear with time t, integral results will appear peak at some points that correspond to the point ω_0, but the cross-terms are not in the straight line, their peaks is much less than peak of the points of the straight line, thus, the RWT can eliminate the interference of cross-terms of WV transform.

Proof:

$$W_z(t,\omega) = 2\pi\delta(\omega - (mt + \omega_0)) \tag{12}$$

$$\begin{aligned} & W_z(t,\omega) = \delta(r^* - t\cos\phi^* - \omega\sin\phi^*) \Rightarrow \\ & W_Z(r,\phi) \\ & = \int_{-\infty}^{\infty} \delta(r^* - (r\cos\phi - s\sin\phi)\cos\phi^* - (r\sin\phi + s\sin\phi)\sin\phi^*)ds \\ & = \int_{-\infty}^{\infty} \delta(r^* - r\cos(\phi - \phi^*) + s\sin(\phi - \phi^*))ds \\ & = \int_{-\infty}^{\infty} \frac{1}{|\sin(\phi - \phi^*)|}\delta(\frac{r^* - r\cos(\phi - \phi^*)}{\sin(\phi - \phi^*)} + s)ds, \text{if } \sin(\phi - \phi^*) \neq 0 \\ & = \frac{1}{|\sin(\phi - \phi^*)|} \end{aligned} \tag{13}$$

If $\phi = \phi^*$ $\sin(\phi - \phi^*) = 0$ then:

$$\begin{aligned} & W_Z(r,\phi) \\ & = \int_{-\infty}^{\infty} \delta(r^* - r)ds \\ & = \begin{cases} 0, & \text{if } r^* \neq r \\ \int_{-\infty}^{\infty} \delta(0)ds & \text{if } r^* = r \end{cases} \end{aligned} \tag{14}$$

From (14), we can note that the RWT must have a peak at the point $\phi = \phi^*$, which is the expected parameter of the signal.

Computations of RWT: In the time-frequency plane of WV distribution, the intercept ω_0 and slope m of ω axis can represent line. Thus, when there needs a line integration along $\omega = \omega_0 + mt$, the integration parameter (r, ϕ) can be replaced with (m, ω_0) and the relationship between the two pairs of parameters can be expressed as:

$$m = -\cot\phi,\ \omega_0 = r/\sin\phi \tag{15}$$

Thus, there is a calculation method from time domain dechirp and frequency domain dechirp for RWT in reference (Wang *et al*., 1998), it respectively adopted dechirp calculation method for RWT of continuous LFM and discrete LFM by using the slope of time-frequency function as variable m and searching the correlation function and power spectrum of the calculating signal. For continuous LFM, the calculation is as follows.

Let multi-component LFM signal z (t) be:

$$z(t) = \sum_{i=1}^{p} e^{j(\omega_{0i}t + \frac{1}{2}m_i t^2)} \tag{16}$$

thus , the time domain dechirp of RWT is:

$$\begin{aligned} W_Z(r,\phi) &= \frac{1}{|\sin\phi|}\int_{-\infty}^{\infty} W_z(t, \omega_0 + mt)dt \Big|_{\substack{m = -\cot\phi \\ \omega_0 = r/\sin\phi}} \\ &= \frac{1}{|\sin\phi|}\left|\int_{-\infty}^{\infty} z(t)e^{j(\omega_0 t + \frac{1}{2}mt^2)}dt\right|^2 \Big|_{\substack{m = -\cot\phi \\ \omega_0 = r/\sin\phi}} \end{aligned} \tag{17}$$

From (17), when $\phi \to 0$, parameter $m \to -\infty$, the time domain dechirp is not suitable for RWT ,hence the RWT should do frequency domain dechirp .For $\phi \to 0$, frequency domain dechirp is as follows:

$$\begin{aligned} W_Z(r,\phi) &= \frac{1}{|\cos\phi|}\int_{-\infty}^{\infty} W_z(t_0 + p\omega, \omega)dt \Big|_{\substack{p = -\tan\phi \\ t_0 = u/\cos\phi}} \\ &= \frac{1}{|\cos\phi|}\left|\frac{1}{2\pi}\int_{-\infty}^{\infty} z(t)e^{j(t_0\omega + \frac{1}{2}pt^2)}dt\right|^2 \Big|_{\substack{p = -\tan\phi \\ t_0 = u/\cos\phi}} \end{aligned} \tag{18}$$

Note that when $\phi \to \pi/2$, frequency domain dechirp is disabled, because of $p \to \infty$, we should do the time domain dechirp.

DESIGN SIMULATION ALGORITHM

According to the above analysis of the WV transform and Radon transform, this study proposes a new simulation algorithm and The algorithm flow is as follows:

- Compute the W_z (t, ω) of multi-component LFM signal z (t). Due to f = ω.2π, it is generally denoted as W_z (t, f)
- According to W_z (t, f), the plane graph of time-frequency can be acquired. From the

time-frequency line, many maximum slope p of time-frequency function can be acquired

- According to p, transforming data matrix $W_z(t, f)$ into $\bar{W}_z(t, f)$ and make the row f become the p times of initial one
- RWT can be obtained by accomplishing Radon transform of $\bar{W}_z(t, f)$

In the third step of algorithm, the reason for transforming data matrix is that it must take same sampling step length to simulate accurately. The principle is as follows:

$$\begin{cases} t = t_m = t_{min} + m\Delta t, & m = 0,1,\ldots,M-1 \\ f = f_n = f_{min} + n\Delta f, & n = 0,1,\ldots,N-1 \end{cases} \quad (19)$$

According to $f = pt+f_0$, we have:

$$n = \frac{pt_m + f_0 - f_{min}}{\Delta f} = \alpha m + \beta \left| \begin{array}{l} \alpha = \frac{p\Delta t}{\Delta f} \\ \beta = \frac{pt_{min}+f_0-f_{min}}{\Delta f} \end{array} \right.$$

SIMULATION EXPERIMENT

The signal length is N = T×fs. Let the multi-component LFM signal z (t) be the sum of 4 pieces of LFM component:

$$z(t) = \sum_{i=1}^{4} e^{j2\pi(f_{0i}t+\frac{1}{2}p_i t^2)} \left| \begin{array}{l} p_1 = 1, f_{01} = 2 \\ p_2 = 1.5, f_{02} = 2.5 \\ p_3 = 2, f_{03} = 1.5 \\ p_4 = 3, f_{04} = 2 \end{array} \right. \quad (21)$$

the plane graph of time-frequency for z (y) is shown in Fig. 2.

From the parameter of signal, according to m = -cotϕ, when the rotation angle ϕ is 135.0°, 146.3°, 153.4° and 161.6°, the amplitude of RWT reach the maximum. From Fig. 3, the rotation angles which we obtained from the analysis of simulation obtain are basically consistent with the rotation angles of the original signal. Figure 4 shows the results of simulation in 3 dimension situation, from Fig 4, the maximum peaks that occur in simulation is consistent with the true value.

As for theconsuming time of simulation, the algorithm that is proposed in reference (Wang *et al*., 1998) takes 159.11s, but the algorithm that is proposed in this study takes only 3.25s. Hence, the execution efficiency is about 49 times of the existing algorithms, it shows that the computational complexity of the algorithm greatly decreases.

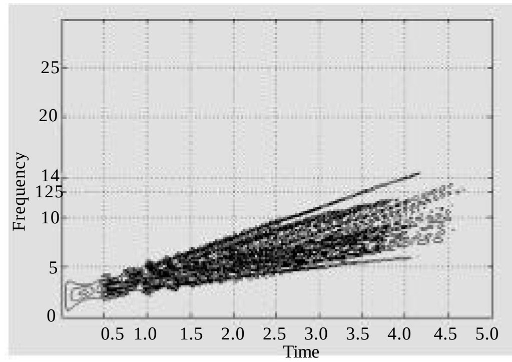

Fig. 2: Plane graph of time-frequency for z (t)

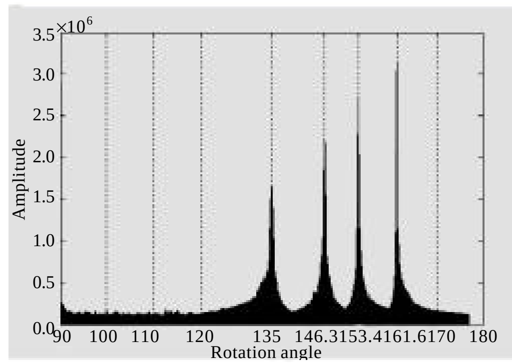

Fig. 3: Relationship between rotation angle ϕ and diameter for RWT

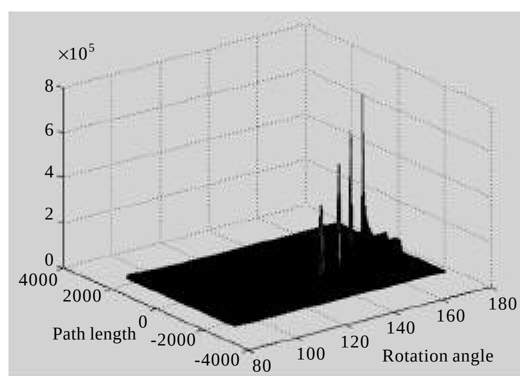

Fig. 4: Three dimensional graph of RWT

CONCLUSION

Radon-Wigner transform is an effective tool, especially when estimating the parameter of multi-component LFM signal, because it combines the ideal time frequency aggregative performance of Wigner-Ville distribution (WVD) with the performance of Radon transform that produces local areas of signal concentration.

A simulation algorithm for parameters estimation based on Radon-Wigner transform is presented by analyzing WVD and Radon transform in detail. Firstly, the algorithm acquires WVD of multi-component LFM signal and seeks the maximum slope of time-frequency straight line of multi-component LFM signal based on the time-frequency distribution. Then, it expands the rows and lines of the two dimensional matrix that is obtained by WVD with the maximum slope of time-frequency straight line and gets a new 2D matrix. Finally, it does Radon transform for the new matrix and accomplishes the Radon-Winger transform of LFM signal. Comparing to the simulation algorithm by dechirping in the time domain, it enhances the performance in terms of speed and accuracy. In addition, the simulation algorithm is validated to be accurate and fast through simulation analysis of multi-component LFM signals.

ACKNOWLEDGMENTS

The authors would like to thank for the support by School of Photoelectric Engineering, Chongqing University of Posts and Telecommunications. The authors also thank for the support from Ou Guo-Jian in the whole work

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