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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (3): 439-443, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.439.443

Consensus Analysis of Multi-agent-based Rural-urban Migration by Relative Lyapunov Function Method

Haiying Ma
School of Economics, Northwest University for Nationalities, Lanzhou 730124, China

Abstract: This note is devoted a theoretic framework to explore the rural-urban migration phenomena by the means of multi-agent system dynamics based on the ideas of consensus analysis which builds a computational approach. Consensus is somewhat analogous to equilibrium and it is the foundation for topics such as formation control and swarm stability in a decentralized viewpoint. The analysis shown in this note suggests that a deeper investigation can be conducted to rural-urban migration phenomena by relative Lyapunov function method.

Key words: Rural-urban migration, consensus, multi-agent system, laplacian matrix

INTRODUCTION

The agent-based study of rural-urban migration is a new developing field of research. In demography, migration studies are mostly related to human population and its dynamics encompassing features such as structures, sizes, distributions and behaviors or phenomena which can change those aspects over space or time and is usually quite unpredictable. Ravenstein (1885) proposed a well empirically grounded description about the general aspects of the human migration phenomenon containing 11 laws with regard to international migration. After Revenstein, several quantitative models of migration flows and the variables that affect those flows were proposed. In related to distances between origin and conventional economic literature, one of the most known was proposed by Todaro (1969). Since then, migration was investigated from a wide range perspective and from different approaches, from classical physical approaches-where migration is mostly destination-to neoclassical economics model where migration emerges from individuals search for more satisfactory economic conditions like higher wages or better job opportunities. In last decades other social factors are also being investigated as related to migration flows such as social networks (Fazito, 2010). However, researches carried out in order to establish the role played by social networks on migration flows are mostly based on surveys, census and official immigration data which has problems and limitations (Massey *et al*., 1994).

However, agent-based model of the complex system place agent as an "autonomous", "proactive", "interacting" entity when individuals are represented as artificial agents in computer architecture. The major motivation to use agent-based models is the possibility of modeling and controlling the model in different granularity levels, from environmental spatial characteristics to behavioral and cognitive individual aspects (Gilbert and Conte, 1995).

This approach enables the economists to produce highly heterogeneous and sophisticated models of complex societies. At the same, a compelling arguments supporting the use of agent-based model in stead of the analytical models was also pointed out (Silveira *et al*., 2006). Even so, the theoretic framework for rural-urban migration is still weak in the existing literature.

This note will be focused on the rural-urban migration phenomena with dynamical consensus analysis, on the basis of the previous work by the authors. The main contributions is the proof of a criterion to model rural-urban migration via studying the structure of the high-order state equation which is a new approach different from those in the literature based on the analytical solution.

PROBLEM FORMULATION

The consensus problem is crucial for study of multi-agent system dynamics. "Consensus" might be the corresponding concept as "equilibrium" in conventional economic system. If a multi-system asymptotically achieves some consensus state, such a situation is just analogous to that the trajectory of an asymptotically stable single-system approaches some origin as $t\to\infty$. Thus, in the venue of multi-system analysis, consensus may play the same fundamental role as equilibrium. However, "consensus" does not tally with the definition of "equilibrium" because even consensus is achieved, the states of agents might still keep on altering. It is essentially a non-equilibrium state.

In dynamic single-system analysis, when we shift the equilibrium point to origin, the "state" of the system actually indicates its difference from the origin. Analogously, in multi-system analysis, often, what are most important to know are not the absolute states of agents but rather the relative states-the differences of states between agents. Consensus means that all differences are zero.

Based on consensus, notions in single-system such as regulator and tracking control may have counterparts in multi-system synthesis.

Now we turn our view to high order linear swarm systems. By far, from the literature we have looked over, other than second-order systems, for instance, (TJP03) and (LF01), study on dynamics of generic high-order swarm systems is absent. The majority of articles take first-order systems as their subject and most typical formula is like:

$$\dot{x}_i = \sum_{j \in N_i} (x_i - x_j)$$

where, N_i is the index set for neighborhood of vertex, i refer to rural sector and j means urban sector in the economy. Such a dual structure is typically used by the economic literature which investigates the rural-urban migration phenomena (Todaro, 1969).

Perhaps there are two main reasons to prevent study of consensus on high-order systems due to heterogeneity of individual migrator. The first reason is the difficulty to construct a macrostate of Lyapunov function for the whole economic system. In first and second order systems, we can conspire to devise the Lyapunov function as:

$$V = \sum_i x_i^T x_i$$

thus $\dot{V}$ takes the form: $-x^T L x$, where L is the Laplacian matrix and x is the stack vector of all agents (TJP03) (10). Then the consensus like conclusion could be drawn based on semi-positive definite property of L. However, in high order systems, the situation is much more complex. The second reason is that for high order case, free agent is not static in phase space. In first order case, if an agent is not connected with other agents, it just stays where it is. While in generic high order case, a free agent would just drift along some vector field defined by its specific dynamics.

Let us consider more general n-th ordered linear multi-agent system:

$$\dot{x}_i = Ax_i + \sum_{j=1}^{N} a_{ij}(t) F(x_i - x_j) \quad (1)$$

In the dynamic equation, $(x_i)_{n\times 1}$ is the state vector of i, $A_{n\times n}$ and $F_{n\times n}$ is coefficient matrix, $a_{ij}(t) \in R^+$ is edge weight of the agent. We could see that for an agent in the system, its dynamics is an addition of contributions from two aspects: The agent's own dynamics and the interaction from its neighbors. As to the consensus condition of such swarm model, we have the following conclusion.

Theorem 1: If the agent is always connected and we can find a symmetric positive definite matrix $P_{n\times n}$ such that both Q = AP+PA and Q = FP+PF are negative positive, then the swarm system achieves consensus.

Proof: The center of the swarm is:

$$x_c = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Let, $e_i = x_i - x_c$ and $\Delta e_{ij} = x_i - x_j$, then we can easily get the error system:

$$\dot{e}_i = Ae_i + \sum_{j=1}^{N} a_{ij} F \Delta e_{ij}$$

Try Lyapunov function candidate as:

$$V = \sum_{i=1}^{N} e_i^T P e_i$$

We get:

$$\dot{V} = \sum_{i=1}^{N} (\dot{e}_i^T P e_i + e_i^T P \dot{e}_i) = \sum_{i=1}^{N} [(e_i^T A^T + \sum_{j=1}^{N} a_{ij} \Delta e_{ij}^T F^T) P e_i + e_i^T P (Ae_i + \sum_{j=1}^{N} a_{ij} F \Delta e_{ij})] =$$
$$\sum_{i=1}^{N} e_i^T (A^T P + PA) e_i + 2 \sum_{i=1}^{N} < e_i, \sum_{j=1}^{N} a_{ij} F \Delta e_{ij} >_P \ (\text{let} < \mathbf{x}, \mathbf{y} >_P = \mathbf{x}^T P \mathbf{y}) = (1) + (2)$$

The hinder part (2) is:

$$2 \sum_{i=1}^{N} < e_i, \sum_{j=1}^{N} a_{ij} F \Delta e_{ij} >_P$$

Consider the relationship of any agent pairs, there is:

$$2 \sum_{i=1}^{N} < e_i, \sum_{j=1}^{N} a_{ij} F \Delta e_{ij} >_P = \sum_{i,j} (< e_i, a_{ij} F \Delta e_{ij} >_P + < e_j, a_{ij} F \Delta e_{ji} >_P) =$$
$$\sum_{i,j} (< e_i, a_{ij} F \Delta e_{ij} >_P - < e_j, a_{ij} F \Delta e_{ij} >_P) = \sum_{i,j} < (e_i - e_j), a_{ij} F \Delta e_{ij} >_P =$$
$$\sum_{i,j} a_{ij} < \Delta e_{ij}, F \Delta e_{ij} >_P$$

From the condition of the theorem, we can figure out that both (1) and (2) is negative. Thus, $\dot{V}<0$. Because the agent is always connected, there must be $e_1 = e_2 = ... = e_{avg} = 0$ as $t\to\infty$ and consensus is achieved.

Corollary: For the n-th ordered linear multi-agent system described by (1), if A = F and both of them are Hurwitz, then the swarm system achieves consensus as $t\to\infty$.

Example: Suppose there is a second-order linear swarm system as:

$$\dot{x}_i = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} x_i + \sum_{j=1}^{N} [a_{ij} \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} (x_i - x_j)]$$

Here, we have:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$

and:

$$F = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

First, choose:

$$Q_1 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

and solve the Lyapunov equation $A^TP+PA = Q_1$.

We get the symmetrical positive definite root:

$$P = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Moreover, according to P, we have:

$$Q_2 = F^TP + PF = \begin{bmatrix} -2 & -1 \\ -1 & -6 \end{bmatrix}$$

It is easy to testify that Q_2 is both symmetrical and Hurwitz and thus it is negative definite. So the condition of theorem1 is satisfied and the swarm system achieves consensus.

MAIN RESULTS

In this note, we consider multi-agent system with N>1 agents, no matter homogenous or heterogeneous. The state of each agent is $x_i \in R^d$, with its dynamics depicted by:

$$\dot{x}_I = f_i(x_i,u_i,t)+\Sigma_{j\in Ni}g_{ij}(x_i,x_j,t)(i,j\in\{1,2,....,N\}) \quad (2)$$

In the above equation, $f_i(\bullet)$ reflects the self-governed component of one agent's dynamics and $g_{ij}(\bullet)$ reflects for the influence from the other agent.

Theorem 2: Consider a dynamical multi-agent system (2). If there exists some injective linear operator, where the image space H is a Hilbert space, such that for any trajectory, $x_0(t)\in co(x_1(t), x_2(t),...,x_N(t))\in R^d$, $T^1x_i(t) = T(x_i-x_0) = \xi i(t)\in H$ and $T^1x_0(t) = 0$. Let, k(t) denote the index of agent possessing the maximal value of $<\xi_k,\xi_k>$ over the entire system.

Definition 1 (relative Lyapunov function): According to the theorem above, for any two sector i and j in the system, the following function determined by both of their images in the Hilbert space H:

$$V_{ij} = \sqrt{<\xi_i - \xi_j, \xi_i - \xi_j>} \in R^+$$

is called the Relative Lyapunov Function between the two sectors.

Theorem 2 is our theoretical foundation. However, it may seem that it is somewhat theoretic to be applicable. Next, we shall discuss its application through some corollaries and example.

Corollary 1: Suppose the dynamics of agents are formulated by:

$$\dot{x}_I = \Sigma_{j\in Ni}g_{ij}(x_i,x_j)(i,j\in\{1,2,...,N\})$$

If there exists some injective linear operator T: $R^d\to H$, where the image space H is Hilbert, such that $Tx_i(t) = \xi_i(t)\in H$ and if at any time for any two sectors i and j:

$$\frac{<Tg_{ij}(x_i,x_j), \xi_j - \xi_i>}{(\|Tg_{ij}(x_i,x_j)\|\|\xi_j - \xi_i\|)} = 1(i,j\in\{1,2,...,N\})$$

and the migration network of the multi-agent system is sufficiently connected, then the system achieves consensus as $t\to\infty$.

Corollary 2: Suppose the dynamics of agents are formulated by $\dot{x}_I = \Sigma_{j\in Ni}g_{ij}(x_i,x_j)(i,j\in\{1,2,...,N\})$ and the network is sufficiently connected. If there exists some relative Lyapunov function ϕ: $R^d\times R^d\to R^+$ between any two agents: $V_{ij} = \phi(x_i,x_j)\in R^+$, $\phi(x,y) = 0$ when x = y, such that the equation below always holds:

$$\frac{\langle g_{ij}(x_i,x_j),\nabla_{x_i}V_{ij}\rangle}{(\|g_{ij}(x_i,x_j)\|\|\nabla_{x_i}V_{ij}\|)} = -1(i,j\in\{1,2,...,N\})$$

then the system is asymptotically swarm stable and it achieves consensus as $t\to\infty$.

If there exists such a relative Lyapunov function, then we can take $\phi(x_i,x_j)$ as the metric of image space H and inner product in H can be induced by this metric. In such an induced space H, the direction of image for vector field $g_{ij}(x_i,x_j)$ is always toward the direction of the shortest distance between images of agents, therefore according to Corollary 1, consensus must be achieved.

Remark: For Corollary 2, it is not necessary that $g_{ij}(x_i,x_j)$ be conservative field. It only needs to have the similar direction with certain gradient field. Such kind of dynamic system may be called quasi-gradient system. If two scalar fields in a space share the same quasi-gradient field, that is, the directions of gradient vectors for both fields are identical everywhere, then the two scalar fields are co-quasi-gradient. For instance, all differentiable functions are co-quasi-gradient fields in, for the gradient of any function always points rightward in the axis.

NUMERICAL EXAMPLE

In this section, numerical instances will be exhibited to illustrate the theoretical results. The rural-urban migration network W is shown in Fig. 1 with:

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.84 & 0.24 & 0.20 & 0.47 & 0.59 \\ 0.25 & 0.93 & 0.25 & 0.35 & 0.55 \\ 0.81 & 0.35 & 0.62 & 0.83 & 0.91 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Evidently it includes no spanning tree. The spectrum of the Laplacian matrix is:

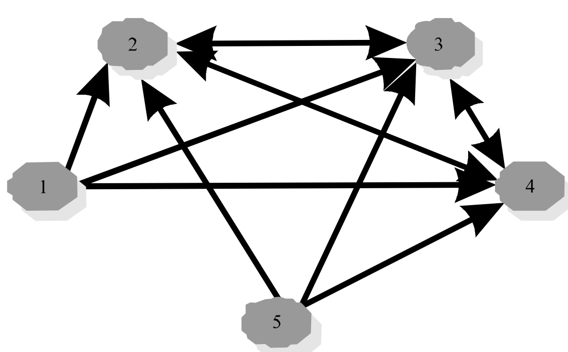

Fig. 1: Migration network

$$\{0\ 0\ 1.31 \smile 2.78+0.195i \smile 2.78-0.195i\}$$

In the first instance, let:

$$F = \begin{bmatrix} 0.60 & 0.69 & 0.08 \\ 0.26 & 0.75 & 0.23 \\ 0.65 & 0.45 & 0.91 \end{bmatrix}$$

Its spectrum is $\{1.47 \quad 0.40+0.12i \quad 0.40-0.12i\}$. According to corollary 1 and 2, the rural-urban migration network is swarm stable but not asymptotically swarm stable.

In the second instance, let:

$$F = \begin{bmatrix} 0.34 & 0.01 & 0.92 \\ 0.78 & 0.60 & 0 \\ 0.68 & 0.39 & 0.46 \end{bmatrix}$$

Its spectrum is $\{1.39\ -0.04\ 0.05\}$. There is a negative eigenvalue. Thus, the system is obviously swarm unstable because the dynamical protocol is repulsive.

Trajectories of vertex states are illustrated by Fig. 2 and 3. Note that thick dots indicate the start positions in Fig. 2 and 3. Two vertices stay still in the state space. This is because that the graph includes no spanning tree and the two vertices receive no information.

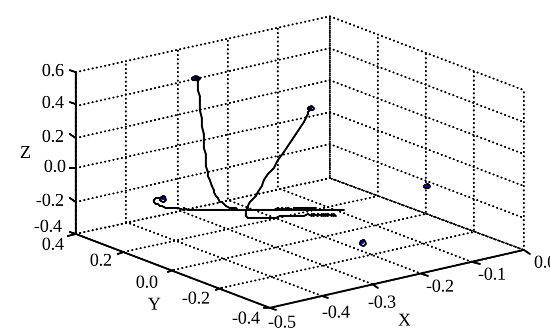

Fig. 2: Trajectories of vertices in first instance

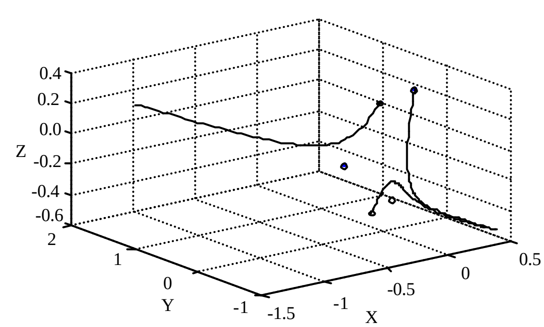

Fig. 3: Trajectories of vertices in second instance

CONCLUSION

This note dealt with a theoretic framework to explore the rural-urban migration phenomena by the means of multi-agent system dynamics based on the ideas of consensus analysis which builds a computational approach. Our idea is unique and different from those in the literature dealing with consensus problems of linear systems. The main thought of our approach is that the convergent property of a multi-agent system can be guaranteed by some dissipative property of an image system in another abstract space. New concepts such as relative Lyapunov function and quasi-gradient field are concomitant with our discussion. The effect of the main result is illustrated by more applicable corollaries and examples extracted from literature. The analysis shown in this note suggests that a deeper investigation can still be conducted to rural-urban migration phenomena by relative Lyapunov function method.

ACKNOWLEDGMENT

This study was supported by the Fundamental Research Funds for the Central Universities of Northwest University for Nationalities (grant No. ZYZ2011010).

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