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# Fatigue S-N Curve Fitting with Inexact Line Search Newton's Method

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**Abstract:** An inexact line search Newton's method for solving the three parameters S-N curve model is proposed in order to describe material fatigue properties better and get high precision S-N curve in this study. This method adds an inexact item based on the classical Newton method and overcomes the disadvantage of high demand for initial value in traditional fitting method. The fitting data for the aluminum alloy 2A12 under different experimental conditions are selected. The comparison between the calculated equation and theoretical equation is carried out under the same experimental conditions. The calculated S-N curve obtained through inexact line search Newton's method is found to be in reasonable agreement with the theoretical one. It provides a new method for three parameters S-N curve fitting.

**Key words:** S-N curve, model, fatigue properties, newton's method, fitting

## INTRODUCTION

Significant research efforts have been devoted to understanding the fatigue behavior of composite materials and the development of techniques for modeling the fatigue life and prediction of the material's behavior under different conditions (Sarfaraz *et al*., 2012). One of the most explicit and straightforward ways to represent experimental fatigue data is the S-N diagram (Zhai and Xiao, 2012). S-N curve model is an important tool for describing the fatigue properties of materials which can reflect the relationship between the cyclic loading and fatigue life. Many researches about S-N curve model have been developed. Zhao *et al*. (2009) proposed a concurrent probability method for estimating probabilistic fatigue S-N curves including the super-long life regime. Zerbst *et al*. (2012) presented an analytical fracture mechanics model for predicting the finite life fatigue strength of components. Ramamurty Raju *et al*. (2007) studied S-N curve for aluminum alloy (Al) A356.2-T6 and estimated the fatigue life under radial fatigue load. Fatemi *et al*. (2005) proposed a bi-linear log-log model for predicting the life of aluminum alloys. Wu and Yao (2008) set up a model which was used to describe the distributions of fatigue life of composites.

The original S-N curve model is used for studying the fatigue performance of metal material. With more and more application of composite material on important parts of aircraft, including aerofoil, empennage, bulkhead, radome and so on (Gibson, 2007). The composite material needs to afford higher cyclic loading. In order to improve the reliability of its structure, it also needs to study the fatigue properties of composite material. So it is an important issue that how to obtain higher accuracy S-N curve. There exist several methods for S-N curve fitting and least square method is the commonly used one. Yang *et al*. (2012) proposed a fast normalized log estimator based on the least squares nonlinear curve fitting. Ji and Yao (2004) proposed a weighted least square method for S-N curve fitting. When there are more parameters in S-N curve model, it has higher requirement for the selection of initial value. It is also the disadvantage of the least square method.

In order to forecast the S-N curve model accurately and conveniently. An inexact line search Newton's method is proposed for solving the three parameters curve fitting. First, the three parameters power function model is transformed into optimization problem. Second, optimization problem is transformed into equations. An inexact Newton's method is proposed for solving the above equations problem at last.

## THREE PARAMETERS POWER FUNCTION MODEL

There are various forms for S-N curve model and three parameters S-N curve model can express long life range S-N curve betterly. Thus, three parameters S-N curve model has been selected in this study.

$$N(S_{max}-S_0)H = C \quad (1)$$

where, $S_0$, H, C are undermined constants, $S_{max}$ is maximum stress and N is fatigue life. It can be obtained after logarithm fetch on Eq. 1:

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$$\lg N = -a - b\lg(S_{max} + c) \tag{2}$$

where, $a = -\lg C$, $b = H$, $c = -S_0$.

The essence of S-N curve fitting is that the experimental data of cyclic loading and fatigue life that is $(N_1, S_1), (N_2, S_2), \ldots, (N_m, S_m)$ are given and the approximate value of three parameters a, b, c can be obtained. Thus, the following function is established:

$$f(a,b,c) = \sum_{i=1}^{m} (\lg N_i + b\lg(S_i + c) + a)^2$$

In order to calculate the approximate value of three parameters a, b, c, it only needs to find the optimality condition for the following optimization problem:

$$\begin{aligned} \min \quad & f(a,b,c) \\ \text{s.t.} \quad & a,b,c \in R \end{aligned} \tag{3}$$

Above optimization problem is convex optimization problem and the optimum point is extreme point. Thus, the optimization problem 3 is equivalent to the following equation problem:

$$\begin{cases} f'_a(a,b,c) = 0 \\ f'_b(a,b,c) = 0 \\ f'_c(a,b,c) = 0 \end{cases}$$

Then solving the following equations:

$$\begin{cases} \sum_{i=1}^{m} (\lg N_i + b\lg(S_i + c) + a) = 0 \\ \sum_{i=1}^{m} (\lg N_i + b\lg(S_i + c) + a)\lg(S_i + c) = 0 \\ \sum_{i=1}^{m} (\lg N_i + b\lg(S_i + c) + a)\dfrac{1}{S_i + C} = 0 \end{cases}$$

The left side of above equations can be rewritten as the form of ternary function:

$$F(a,b,c) = \begin{pmatrix} F_1(a,b,c) \\ F_2(a,b,c) \\ F_3(a,b,c) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{m} (\lg N_i + b\lg(S_i + c) + a) \\ \sum_{i=1}^{m} (\lg N_i + b\lg(S_i + c) + a)\lg(S_i + c) \\ \sum_{i=1}^{m} (\lg N_i + b\lg(S_i + c) + a)\dfrac{1}{S_i + C} \end{pmatrix}$$

Then, the optimization problem is transformed into the following equation:

$$F(a,b,c) = 0 \tag{4}$$

## AN INEXACT LINE SEARCH NEWTON'S METHOD

In order to solve the above Eq. 4, a class of Newton algorithm is applied.

$$JF(a,b,c) = \begin{pmatrix} m & \sum_{i=1}^{m} \lg(S_i + c) & b\sum_{i=1}^{m} \dfrac{1}{(S_i + c)\ln 10} \\ \sum_{i=1}^{m} \lg(S_i + c) & \sum_{i=1}^{m} [\lg(S_i + c)]^2 & \sum_{i=1}^{m} \dfrac{2b\lg(S_i + c) + \lg N_i + a}{(S_i + c)\ln 10} \\ \sum_{i=1}^{m} \dfrac{1}{(S_i + c)} & \sum_{i=1}^{m} \dfrac{\lg(S_i + c)}{(S_i + c)} & \sum_{i=1}^{m} \dfrac{b - \ln 10(\lg N_i + b\lg(S_i + c) + a)}{(S_i + c)\ln 10} \end{pmatrix}$$

According to the inexact line search Newton's method, the proceed as follows:

**Data:** Given $z^0 = (z^0, b^0, c^0)$, $\sigma>1, p>1, \gamma\in(0, 1); \{\eta_k\}$ be a sequence of nonnegative scalars.

**Step 1:** Set $_{k=0}$

**Step 2:** If $F(a^k, b^k, c^k) = 0$, let $(a, b, c) = (a^k, b^k, c^k)$. Otherwise, go to Step 3; (In general, $F(a^k, b^k, c^k) = 0$ cannot satisfy. Usually, let $F(a^k, b^k, c^k) = \varepsilon$, where $\varepsilon$ is small enough for satisfying the accuracy)

**Step 3:** Calculate a vector $d^k$ such that:

$$F(z^k) + JF(z^k)d = rk \tag{5}$$

where $r^k$ is a vector satisfying:

$$\|r^k\| \le h_k \|F(z^k)\|$$

If Eq. 5 cannot satisfy the following condition:

$$[JF(z^k)]^T F(z^k) d^k \le -\sigma \|d^k\|^p$$

Let $d^k = -F(z^k)$.

**Step 4:** Find minimum nonnegative integer $i_k$, set $i = i_k$:

$$\|F(z^k + 2^{-i}d)\|^2 \le \|F(z^k)\|^2 + \gamma 2^{-i}[F(z^k)]^T d^k$$

Set $\tau_k \equiv 2^{-i_k}$.

**Step 5:** Let $z^{k+1} \equiv z^k + \tau_k d^k$ and $k\kappa k+1$, go to Step 1

Then, the three parameters a, b, c in Eq. 2 can be obtained.

## NUMERICAL EXAMPLES VALIDATE

In order to verify the effectiveness of the inexact line search Newton's method proposed for S-N curve fitting

Table 1: Experimental conditions

| Material | 2A12 |
|---|---|
| Condition/heat treatment | T4 |
| Specification | δ2.5 mm board |
| Tensile Strength $\sigma_b$ | 457 Mpa |
| The percentage elongation stress $\sigma_{P0.2}$ | 336 Mpa |
| Elongation $\delta_5$ | 18.7% |
| Sampling direction | L |
| Stress concentration factor | $K_t = 1$ |
| Test frequency | 116 Hz |
| Experiment environment | Laboratory Air |
| Stress ratio R | 0.6, 0.02 |
| Loading method | Axial |

Table 2: Experimental Data ($K_t$=1)

| Stress ratio R = 0.02 | | Stress ration R = 0.6 | |
|---|---|---|---|
| $S_{max}$ | N | $S_{max}$ | N |
| 275 | $1.2\times10^5$ | 358 | $3.25\times10^5$ |
| 185 | $4.5\times10^5$ | 324 | $7.0\times10^5$ |
| 161 | $6.0\times10^5$ | 292 | $1.8\times10^6$ |
| 129 | $4.0\times10^6$ | 274 | $2.7\times10^6$ |
| 124 | $10.0^7$ | 253 | $7.0\times10^6$ |

Table 3: Comparison between the calculated equation and the theoretical one under different stress ratio

| Stress ratio R | S-N curve fitting equation |
|---|---|
| 0.6 | Theoretical: |
| | $\text{Lg}\, N = 14.060\text{-}3.7745\lg(S_{max}\text{-}173.2)$ |
| | Calculated: |
| | $\text{Lg}\, N = 25.2096\text{-}7.7835\lg(S_{max}\text{-}18.1)$ |
| 0.02 | Theoretical: |
| | $\text{Lg}\, N = 7.9543\text{-}1.3297\lg(S_{max}\text{-}119.5)$ |
| | Calculated: |
| | $\text{Lg}\, N = 8.2604\text{-}2.2659\lg(S_{max}\text{-}100.4)$ |

Table 4: Experimental conditions

| Material | 2A12 |
|---|---|
| Condition/heat treatment | T4 |
| Specification | δ3 mm board |
| Tensile Strength $\sigma_b$ | 471 Mpa |
| The percentage elongation stress $\sigma_{P0.2}$ | 362 MPa |
| Elongation $\delta_5$ | 15.4% |
| Sampling direction | L |
| Stress concentration factor | $K_t = 3$ |
| Test frequency | 10 Hz |
| Experiment temperature | room temperature |
| Stress ratio R | 0.1 |
| Loading method | Axial |
| Specimen surface | aluminum coating |
| Sampling direction | L |

Table 5: Experimental Data ($K_t$ = 3, R = 0.1)

| $S_{max}$ | N |
|---|---|
| 132.3 | $2.1\times10^4$ |
| 119.4 | $4.6\times10^4$ |
| 89.5 | $1.2\times10^5$ |
| 67.7 | $3.0\times10^5$ |
| 50.0 | $1.0\times10^6$ |

Table 6:Comparison between the calculated equation and the theoretical one

| Stress ratio R | S-N curve fitting equation |
|---|---|
| 0.1 | Theoretical: |
| | $\lg N = 11.0428\text{-}3.12611 \lg(S_{max}\text{-}6.65)$ |
| | Calculated: |
| | $\lg N = 11.9383\text{-}3.5108 \lg(S_{max}\text{-}0.035)$ |

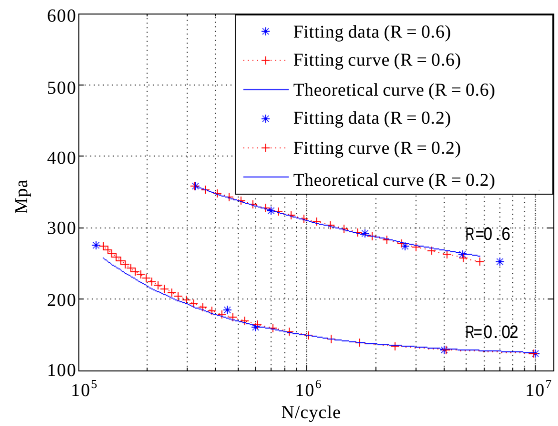


Fig. 1: S-N Curve Fitting of Aluminum Alloy 2A12-T4 Sheet Smooth Specimen in Laboratory Air Environment

in this study, fatigue properties experimental data for aluminum alloy 2A12 has been selected for verification (Anonymous, 2002).

**Example 1:** The type of high frequency fatigue testing machine is AMSLER. The experimental conditions for aluminum alloy 2A12 are shown in Table 1. The experimental environment is laboratory air and axial loading way is selected. The experimental data for aluminum alloy 2A12 under different stress ratio are shown in Table 2.

According to the inexact line search Newton's method proposed in this study, the calculated equation of the three parameters S-N equation is obtained. The comparison between the calculated equation and the theoretical one are shown in Table 3. The range of application is less than $10^7$. When stress ratio R = 0.6, there exist big error between calculated value and theoretical value for parameter c. The deviation between the fitting curve and the theoretical curve is larger while N is larger than $2\times10^6$ in Fig. 1. When stress ratio R = 0.02, the calculated three parameters show reasonable agreement with the theoretical one and the deviation between the fitting curve and the theoretical curve become larger when N is smaller than $3\text{x}10^5$.

**Example 2:** In example 2, the experimental environment has been changed into NaCl solution (3.5%). The experimental conditions are shown in Table 4 and experimental data for aluminum alloy 2A12 fatigue test are shown in Table 5. The comparison between the calculated equation and the theoretical one are shown in Table 6.

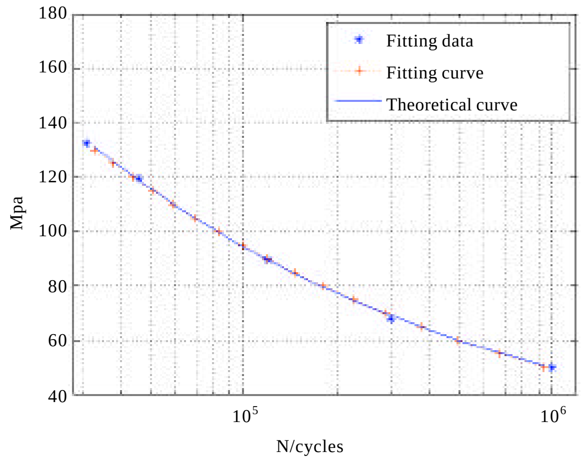


Fig. 2: S-N curve fitting of aluminum alloy 2a12-t4 sheet smooth specimen in saline solution environment

The range of application is between $4\times10^4$~$10^6$. The calculated curve show reasonable agreement with the theoretical curve in Fig. 2.

## CONCLUSION

An inexact line search Newton's method is proposed for three parameters S-N curve fitting in this study. The fitting data under different experimental condition have been selected. Three parameters can be obtained and the S-N equation can be also established with less experimental data. The inexact line search Newton's method overcomes the disadvantage of high demand for initial value. It can be seen that the calculated curve show reasonable agreement with the theoretical one. Thus, it is suitable for the application in engineering.

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