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# Controllability of Wireless Sequence Control System with Time Delays Based on Semi-tensor Product

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**Abstract:** The sequence control system based on wireless network with smart nodes is proposed. Because the data is transmitted through wireless network, the controllability of the system will be influenced by bit errors and time delays in data transmission. Although this problem has been studied in continuous control system, there is no published study solving it in sequence control system. To settle this problem, the system is modeled as a Probabilistic Boolean Control Network (PBCN) with time delays. Then, with the introduction of the Semi-Tensor Product (STP), the criteria of the controllability are derived when the system is under different controls. Base on the derived criteria, related examples are presented for illustration and demonstration of the proposed solution.

**Key words:** Probabilistic Boolean control network, semi-tensor product, wireless network, sequence control, time delays

## INTRODUCTION

In recent years, Wireless Sensor Network (WSN) has been developed greatly and applied in the industrial field gradually (Willig, 2008). In our former work, the wireless network is adopted for the transmission of controls in sequence control system (Wang *et al*., 2012). The most relevant feature of a WSN is that it is a dynamic distributed system, in which complex tasks are performed through the coordinated action of a large number of small autonomous nodes. For the wireless nodes has the ability of information storage, computation and wireless communication, we introduce the sequence control system with smart wireless nodes which can not only transmit data but also generate controls. So, it's a partially intelligent control system. And it's obvious that the controllability of the system will be influenced by the bit errors and time delays in data transmission. And the problem caused by bit errors is discussed in (Wang and Bai, 2013). The approach of Semi-Tensor Product (STP) is introduced to settle the controllability of sequence control system based on wireless network and it shows the advantage of the method. For the system is new designed, the model of the system is different that the system is modeled as a Probabilistic Boolean Control Network (PBCN) with time delays. Then, by using STP, the performance of the system is carried out and the controls are designed to ensure the controllability of the system.

The Boolean network was first introduced by Kauffman to describe genetic circuits (Kauffman, 1969). And Boolean Control Network (BCN) is to manipulate the Boolean network. In (Shmulevich *et al*., 2002) a generalized uncertainty model of Boolean network-PBCN is proposed. Semi-tensor product is proposed by Cheng to convert a logical function into an algebraic function and some classical mathematic theory can be applied in the logic system (Cheng and Qi, 2009) Because of the advantage of semi-tensor product, the approach of STP has been introduced in the research of PBCN, such as (Li and Sun, 2011a) studies the controllability of PBCN, (Ching *et al*., 2009) gives the optimal control policy for probabilistic Boolean networks, (Zhao and Cheng, 2012) considers the controllability and stabilizability of probabilistic logical control networks, (Li and Sun, 2011b) considers the controllability of BCN with state delays and so on. Although there have been some related achievements, it's a new challenging problem to study the controllability of PBCN with time delays.

The rest of this study is organized as follows. Section 2 presents the preliminaries about STP and PBCN. In Section 3, the main results of the controllability of PBCN with time delays are presented. In Section 4, the sequence control system based on wireless network with smart node is modeled as a PBCN and the controllability is studied under different controls by using STP. Section 5 is a brief conclusion.

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## PRELIMINARIES

The main mathematic approach is semi-tensor product which is a generalized multiplication of matrices and can be applied for any matrices.

First of all, the notations about logic should be illustrated. The logical variable in sequence control system takes value from D = {1, 0}, where 1~C and 0~O, represent "Close" and "Open", respectively. And we use two vectors to represent the two logical values as C~1~$\delta_2^1$, O~0~$\delta_2^2$, where $\delta_2^i$ denotes the ith column of the identity matrix $I_n$. And the set is denoted by $\Delta_n = \{\delta_n^k \mid 1 \le k \le n\}$ (Cheng *et al.*, 2011).

For notational ease, logic matrix $M = [\delta_n^{i_1}, \delta_n^{i_2}, \ldots, \delta_n^{i_m}]$ is expressed as M = $\delta_n[i_1, i_2, \ldots, i_m]$. Col(M) represents the column set with the elements of M and $Col_{\Delta_{2^n}}(M)$ means that the elements are in Col (M) and in $\Delta_{2^n}$.

As it's well know, there is logical operation in sequence control system and PBCN. The significance of STP is that any logical function can be converted into an algebraic function by STP. According to the theorem referred in (Cheng and Qi, 2010) that any logical function L ($A_1, \ldots, A_n$) with logical arguments $A_1, \ldots, A_n \in \Delta$ can be expressed in a multi-linear form as:

$$L(A_1, \ldots, A_n) = M_L A_1 A_2 \ldots A_n$$

where, $M_L$ is unique, called the structure matrix of L.

## MAIN RESULTS

**PBCN with time delays:** We can derive the PBCN with time delays as follows:

$$\begin{cases} s_1(t+1) = f_1(u_1(t), \cdots, u_m(t), s_1(t-\tau), \cdots, s_n(t-\tau)) \\ s_2(t+1) = f_2(u_1(t), \cdots, u_m(t), s_1(t-\tau), \cdots, s_n(t-\tau)) \\ \vdots \\ s_n(t+1) = f_n(u_1(t), \cdots, u_m(t), s_1(t-\tau), \cdots, s_n(t-\tau)) \end{cases} \quad (1)$$

where, $f_i$: $D^{n+m} \rightarrow D$, i=1,2,…,n are the logic functions; $u_i(t)$, $s_i(t) \in \Delta_2$; t=0,1,2,… and $\tau$ is a finite positive integer.

As a PBCN, $f_i$ contains several different functions and fj i denotes the j-th function, where the probability of $f_i$ being fj i is pj i.

The matrix K is defined to denote the index set of possible model. When the choice of functions is independent, each row of K represents a possible control model with probability:

$$P_i = \prod_{j=1}^{n} p_j^{K_{ij}}$$

Define $x(t) = \ltimes_{i=1}^{n} s_i(t)$, further multiplying Eq. 1 yields:

$$x(t+1) = L_i u(t) x(t-\tau) \quad (2)$$

where, $L_i = M_1^{K_{i1}}(I_{2^{n+m}} \otimes M_2^{K_{i2}})\Omega_{n+m} \cdots (I_{2^{n+m}} \otimes M_n^{K_{in}})\Omega_{n+m}$ and $\Omega_n = \ltimes_{i=1}^{n} I_{2^{i-1}} \otimes (W_{[2,2^{n+1-i}]} M_r)$, $W_{[i,j]}$ is a swap matrix and $M_r$ is the power reducing matrix $M_r = \delta_4[1\ 4]$ (Cheng *et al.*, 2010).

And then, we transform Eq. 2 to:

$$x(t+1) = L_i W_{[2^n, 2^m]} x(t-\tau) u(t) \quad (3)$$

Denoting that $\tilde{L}_i = L_i W_{[2^n, 2^m]}$, the overall expected value of x(t+1) satisfies:

$$Ex(t+1) = \tilde{L} Ex(t-\tau) u(t) \quad (4)$$

Where:

$$\tilde{L} = \sum_{i=1}^{N} P_i \tilde{L}_i$$

**Controllability:** In the following section, the controllability will be discussed when the system under different kinds of controls.

**A. Free boolean sequence control: Theorem 1:** Consider the PBCN shown in Eq. 1 with the free Boolean sequence control. $x_d$ is reachable from x(m-$\tau$) with probability one, iff:

$$x_d \in \bigcup_{j=1}^{r} Col_{\Delta_{2^n}} \{\tilde{L}^j x(m-\tau)\}$$

where, d-k-k$\tau$ = m-$\tau$ and m$\in$\{0,1,…, $\tau$\}.

**Proof:**

$$\begin{aligned} Ex(d) &= \sum_{i=1}^{N} P_i \tilde{L}_i Ex(d-1-\tau) u(d-1) \\ &= \tilde{L} Ex(d-1-\tau) u(d-1) \\ &= \tilde{L}^2 Ex(d-2-2\tau) u(d-2-\tau) u(d-1) \\ &\vdots \\ &= \tilde{L}^k Ex(d-k-k\tau) u(d-k-(k-1)\tau) \cdots u(d-1) \end{aligned}$$

Suppose that d-k-k$\tau$ = m-$\tau$, where m$\in$\{0,1,…,$\tau$\}, we have:

$$Ex(d) = \tilde{L}^k x(m-\tau) u(d-k-(k-1)\tau) \cdots u(d-1)$$

Referring to the theorem presented in (Wang *et al.*, 2012) that if there is a smallest positive integer r that

$Col\{\widetilde{L}^{r+1}x_0\} \subset Col\{\widetilde{L}^s x_0 \mid s=1,2,\ldots,r\}$, the reachable set with probability one is:

$$\bigcup_{j=1}^{r} Col_{\Delta_{2^n}}\left\{\widetilde{L}^j x_0\right\}$$

So, the reachable set from x(m-τ) under the sequence control u(d-k-(k-1)τ)... u(d-1) with probability one is:

$$\bigcup_{j=1}^{r} Col_{\Delta_{2^n}}\left\{\widetilde{L}^j x(m-\tau)\right\}$$

**Close-loop control:** When the system is a close-loop, the control u(t) is generated according to the delayed states and can be expressed as:

$$u(t) = Hx(t-\tau)$$

**Theorem 2:** Consider the PBCN with the close-loop control and the initial state x(m-τ), m∈{0,1,..., τ}. If there is a smallest positive integer r satisfying that:

$$\widetilde{G}^{r+1}x(m-\tau)=\widetilde{G}^s x(m-\tau)$$

where, s∈{1,2,..., r} and $x_d$ is reachable from x(m-τ) with probability one, such that:

$$x_d \in \bigcup_{j=1}^{r} Col_{\Delta_{2^n}}\left\{\widetilde{G}^j x(m-\tau)\right\}$$

where, d-k-kτ = m-τ and m∈{0,1,..., τ}.

**Proof:** According to Eq. 2, we have:

$$x(t+1) = LiHx(t-\tau)x(t-\tau) = LiH\Omega nx(t-\tau) = Gix(t-\tau)$$

So:

$$Ex(t+1)=\sum_{i=1}^{N}P_iG_iEx(t-\tau)=\widetilde{G}Ex(t-\tau)$$

$$\begin{aligned} Ex(d) &= \sum_{i=1}^{N}P_iG_iEx(d-1-\tau) \\ &= \widetilde{G}Ex(d-1-\tau) \\ &= \widetilde{G}^2Ex(d-2-2\tau) \\ &\vdots \\ &= \widetilde{G}^kEx(d-k-k\tau) \end{aligned}$$

Suppose that d-k-kτ = m-τ, where k∈{0,1,...}, m∈{0,1,...,τ}. Then, there is:

$$Ex(d)=\widetilde{G}^kEx(m-\tau)=\widetilde{G}^kx(m-\tau)$$

Because there are τ+1 initial states, for any d>0, Ex(d) has a periodic relationship with the initial states and the period is T = τ+1:

$$Ex(d+nT)=\widetilde{G}^{k+n}x(m-\tau)$$

For the state x(a1) which is corresponding to initial state x(m1-τ), we have:

$$Ex(a_1)=\widetilde{G}^kx(m_1-\tau)$$

If there is:

$$\widetilde{G}^{r+1}x(m_1-\tau)=\widetilde{G}^sx(m_1-\tau)$$

It leads to:

$$\begin{aligned} Ex(a_1+(r+1)T) &= \widetilde{G}^{k+r+1}x(m_1-\tau) \\ &= \widetilde{G}^{k+s}x(m_1-\tau) \\ &= Ex(a_1+sT) \end{aligned}$$

And:

$$\begin{aligned} Ex(a_1+(r+2)T) &= \widetilde{G}^{k+r+2}x(m_1-\tau) \\ &= \widetilde{G}^{k+s+1}x(m_1-\tau) \\ &= Ex(a_1+(s+1)T) \\ &\vdots \\ Ex(a_1+(r+r-s+2)T) &= Ex(a_1+(s+r-s+1)T) \\ &= Ex(a_1+(r+1)T) \\ &= Ex(a_1+sT) \end{aligned}$$

Let the destination state x(d1) = x(a1+sT). it can be found that after r periods there are no more new columns. And the reachable set from x(m1-τ) with probability one is:

$$\bigcup_{j=1}^{r} Col_{\Delta_{2^n}}\left\{\widetilde{G}^j x(m_1-\tau)\right\}$$

The deduction is similar when the destination states corresponding to other initial states, so the conclusion can be derived.

## ANALYSIS OF SEQUENCE CONTROL SYSTEM

**System introduction:** Figure 1 shows the sequence control system with smart node. For simplicity, we present

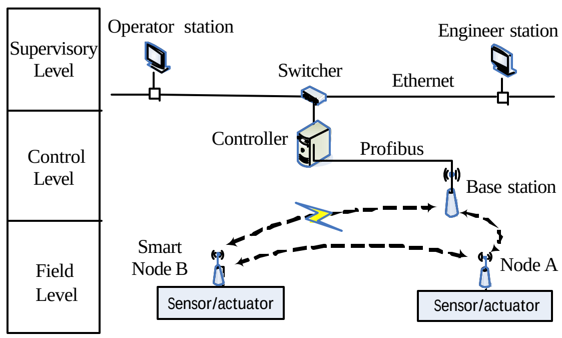


Fig. 1: Sequence control system with smart node

the example with one smart node. However, the approach is applicable to the system with n smart nodes.

In traditional way, all the wireless nodes have to communicate with base station to receive the control instructions to drive the actuator. However, when there is a barrier or the scale of the network is too large, the base station can't communicate with every nodes directly, such as Node B. One normal solution is using router. For example, Node A plays as a router, relaying the data between base station and Node B (Al-Karaki and Kamal, 2004). And the solution is feasible. However, the relay of data will result in high delays and errors of data (Eryilmaz and Srikant, 2006).

In this study, a new solution is proposed, introducing smart node. The feature is that smart node stores the control logic related to the connected actuator. So the smart node doesn't have to wait for the control instructions sent from controller, but generate it in the node locally. It's obvious that Node B will act more quickly and the workload of Node A will decrease that Node A doesn't relay the control instructions from controller to Node B. So, the performance of real time of the system will be improved and the workload of wireless nodes will be more balanced. However, there is problem introduced at the same time. For the control logic executed in the wireless smart node, the controllability of the system will be influenced by bit errors and time delays.

**Analysis of the system:** Assume that sample period is larger than transmission delay and one node can send its data in one period. And data can be transmitted successfully in one sample period by retransmission. So Node B sends its state to Node A in one sample period and Node A sends the states of Node A and Node B to controller in the next sample period. And it cost 2 sample periods for controller to receive the states of the device in field and it can be set that $\tau = 2$. To ensure synchronization of control, Node B will delay changing the state after $\tau$. Besides that, when there are errors in data transmission, both the controller and smart node will execute different functions.

The system can be expressed as:

$$\begin{cases} s_1(t+1) = f_1(u_1(t), s_1(t-2), s_2(t-2)) \\ s_2(t+1) = f_2(u_2(t), s_1(t-2), s_2(t-2)) \end{cases} \quad (5)$$

The functions in controller and smart node are listed, respectively as follow:

$$\begin{cases} f_1^1 = u_1(t) \vee (s_1(t-2) \leftrightarrow s_2(t-2)) \\ f_1^2 = u_1(t) \wedge (s_1(t-2) \wedge s_2(t-2)) \end{cases} \quad (6)$$

$$\begin{cases} f_2^1 = s_1(t-2) \vee s_2(t-2) \\ f_2^2 = s_1(t-2) \wedge s_2(t-2) \end{cases} \quad (7)$$

The probability of the execution of $f_i^j$ is decided by the bit error in wireless transmission. And we can suppose that the probabilities of each function are 0.9, 0.1, 0.9, 0.1, respectively. And matrix K and the control probabilities are:

$$K = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{bmatrix} \qquad \begin{aligned} P_1 &= 0.9 \times 0.9 = 0.81 \\ P_2 &= 0.9 \times 0.1 = 0.09 \\ P_3 &= 0.9 \times 0.1 = 0.09 \\ P_4 &= 0.1 \times 0.1 = 0.01 \end{aligned}$$

In the following parts, the controllability of the system under different controls will be studied.

**Free control sequence:** Let u(t) = u1(t). According to Eq. 2, we have:

$$\begin{aligned} x(t+1) &= M_d(I_2 \otimes M_e)(I_{2^3} \otimes M_d)\left[I_2 \otimes (I_2 \otimes W_{[2]})M_r\right] \\ &\quad u(t)x(t-2) \\ &= \delta_4[1,1,1,2,3,1,1,4]u(t)x(t-2) \\ &= L_1 u(t)x(t-2) \end{aligned}$$

Similarly, we have $L_2 = \delta_4[1, 2, 2, 2, 3, 2, 2, 4]$, $L_3 = \delta_4[1, 3, 3, 4, 3, 3, 3, 4]$, $L_4 = \delta_4[1, 4, 4, 4, 3, 4, 4, 4]$.

According to Eq. 3 and 4, we have:

$$\tilde{L} = \sum_{i=1}^{N} P_i L_i W_{[4,2]} = \begin{bmatrix} 1 & 0 & 0.81 & 0.81 & 0.81 & 0.81 & 0 & 0 \\ 0 & 0 & 0.09 & 0.09 & 0.09 & 0.09 & 0.9 & 0 \\ 0 & 1 & 0.09 & 0.09 & 0.09 & 0.09 & 0 & 0 \\ 0 & 0 & 0.01 & 0.01 & 0.01 & 0.01 & 0.1 & 1 \end{bmatrix}$$

We have $Ex(t+1)=\tilde{L}Ex(t-\tau)u(t)$. Suppose that i = 2, s = 3, $s_1(2-\tau) = \delta^1_2$, $s_2(2-\tau) = \delta^1_2$, $x(2-\tau) = \delta^1_4$ and carry on the calculation further. Then according to Theorem 1, when k = 1, there is:

$$Ex(s+i)=\tilde{L}^2Ex(2-\tau)u(1)u(4)=\begin{bmatrix}1 & 0 & 0.81 & 0.81\\ 0 & 0 & 0.09 & 0.09\\ 0 & 1 & 0.09 & 0.09\\ 0 & 0 & 0.01 & 0.01\end{bmatrix}u(1)u(4)$$

It can be found that $\delta^1_4$ and $\delta^1_3$ can be reached. When $\delta^1_3$ is set as the destination state, we have to choose the control sequence which satisfies u(1)u(4) = $\delta^1_2$, meaning that $u_1(1) = u(1) = \delta^1_2$, $u_1(4) = u(4) = \delta^2_2$.

**Close-loop control:** When the controls are generated based on the states fed back from the wireless nodes, it forms the close-loop control. Based on the system and the functions presented from Eq. 5-7, suppose that the control is:

$$u_1(t)=u(t)=s_1(t-2)\leftrightarrow s_2(t-2)=M_e x(t-\tau)$$

So, we have $Ex(t+1)=\tilde{G}Ex(t-2)$, where:

$$\tilde{G}=\sum_{i=1}^{4}P_iL_iM_e\Omega_2$$

And it can be derived:

$$\tilde{G}=\begin{bmatrix}0 & 0.81 & 0.81 & 0\\ 0 & 0.09 & 0.09 & 0\\ 1 & 0.09 & 0.09 & 0\\ 0 & 0.01 & 0.01 & 1\end{bmatrix}$$

For k steps of state transitions, we have $\tilde{G}^k$ when $k\rightarrow\infty$, $\tilde{G}^k$ will converge to a constant and it can be found that the destination state will be $\delta^4_4$, no matter what the initial state is.

## CONCLUSION

In this study, the controllability of PBCN with time delays is investigated. And the sequence control system based on wireless network with smart nodes is proposed. With the derived results in theory, the controllability of the proposed system is analysed. And examples are also presented for illustration and demonstration of the approach. However, there are other problems still not yet considered, such as the variant time delays in states. Because the delay will vary greatly in the wireless network with large scale, the performance of the sequence control system will be influenced. How to settle this problem will be the focus in our further research.


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