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Control of Hopf Bifurcation in Internet Congestion Control Model via Time-delayed Feedback Control

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Abstract: In this study, we are concerned with controlling Hopf bifurcation in a dual model of Internet congestion control algorithms. The stability of this system depends on a communication delay parameter, and Hopf bifurcation occurs when the communication delay passes through a critical value. Comparing with previous work, a time-delayed feedback method is proposed to postpone the onset of undesired Hopf bifurcation and improve stability of the dual model. Theoretical analysis and numerical simulation are provided to verify the effectiveness of our method.

Key words: Internet congestion control, jopf bifurcation, bifurcation control, time-delayed feedback

INTRODUCTION

Nowadays, Internet congestion control algorithms model have attracted considerable attention. As we know, congestion is an unavoidable phenomenon in the network, which may induce the loss of packets and the increasing of delay. So the stability of network congestion control plays a key role in the Internet congestion control system. When the congestion control system loses its stability, it may cause some dynamic behavior such as bifurcation and chaos, which has been intensively studied in recent years (Van Gorder and Choudhury, 2011; Ding *et al*., 2009; Liu *et al*., 2011).

In reality, the complex dynamic behaviors often degrade the performance of networks. Hence, there are a lot of methods to delay or even avoid this kind of behaviors. For instance, Guo *et al.* (2008) used a dynamic delayed feedback controller for the second-order Internet congestion control system. Ding *et al.* (2008) presented a hybrid control of bifurcation and chaos method in stroboscopic model of Internet congestion control system. There are many other methods (Nguyen and Hong, 2012; Liu *et al*., 2012).

The rest of the article is organized as follows. In Sec. 2, we introduce a time-delayed feedback method in the fair dual model of Internet congestion control system. Sec. 3 is devoted to the direction and stability analysis of the Hopf bifurcation on the system with time-delayed feedback control. Numerical examples to verify the theoretic analysis are given in Sec. 4. The last section gives a brief conclusion.

EXISTENCE OF HOPF BIFURCATION OF SYSTEM WITH CONTROL

The dynamical representation of a dual congestion control system is as follows:

$$\dot{p}(t) = kp(t)(x(t-\tau) - c) \tag{1}$$

where, x (t) = f (p (t)) is a nonnegative continuous, strictly decreasing demand function and has at least third-order continuous derivatives. The scalar c is the capacity of the bottleneck link and the variable p is the price at the link. k is a gain parameter.

We add a time-delayed force h (p(t)-p (t-τ)) to the model (1) and then get the following controlled system:

$$\dot{p}(t) = kp(t)(x(t-\tau) - c) + h(p(t) - p(t-\tau)) \tag{2}$$

where, the feedback gain h is negative real number. It is obvious that the controlled system has the same equilibrium point as the original model (1).

The initial condition of Eq. 2 is specified by a real-valued continuous function:

$$p(s) = \phi(s),\ s \in [-\tau, 0]$$

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Let p* be an equilibrium point of (2). Then p* satisfy:

$$x(p^*) = c \tag{3}$$

Define u (t) = p (t)-p* and take a Taylor expansion of Eq. 2 including the linear, quadratic and cubic terms, we obtain:

$$\begin{aligned}\dot{u}(t) &= hu(t) + b_2u(t-\tau) + b_4u(t)u(t-\tau) + \\ & b_5u^2(t-\tau) + b_8u(t)u^2(t-\tau) + \\ & b_9u^3(t-\tau) + O(u^4)\end{aligned} \tag{4}$$

Where:

$$\begin{aligned}&b_2 = kp^*x'(p^*) - h, b_4 = \frac{1}{2}kx'(p^*),\\ &b_5 = \frac{1}{2}kp^*x''(p^*), b_8 = \frac{1}{6}kx''(p^*),\\ &b_9 = \frac{1}{6}kp^*x'''(p^*)\end{aligned}$$

The linearized equation of Eq. 4 is:

$$\dot{u}(t) = hu(t) + b_2u(t-\tau) \tag{5}$$

and its characteristic equation is:

$$\lambda - h - b_2e^{-\lambda\tau} = 0 \tag{6}$$

By means of simple analysis, we get:

- **Corollary 1:** When the communication delay τ is smaller than the critical value $\tau_0 = -\pi/(2b)$, the equilibrium point p* of system (1) is asymptotically stable. When the delay τ passes through τ_0, there is a Hopf bifurcation of system (1) at its equilibrium point p*

STABILITY AND DIRECTION OF BIFURCATING PERIODIC SOLUTIONS

In this section, we determinate the direction of the bifurcation and the stability of bifurcating periodic solutions of controlled system (2).

We use the similar method in the study by Ding *et al*. (2009), the main results are as follows:

- **Theorem 1:** For controlled system (2), the following results hold:

- μ_2 determines the direction of the Hopf bifurcation. If $\mu_2>0$ (<0), the Hopf bifurcation is supercritical (subcritical) and the bifurcating periodic solutions exist for $\tau>\tau_0$ ($\tau<\tau_0$)
- β_2 determines the stability of the bifurcating periodic solution. If $\beta_2>0$ (>0), the bifurcating periodic solutions are stable (unstable)
- T_2 determines the period of the bifurcating periodic solution. If $T_2 > 0$ (<0), the period increases (decreases)

Where:

$$\begin{aligned}&C_1(0) = \frac{i}{2\omega_0}\left(g_{20}g_{11} - 2|g_{11}|^2 - \frac{1}{3}|g_{02}|^2\right) + \frac{g_{21}}{2}\\ &\mu_2 = -\frac{\operatorname{Re}\{C_1(0)\}}{\operatorname{Re}\lambda'(0)}\\ &T_2 = -\frac{\operatorname{Im}\{C_1(0)\} + \mu_2\operatorname{Im}\lambda'(0)}{\omega_0}\\ &\beta_2 = 2\operatorname{Re}\{C_1(0)\}\end{aligned} \tag{7}$$

Here C_1 (0) is the Lyapunov coefficient, and g_{20}, g_{11}, g_{02}, g_{21}, are defined by:

$$\begin{cases} g_{20} = 2\bar{B}\left(b_4\exp(-i\omega_0\tau) + b_5\exp(-i2\omega_0\tau)\right)\\ g_{11} = \bar{B}\left[b_4\left(\exp(i\omega_0\tau) + \exp(-i\omega_0\tau)\right) + 2b_5\right]\\ g_{02} = 2\bar{B}\left(b_4\exp(i\omega_0\tau) + b_5\exp(i2\omega_0\tau)\right)\\ g_{21} = 2\bar{B}\Big[b_4W_{11}(0)\exp(-i\omega_0\tau) + \dfrac{W_{20}(0)}{2}\exp(i\omega_0\tau)\\ +W_{11}(-\tau) + \dfrac{W_{20}(-\tau)}{2}\\ +b_5\left(2W_{11}(-\tau)\exp(-i\omega_0\tau) + W_{20}(-\tau)\exp(i\omega_0\tau)\right)\\ +b_8\left(\exp(-i2\omega_0\tau) + 2\right) + 3b_9\exp(-i\omega_0\tau)\Big] \end{cases} \tag{8}$$

$$B = \frac{1}{1 + b_2\tau\exp(i\omega_0\tau)} \tag{9}$$

We still need the values of W_{11} (θ)and W_{20} (θ) for $\theta\in[-\tau, 0)$:

$$\begin{aligned}W_{20}(\theta) = &-\frac{g_{20}}{i\omega_0}q(0)\exp(i\omega_0\theta) - \\ &\frac{\bar{g}_{02}}{i3\omega_0}\bar{q}(0)\exp(-i\omega_0\theta) + E_1\exp(i2\omega_0\theta)\end{aligned}$$

$$\begin{aligned}W_{11}(\theta) = &\frac{g_{11}}{i\omega_0}q(0)\exp(i\omega_0\theta) - \\ &\frac{\bar{g}_{11}}{i\omega_0}\bar{q}(0)\exp(-i\omega_0\theta) + E_2\end{aligned}$$

Where:

$$E_1 = \frac{\Phi_1}{h + b_2 \exp(-i2\omega_0\tau) - i2\omega_0}, \qquad E_2 = \frac{\Phi_2}{h + b_2} \tag{10}$$

NUMERICAL SIMULATIONS

In this section, we consider the fair dual which give a proportionally fair resource allocation, i.e., x (t) = 1/p(t).

Let the link capacity is 1.25 Mbps and the time unit is 40 ms. If the packet sizes are 1000 bytes each, the link capacity can be expressed as c = 50 packets per time unit. In addition, let the gain parameter k = 0.1.

We first choose h = 0. By direct calculation we can get:

$$p^* = 0.02,\ \omega_0 = 0.5,\ \tau_0 = 3.1416$$
$$\mu_2 = 5259.2,\ T_2 = 2125,\ \beta_2 = -758.38$$

The dynamic behavior of the uncontrolled model (1) is illustrated in Fig. 1-3. From Corollary 1, it is obvious that when $\tau<\tau_0$, trajectories converge to the equilibrium point (Fig. 1), while as τ is increased to pass τ_0, p* loses its stability and a Hopf bifurcation occurs (Fig. 2 and 3). Since, $\beta_2<0$, the periodic orbits are stable. As $\mu_2>0$, the Hopf bifurcation is supercritical and the bifurcation periodic solutions exist when $\tau>\tau_0$. The period of the periodic solutions increases as τ increases due to $T_2>0$ (compare Fig. 2 and 3).

Now we consider the problem of controlling the Hopf bifurcation in system (1). By choosing h = -0.1, we obtain:

$$p^* = 0.02,\ \omega_0 = 0.3873,\ \tau_0 = 4.7082$$
$$\mu_2 = 27606,\ T_2 = 5572.9,\ \beta_2 = -1508.9$$

Note that the controlled system (2) has the same equilibrium point as that of the original system (1), but the critical value τ_0 increases from 3.1416 to 4.7082, implying that the onset of Hopf bifurcation is delayed. It is seen from Fig. 4 and 5 that when τ = 3.4, the controlled system (2) converges to the equilibrium point p*. When τ passes the critical value τ_0 = 4.7082, a Hopf bifurcation occurs.

The relationship between the critical value τ_0 and the control feedback gain h is shown in Fig. 6. From this figure we know that when decreasing h, the critical value

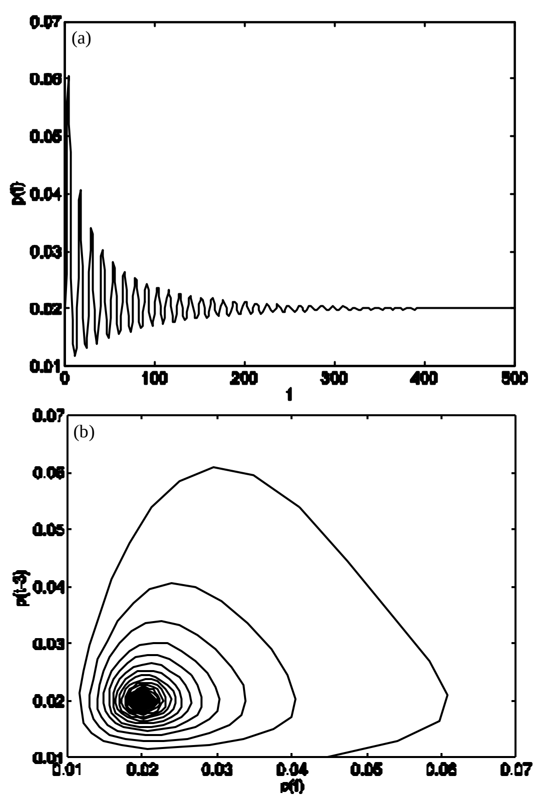

Fig. 1: Waveform plot and phase portrait of uncontrolled system (1) with τ = 3

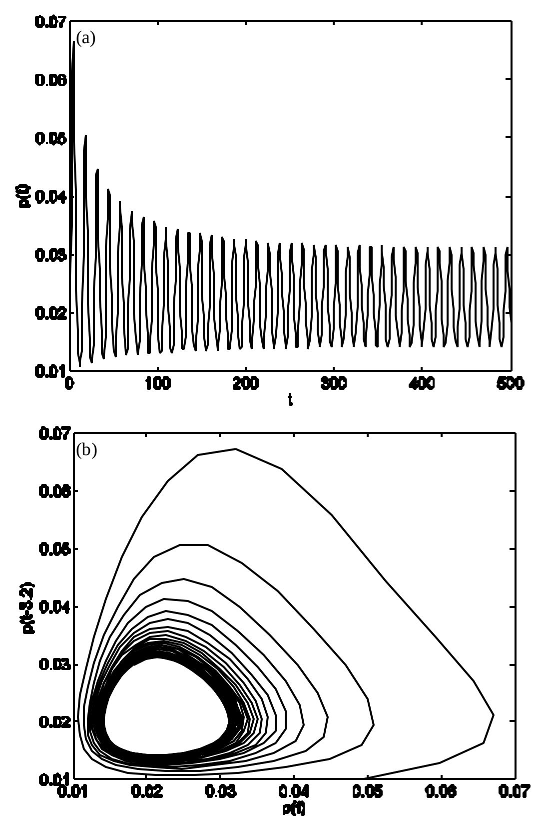

Fig. 2: Waveform plot and phase portrait of uncontrolled system (1) with τ = 3.2

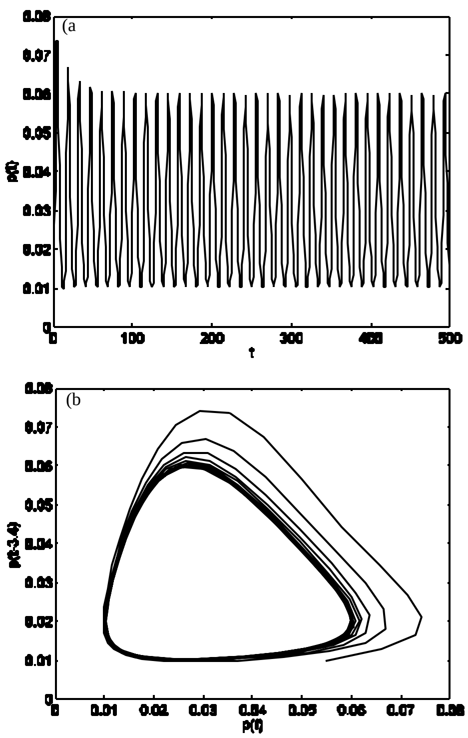

Fig. 3(a-b): Waveform plot and phase portrait of uncontrolled system (1) with τ = 3.4

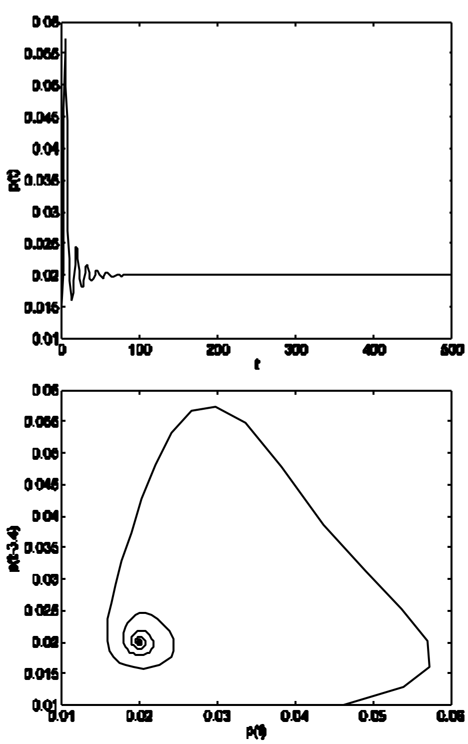

Fig. 4(a-b): Waveform plot and phase portrait of controlled system (2) with τ = 3.4 and h = -0.1

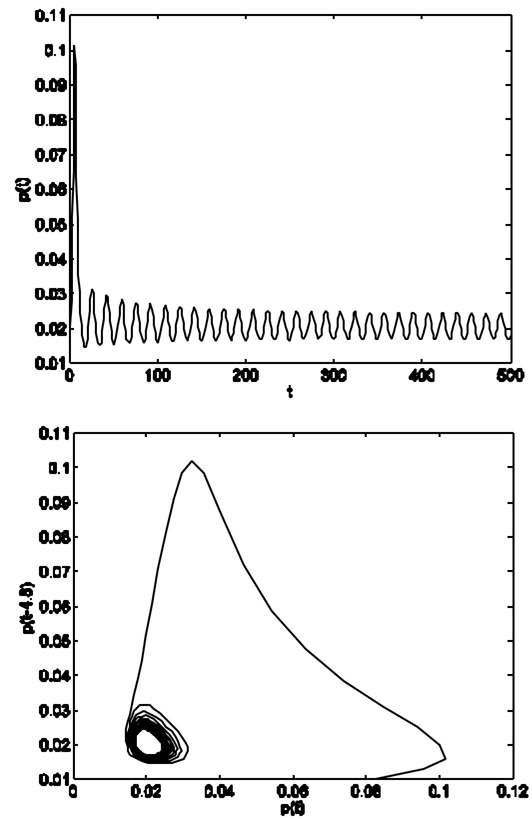

Fig. 5(a-b): Waveform plot and phase portrait of controlled system (2) with τ = 4.8 and h = -0.1

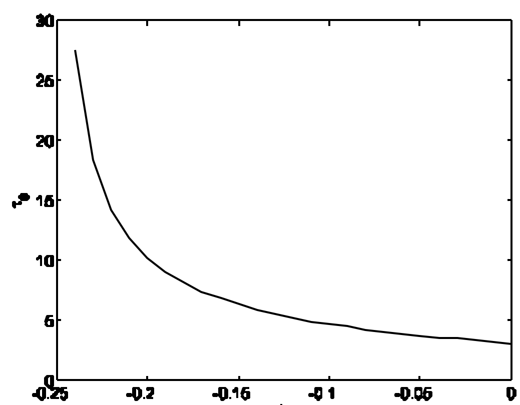

Fig. 6: Relationship between τ_0 with h

increases, therefore we can get a larger stability range of the system. For example, by choosing h = -0.15, the critical value τ_0 = 6.3679. In detail, we choose τ = 5 and h = -0.15. The controlled system (2) converges to the equilibrium point, as shown in Fig. 7.

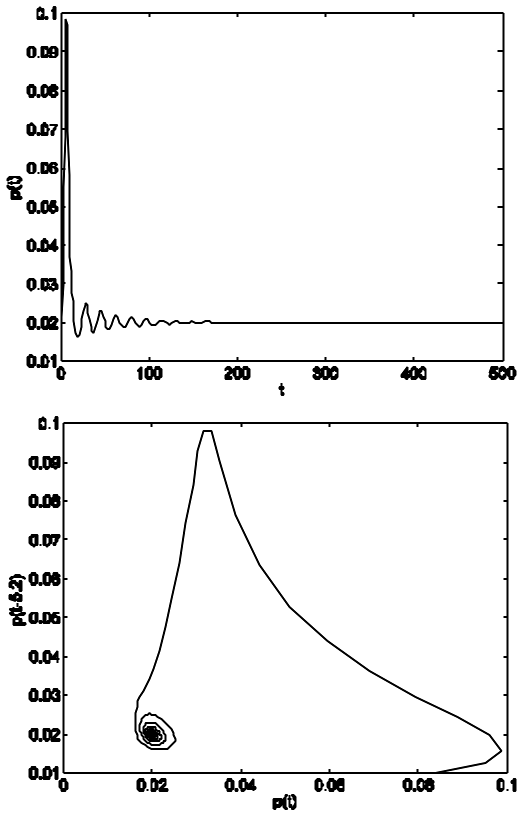

Fig. 7(a-b): Waveform plot and phase portrait of controlled system (2) with $\tau = 5.2$ and $h = -0.15$

CONCLUSION

A fair dual model of Internet congestion control system with a time-delayed feedback controller has been studied in this study. The system loses stability and a Hopf bifurcation occurs when the delay passes a critical value. It has been shown that the time-delayed feedback controller can effectively control Hopf bifurcation. Numerical simulations have verified the validity of this control method.

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