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T_H-interval-valued Fuzzy Rings and Their Homomorphism Properties

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Abstract: In this study, the concept of T_H-interval-valued fuzzy ring is first introduced based on the notion of fuzzy ring, then some meaningful properties of T_H-interval-valued fuzzy ring are investigated. The results show that fuzzy ring and interval-valued fuzzy ring are special cases of T_H-interval-valued fuzzy ring.

Key words: Interval-valued fuzzy set, T_H-interval-valued fuzzy ring, homomorphism

INTRODUCTION

The research on interval-valued fuzzy set has attracted many authors' attention. For example, Biswas (1994) first introduced the concept of interval-valued fuzzy set to discuss fuzzy algebra and studied interval-valued fuzzy group. Sun and Gu (1998) investigated the properties of fuzzy algebra based on interval-valued fuzzy set. Wang and Zhou (1997) and Li and Wang (2000) proposed the concept of T_H-interval-valued fuzzy group and S_H-interval-valued fuzzy group and studied their properties.

In this study, we first introduce the concept of T_H-interval-valued fuzzy ring. Then, the corresponding properties of T_H-interval-valued fuzzy ring are studied. Through the remarks below, one would find that the results of this study are an interesting and meaningful extension of fuzzy ring and interval -valued fuzzy ring.

PRELIMINARIES

Here, some basic notions and notations of fuzzy set are reviewed, the detailed descriptions could be found by Zadeh (1965), Meng (1993) and Kumar *et al.* (1992).

Definition 1: Let X be a crisp set. The mapping A: $X\rightarrow[I]$ is called an interval-valued fuzzy set based on X. We denote by (X) the set of all the fuzzy set on X. For any $A\in IF(X)$, if we define that $A(X) = [A(x), A^+(x)]$, $A^-(x)\leq A^+(x)$, $x\in X$, then $A^-: X\rightarrow I$ and $A^+: X\rightarrow I$ are called the lower fuzzy set and the upper fuzzy set of X, respectively.

For any $x\in X$, $A,B\in IF(X)$, one can define that $(A\cup B)(x) = A(x)\vee B(x)$, $(A\cap B)(x) = A(x)\wedge B(x)$.

For any $A\in IF(X)$, $[\lambda_1, \lambda_2]$, we have that $A_{[\lambda 1,\ \lambda 2]} = \{x\in X|A^-(x)\geq\lambda_1, A^-(x)\geq\lambda_2\}$, $A_{[\lambda 1,\ \lambda 2]}$ is called the $[\lambda_1, \lambda_2]$- cut sets of A. Obviously, we have that $A_{[\lambda 1,\ \lambda 2]} = A^-\lambda_1\cap A^+\lambda_2$, $A_{[0,\ 0]} = X$.

For any $A, B\in IF(X)$ and $[\lambda_1, \lambda_2]\in[I]$, the following two equations are satisfied:

$$(A\cup B)_{[\lambda_1,\lambda_2]} = A_{[\lambda_1,\lambda_2]}\cup B_{[\lambda_1,\lambda_2]}\cup(A^-_{\lambda_1}\cap B^+_{\lambda_2})\cup(A^-_{\lambda_1}\cap B^+_{\lambda_2})$$

$$(A\cap B)_{[\lambda_1,\lambda_2]} = A_{[\lambda_1,\lambda_2]}\cap B_{[\lambda_1,\lambda_2]}$$

Definition 2: A mapping $T : I\times I\rightarrow I$ is said to be an idempotent norm, if the following conditions satisfy, where $a, b, c, d\in I$:

(1) If $a\leq c, b\leq d$ then $T(a,b)\leq T(c,d)$
(2) $T(a,b) = T(b,a)$
(3) $T(T(a,b),c) = T(a,T(b,c))$
(4) $T(a,0) = 0$, $T(a,1) = a$
(5) $T(a,a) = a$

Proposition 1: If T is an idempotent norm, then for any a, b, c, $d\in I$ we have:

$$T(a,b)\wedge T(c,d)\geq T(a\wedge c,b\wedge d)$$

Proof: Noting that $a\geq a\wedge c$, $b\geq b\wedge d$ according to (1) in Definition 2, $T(a,b)\geq T(a\wedge c,b\wedge d)$ is satisfied; Analogically, $T(c,d)\geq T(a\wedge c,b\wedge d)$; Thus, $T(a,b)\geq T(a\wedge c,b\wedge d)$.

Definition 3: A mapping $T_H : [I]\times[I]\rightarrow[I]$ is said to be an idempotent interval norm, if for any $\bar{a}, \bar{b}\in[I]$, $T_H = [T(a^-, b^-), T(a^+, b^+)]$ is satisfied, where T is an idempotent norm.

Proposition 2: For any $\bar{a}, \bar{b}, \bar{c}, \bar{d}\in[I]$, if $\bar{a}\leq\bar{b}$, $\bar{c}\leq\bar{d}$, then $T_H(\bar{a}\leq\bar{b})$, $T_H(\bar{c}\leq\bar{d})$.

Proof: If $\bar{a}\leq\bar{b}$, $\bar{c}\leq\bar{d}$ then we have that $a^-\leq c^-$, $a^+\leq c^+$; $b^-\leq d^-$, $b^+\leq d^+$. According to Definition 2, $T(a^-,b^-)\leq T(c^-,d^-)$ and $T(a^+,b^+)\leq T(c^+,d^+)$. Therefore:

$$[T(a^-,b^-), T(a^+,b^+)] \le [T(c^-,d^-), T(c^+,d^+)]$$

Thus, we have that $T_H(\bar{a} \le \bar{b}), T_H(\bar{c} \le \bar{d})$.

Proposition 3: For any $\bar{a}, \bar{b}, \bar{c}, \bar{d} \in [I]$ we have that $T_H(\bar{a} \le \bar{b}) \wedge T_H(\bar{c} \le \bar{d}) \ge T_H(\bar{a} \wedge \bar{b}, \bar{c} \wedge \bar{d})$.

Proof: Noting that $\bar{a} \ge \bar{a} \wedge \bar{c}$, $\bar{b} \ge \bar{b} \wedge \bar{d}$, according to Proposition 2, $T_H(\bar{a} \le \bar{b}) \wedge T_H(\bar{c} \le \bar{d}) \ge T_H(\bar{a} \wedge \bar{b}, \bar{c} \wedge \bar{d})$ is satisfied. Analogically, $T_H(\bar{c}, \bar{d}) \wedge T_H(\bar{c} \le \bar{d}) \ge T_H(\bar{a} \wedge \bar{b}, \bar{c} \wedge \bar{d})$; Therefore, $T_H(\bar{a} \le \bar{b}) \wedge T_H(\bar{c} \le \bar{d}) \ge T_H(\bar{a} \wedge \bar{b}, \bar{c} \wedge \bar{d})$. From Proposition 3, we can conclude that for any $\bar{a} \in [I]$, $T_H(\bar{a}, \bar{a}) = \bar{a}$, $T_H(\bar{a}, \bar{1}) = \bar{a}$, $T_H(\bar{a}, \bar{0}) = \bar{0}$.

T_H-INTERVAL-VALUED FUZZY RING

Here, we shall introduce the concept of T_H-Interval-valued fuzzy ring based on the concept of fuzzy j ring.

Definition 4: Let R be a ring and A is a fuzzy set on R. A is said to be a fuzzy subring of R, if the following two conditions are satisfied.

(1) For any a, b∈R, $A(a+b) \ge A(a) \wedge A(b)$, $A(ab) \ge A(a) \wedge A(b)$
(2) For any a∈R, $A(-a) \ge A(a)$

Definition 5: Let R be a ring and T_H an idempotent interval norm. For any A∈IF(X), if the following two conditions are satisfied, then A is said to be a T_H-interval-valued fuzzy ring of R:

(1) For any x,y∈R, $A(x+y) \ge T_H(A(x),A(y))$ $A(xy) \ge T_H(A(x),A(y))$
(2) For any x∈R, $A(-x) \ge A(x)$

Theorem 1: Let R be a ring and A an interval-valued fuzzy set of R. Then A^- and A^+ are T-type fuzzy ring of R if and only if A is a T_H-interval-valued fuzzy ring of R.

Proof: If A is a T_H- interval-valued fuzzy ring of R, then, according to Definition 5, for any x,y∈R, we have $A(xy) \ge T_H(A(x),A(y))$ that is to say, $[A^-(x+y), A^+(x+y)] \ge T_H([A^-(x),A^+(x)], [A^-(y),A^+(y)]) = [T(A^-(x),A^-(y)), T(A^+(x), A^+(y))]$.

Then, we have that $A^-(x+y) \ge T(A^-(x),A^-(y))$, $A^+(x+y) \ge T(A^+(x),A^+(y)$. Thus, A^- and A^+ are T-type fuzzy ring of R. On the contrary, if $A(xy) \ge T_H(A(x), A(y))$ then, we have $A^-(xy) \ge T(A^-(x),A^-(y)) A^+(xy) \ge T(A^+(x),A^+(y))$; if $A(-x) \ge A(x)$ then, we have that $A^-(-x) \ge A^-(-x)$ and $A^+(-x) \ge A^+(-x)$. Thus, A is a T_H-interval-valued fuzzy ring of R.

Theorem 2: Let R be a ring. If A_1 and A_2 are two interval-valued fuzzy rings of R, then $A_1 \cap A_2$ is a T_H interval-valued fuzzy ring of R.

Proof:

- For any a,b∈R,we have that: $(A_1 \cap A_2)(a+b) = A_1(a+b) \wedge A_2(a+b)$:

$$\ge T_H(A_1(a),A_1(b)) \wedge T_H(A_2(a), A_2(b))$$
$$\ge T_H(A_1(a),A_2(a), A_1(b), A_2(b))$$
$$= T_H((A_1 \cap, A_2(a), (A_1 \cap A_2(b))$$

and $(A_1 \cap A_2)(ab) = A_1(ab) \wedge A_2(ab)$:

$$\ge T_H(A_1(a),A_1(b)) \wedge T_H(A_2(a), A_2(b))$$
$$\ge T_H(A_1(a),A_2(a), A_1(b) \wedge A_2(b))$$
$$= T_H((A_1 \cap, A_2(a), (A_1 \cap A_2(b))$$

- For any a∈R,we have that:

$$(A_1 \cap A_2)(-a) = A_1(-a) \wedge A_2(-a) \ge A_1(a) \wedge A_2(a) = (A_1 \cap A_2)(-a)$$

Thus, according to Definition 5, $A_1 \cap A_2$ is a T_H interval-valued fuzzy ring of R.

HOMOMORPHISM PROPERTIES

Here, we will discuss the homomorphism properties about the T_H interval-valued fuzzy ring.

Definition 6: Let R_1 and R_2 be two rings and a mapping φ between R_1 and R_2 is given as $\varphi: R_1 \to R_2$. Suppose $\bar{\varphi}$, $\bar{\varphi}^{-1}$ are two mappings between $IF(R_1)$ and $IF(R_2)$, they are given as $\bar{\varphi}$: $IF(R_1) \to IF(R_2)$ and $\bar{\varphi}^{-1}$: $IF(R_1) \to IF(R_2)$, we denote:

$$\hat{\varphi}(A)(y) = \begin{cases} \sup\limits_{x \in \varphi^{-1}(y)} A(x), \varphi^{-1}(y) \ne \Phi, \\ [0,0], \varphi^{-1}(y) = \Phi, \end{cases} \quad y \in R$$

$$\bar{\varphi}^{-1}(B)(x) = B(\varphi(x)), \forall x \in R_1$$

where, $A \in IF(R_1)$, $B \in IF(R_2)$, $\bar{\varphi}^{-1}(y) = \{x \in R_1 | \varphi(x) = y\}$ Then $\bar{\varphi}$ and $\bar{\varphi}^{-1}$ are respectively said to be interval-valued fuzzy transformation and interval-valued fuzzy inverse transformation generated by φ . From Definition 5, we can easily get the following two equations:

$$\hat{\varphi}(A)(y) = \left[\bigvee_{x \in \varphi^{-1}(y)} A^-(x), \bigvee_{x \in \varphi^{-1}(y)} A^+(x) \right]$$
$$= \left[\hat{\varphi}(A^-)(y), \hat{\varphi}(A^+)(y) \right], \text{ for any } y \in R_2$$

$$\hat{\varphi}^{-1}(B)(x) = \left[B^-(\varphi(x)), B^+(\varphi(x))\right] = \left[\hat{\varphi}^{-1}(B^-)(x), \hat{\varphi}^{-1}(B^+)(x)\right], \text{ for any } x \in R_1$$

Theorem 3: Let R_1 and R_2 be two rings, ϕ: $R_1 \to R_2$ a homomorphism mapping from $R_1$to R_2 and $\bar{\varphi}$ interval-valued fuzzy transformation generated by φ, If A_1 is a T_H-interval-valued fuzzy ring of R_1, $A_2 = \bar{\varphi}(A_1)$, then A_2 is a T_H-interval-valued fuzzy ring of R_2.

Proof:

- Firstly, if for any $y \in R_1$, there is $\varphi^{-1}(y) = \phi$, then we can conclude that $\varphi^{-1}(-y) = \phi$. In fact, if there exists $x_0 \in \varphi^{-1}(-y)$, then there must have $\varphi(x_0) = -y$. Since φ is a homomorphism mapping, we have that $\varphi(-x_0) = -\varphi(x_0) = y$. Consequently, $-x_0 \in \varphi^{-1}(-y)$. This is contradicted with hypotheses. By Definition 4.1, it follows that:

$$A_2(-y) = \bar{\varphi}(A_1)(-y) = [0,0] = \bar{\varphi}(A_1)(y) = A_2(y)$$

Secondly, if $\varphi^{-1}(-y) \neq \phi$, we have that:

$$\begin{aligned} A_2(-y) &= \bar{\varphi}(A_1)(-y) = \bigvee_{x \in \varphi^{-1}(-y)} A_1(x) \\ &= \bigvee_{\varphi(x)=-y} A_1(x) = \bigvee_{\varphi(-x)=y} A_1(-(-x)) \\ &\geq \bigvee_{-x \in \varphi^{-1}(y)} A_1(-x) = \bigvee_{z \in \varphi^{-1}(y)} A_1(z) \\ &= \bar{\varphi}(A_1)(y) = A_1(y) \end{aligned}$$

- Suppose φ: $R_1 \to R_2$ be an epimorphism mapping. If there exist $y_1, y_2 \in R_2$ such that:

$$\begin{aligned} A_2(y_1+y_2) &= \bar{\varphi}(A_1)(y_1+y_2) \\ &< T_H(\bar{\varphi}(A_1)(y_1), \bar{\varphi}(A_1)(y_2)) \\ &= T_H(A_2(y_1), A_2(y_2)) \end{aligned}$$

Thus, we can conclude that there exist $x_1, x_2 \in R_1$ satisfying $\varphi(x_1) = y_1$, $\varphi(x_2) - y_2$ and $A_2(y_1+y_2)$:

$$= \bar{\varphi}(A_1)(y_1+y_2) < T_H(A_1(x_1), A_1(x_2))$$

Noting that φ is an epimorphism mapping, then we have that $\varphi(x_1+x_2) = \varphi(x_1)+\varphi(x_2) = y_1+y_2$ such that:

$$A_2(y_1+y_2) = \hat{\varphi}(A_1)(y_1+y_2) = \bigvee_{\varphi(z)=y_1+y_2} A_1(z) \geq A_1(x_1+x_2)$$

Hence, $A_1(x_2+x_1) < T_H(A_1(x_1), A_1(x_2))$.

This is contradicted with the fact that A_1 is a T_H-interval-valued fuzzy ring of R_1. Therefore, for all $y_1, y_2 \in R_2$ we have:

$$\begin{aligned} A_2(y_1+y_2) &= \hat{\varphi}(A_1)(y_1+y_2) \\ &\geq T_H(\hat{\varphi}(A_1)(y_1), \hat{\varphi}(A_1)(y_2)) \\ &= T_H(A_2(y_1), A_2(y_2)) \end{aligned}$$

- If there exist $y_1, y_2 \in R_2$ such that:

$$\begin{aligned} A_2(y_1y_2) &= \hat{\varphi}(A_1)(y_1y_2) \\ &< T_H(\hat{\varphi}(A_1)(y_1), \hat{\varphi}(A_1)(y_2)) \\ &= T_H(A_2(y_1), A_2(y_2)) \end{aligned}$$

then, we can obtain that there exist $x_1, x_2 \in R_1$ satisfying $\varphi(x_1) = y_1$, $\varphi(x_2) = y_2$ and:

$$\begin{aligned} A_2(y_1y_2) &= \hat{\varphi}(A_1)(y_1y_2) < T_H(\hat{\varphi}(A_1)(y_1), \hat{\varphi}(A_1)(y_2)) \\ &= T_H(A_1(x_1), A_1(x_2)) \end{aligned}$$

Noting that φ is an epimorphism mapping, then we have $\varphi(x_1 x_2) = \varphi(x_1)\,\varphi(x_2) = y_1 y_2$ such that:

$$A_2(y_1y_2) = \hat{\varphi}(A_1)(y_1y_2) = \bigvee_{\varphi(z)=y_1y_2} A_1(z) \geq A_1(x_1x_2)$$

Consequently, $A_1(x_2+x_1) < T_H(A_1(x_1), A_1(x_2))$.

This is contradicted with the fact that A_1 is a T_H-interval-valued fuzzy ring of R_1.

Hence, for all $y_1, y_2 \in R_2$ we have:

$$\begin{aligned} A_2(y_1y_2) &= \hat{\varphi}(A_1)(y_1y_2) \geq T_H(\hat{\varphi}(A_1)(y_1), \hat{\varphi}(A_1)(y_2)) \\ &= T_H(A_2(y_1), A_2(y_2)) \end{aligned}$$

Therefore, according to Definition 3.2, we have that A_2 is a T_H-interval-valued fuzzy ring of R_2.

Theorem 4: Let R_1 and R_2 be two rings, ϕ: $R_1 \to R_2$a homomorphismz mapping and $\hat{\varphi}^{-1}$ an interval-valued fuzzy inverse transformation generated by φ. If A_2 is a T_H-interval-valued fuzzy ring of R_2, $A_1 = \hat{\varphi}^{-1}(A_2)$, then A_1 is a T_H-interval-valued fuzzy ring of R_1.

Proof: For any $x, y \in R$, we have:

$$\begin{aligned} A_1(x+y) &= \hat{\varphi}^{-1}(A_2)(x+y) = A_2(\varphi(x+y)) = A_2(\varphi(x)+\varphi(y)) \\ &\geq T_H(A_2(\varphi(x)), A_2(\varphi(y))) = T_H(\varphi^{-1}(A_2)(x), \varphi^{-1}(A_2)(y)) \\ &= T_H(A_1(x), A_1(y)) \end{aligned}$$

$$\begin{aligned} A_1(xy) &= \hat{\varphi}^{-1}(A_2)(xy) = A_2(\varphi(xy)) = A_2(\varphi(x)\varphi(y)) \geq T_H(A_2(\varphi(x)), A_2(\varphi(y))) \\ &= T_H(\varphi^{-1}(A_2)(x), \varphi^{-1}(A_2)(y)) = T_H(A_1(x), A_1(y)) \end{aligned}$$

$$\begin{aligned} A_1(-x) &= \hat{\varphi}^{-1}(A_2)(-x) = A_2(\varphi(-x)) = A_2(-\varphi(x)) \\ &\geq A_2(\varphi(x)) = \varphi^{-1}(A_2)(x) = A_1(x) \end{aligned}$$

Therefore, according to Definition 5, we conclude that A_1 is a T_H-interval-valued fuzzy ring of R_1.

CONCLUSION

In this paper we have studied the problem of fuzzy ring. We have proposed the notion of T_H-interval-valued fuzzy ring. And based on the notion, the corresponding homomorphism properties have been researched and some interesting results have been got. In the future we may involve in the investigation of the isomorphic properties of T_H-interval-valued fuzzy ring.

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