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ISSN 1812-5638

# INFORMATION TECHNOLOGY JOURNAL



Information Technology Journal 12 (19): 4968-4976, 2013 ISSN 1812-5638 / DOI: 10.3923/itj.2013.4968.4976 © 2013 Asian Network for Scientific Information

# Short-term Production Scheduling Optimization Integrated with Raw Materials Mixing Process in Petrochemical Industry

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**Abstract:** In petrochemical plants, schedulers have to determine how much newly received raw material should be piped into reception tanks which stored previously received raw material and mix raw material with desirable composition proportion to produce products with due dates. To address the challenge, we established an optimization model to deal with short-term production scheduling integrated with raw materials mixing process and introduced a strategy to make due dates of products demands flexible that can be adjusted at minimum costs while original ones cannot be met. A novel bounding algorithm is developed to determine hard bounds of variables in bilinear terms of the model, which makes the proposed mixed integer nonlinear programing (MNILP) model can be solved effectively by using the global solution approach based on piecewise defined convex envelops. Finally, a study case of petrochemical production is presented to illustrate the approaches proposed in the paper.

Key words: Short-term scheduling, MINLP, Convex envelops, Piecewise linear relaxation

# INTRODUCTION

Petrochemical plants receive raw material from upstream sources such as refineries and produce chemical products to meet market demands. The stored raw materials in plants are inevitably mixed with those newly received before fed into production units, which will lead to changes of composition proportion of the mixture in tanks. Schedulers have to determine which tanks are selected as reception tanks and how much raw material should be piped into selected tanks, while arrange subsequent production operations to meet demands.

Material mixing scheduling and production scheduling have achieved great progress in respective domain. The process of chemical raw materials mixing scheduling is similar with crude oil scheduling, which can share similar methodologies and strategies. Considering the properties constrains of crude oil mixing process, a MINLP model of scheduling optimization is built and MILP-based solution approaches are developed in (Moro and Pinto, 2004; Reddy et al., 2004) Karuppiah et al. (2008) presented an approach to find the global optimal solution of crude oil scheduling. As to Production scheduling, it determines the load of production units and which raw material tanks are used at each time point to meet products demands while consider

the capability of production units and tanks. Researches of production scheduling mainly focus on model description methods and solving strategies. The overall scheduling problems such as integration of material mixing and production process are always confronted with difficulties of complex modeling and inefficient algorithms, but there are some efforts and progress have been made (Zhang and Zhu, 2000; Jia and Ierapetritou, 2004; Zhang and Zhu, 2006).

Considering the constraints of raw materials components balance, mixing scheduling is a pooling problem which has binary terms in mathematical models. The standard pooling problem is descripted in detail by using different formulations and have varying implications for problem size and relaxation tightness (Misener and Floudas, 2009). Piecewise Linear Relaxing (PLR) is an efficient approach to relax the binary terms on the basis of convex envelops. PLR has numbers of formulation descriptions but they are mathematically equivalent. The achievements of PLR researches in recent years provide a powerful methodology to cope with nonlinear terms with the way of the global optimization (Gounaris et al., 2009). The rest of this paper is organized as follows. The scheduling problem description is described in section 2. Then optimization model is established in section 3. A novel solution strategy for

Corresponding Author: Yucheng WU, Department of Control Science and Engineering, Institute of Cyber-System and Control, State Key Laboratory of Industrial Control Technology, Zhejiang University, Hangzhou 310027, China MINLP model on the basis of piecewise linear relaxation is developed in section 4. We present a case study to demonstrate proposed approaches in section 5. Finally, conclusions of this paper are presented in section 6.

# PROBLEM DISCRIPTION

In petrochemical plants, raw materials are supplied from different sources with the batch mode and piped into selected tanks mixing with those stored previously. Each batch of raw material has expected amount, composition, arrival date and other properties to satisfy the products demand in the planning period. The mixture of raw materials is fed into production units from selected tanks and produce products according to demands with specified due dates. The products are stored in products tanks for final delivery with batch mode. A typical petrochemical production flowsheet is depicted illustratively in Fig. 1.

The following information of scheduling problem is given previously:

- Batch information of raw materials supplement: batch size, arrival date and material composition proportion
- Basic information of material tanks: capacity, initial amount of material and its composition proportion
- Basic information of units: production capacity
- Basic information of products tanks: capacity, initial amount of the product
- Batch information of products demands: batch size, specified due dates

The operations arranged by short-term scheduling in the time horizon list as follows:

- The reception arrangement of material tanks: reception time interval of each tanks, reception amount of each tanks
- The feeding arrangement of material tanks: feeding time of each tanks, feeding flowrate of each tanks
- The arrangement of production units: load, changeover of the load
- Once the proportion of the arrived raw materials composition deviates from the expectation, the specified due dates of products demands may not be satisfied
- Scheduling need to determine which products demands should be delayed with minimize losses by moving unrealizable due dates to the later time intervals

Furthermore, the following common policies should be obeyed in the petrochemical production:

- Raw material tanks cannot receive and feed simultaneously
- Only one tank can be selected as feed tanks at each time interval

# MATHEMATIC MODELS

Scheduling optimization problems can be modeled as discrete-time and continuous-time representation. Although the continuous-time formulations can reduce the size of model, modeling of discrete-time representation is still attractive because resource constraints are much easier to handle and formulations are tight in general under this approach (Pinto *et al.*, 2000; Floudas and Lin, 2004). The proposed model is established based on the discrete-time formulation and the scheduling time

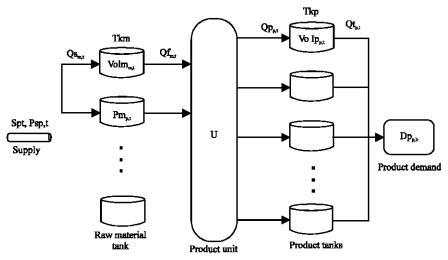


Fig. 1: Illustrative production flowsheet

horizon is divided into a number of time intervals with uniform duration. Events such as changing production load, shifting reception and feeding tanks and moving an unrealizable due date only start and end with the boundary of these time intervals.

**Supply constraints:** Constraint (1) denotes the amount of raw materials piped into the selected tank cannot exceed the batch size of the raw materials supply at each time interval:

$$Qs_{mt} \leq Ki_{mt} * Sp_{t} \forall m \in Tm, t \in T$$
(1)

where, constraint (2) expresses the mass balance that the amount of the supply at the time t is equal to the sum of the amount of raw materials received by all tanks:

$$Sp_{t} = \sum_{m \in Tm} Qs_{m,t}, \ \forall t \in T$$
(2)

**Raw material tanks constraints:** Constraints (3a)-(3b) express the mass balance of raw material tanks:

$$\begin{split} & \operatorname{Volm}_{m,t} = \operatorname{Vol}_{m}^{Int} + \operatorname{Qs}_{m,t} - \operatorname{Qf}_{m,t}, \\ & \forall m \in Tm \text{ , } t \in T \text{ , } t = 1 \end{split} \tag{3a}$$

$$\begin{aligned} & \text{Volm}_{m,t} = \text{Volm}_{m,t-1} + \text{Qs}_{m,t} - \text{Qf}_{m,t}, \\ & \forall m \in \text{Tm} \text{, } t \in \text{T} \text{, } t \neq 1, \end{aligned}$$

Components balance is expressed in constraints (4a)-(4b):

$$Pm_{m,p,t}(Vol_{m}^{Int} + Qs_{m,t}) = Pm_{m}^{Int} * Vol_{m}^{Int} + Ps_{p,t} * Qs_{m,t},$$

$$\forall m \in Tm, \ p \in P, t = 1.$$

$$(4a)$$

$$\begin{aligned} &Pm_{m,p,t}(Volm_{m,t-1} + Qs_{m,t}) \\ &= Pm_{m,p,t-1} *Volm_{m,t-1} + Ps_{p,t} *Qs_{m,t}, \\ &\forall m \in Tm, \ t \in T, \ p \in P, \ t \neq 1 \end{aligned}$$

Constraints (5)-(6) express that the capacity limitations and feeding flowrate limitations of material tanks:

$$\begin{split} & \operatorname{Volm}_{\scriptscriptstyle{\mathfrak{m}}}^{\operatorname{Min}} \leq \operatorname{Volm}_{\scriptscriptstyle{\mathfrak{m}},t} \leq \operatorname{Volm}_{\scriptscriptstyle{\mathfrak{m}}}^{\operatorname{Max}}, \\ & \forall m \in \operatorname{Tm} \,, \, t \in \mathrm{T}, \end{split}$$

$$\begin{aligned} 0 &\leq Q f_{m,t} \leq K o_{m,t} * Q f^{Max}, \\ &\forall m \in Tm, t \in T, \end{aligned} \tag{6}$$

Constraint (7) forces that at most one tank can be selected as a feeding tank at the one time interval. Constraint (8) imposes tanks cannot receive raw materials and feed to production units simultaneously:

$$\sum_{m} Ko_{m,t} \le 1, \forall m \in Tm, t \in T,$$
(7)

$$Ki_{m,t} + Ko_{m,t} \le 1, \forall m \in Tm, t \in T$$
(8)

Constraints (9a)-(9b) records the shift times of material tanks while feeding raw material to production units:

$$Ch_{m,t} \ge Ko_{m,t-1} - Ko_{m,t}, \ \forall m \in Tm \ , \ t \in T, t \neq 1$$

$$Ch_{m,t} \ge Ko_{m,t} - Ko_{m,t-1}, \forall m \in Tm, t \in T, t \neq 1$$
 (9b)

**Production unit constraints:** Constraint (10) expresses the mass balance that the total amount of raw materials fed to the production units is equal to the total amount of products produced at time interval t:

$$\sum_{m} Qf_{m,t} = \sum_{p} Qp_{p,t}, \ \forall m \in Tm, \ t \in T, \ p \in P \tag{10}$$

Constraint (11) expresses the amount of products produced at each time interval is equal to the total amount of the relative components of all materials fed to the units:

$$Qp_{p,t} = \sum_{m} Qf_{m,t} * Pm_{m,p,t}, \forall m \in Tm, t \in T, p \in P$$
(11)

Stability is crucial to petrochemical production. It demands minimizing changes of the production load. Changes of load can be represented as absolute terms of the flowrate difference between current time interval and the previous one:

$$\sum_{m \in Tm} Qf_{m,t} - \sum_{m \in Tm} Qf_{m,t-l} \left|, \forall t \in T, t \neq 1 \right.$$

Absolute terms can be transformed to linear representations by introducing instrumental variables Ld>0, LV>0 and the changes of the production load are expressed as following constraints (12a)-(12c):

$$Ld_t - Lv_t \geq \sum_{m \in Tm} Qf_{m,t}, t = 1$$
 (12a)

$$Ld_t - Lv_t \geq \sum_{m \in Tm} Qf_{m,t} - \sum_{m \in Tm} Qf_{m,t-1}, \ \forall t \in T, t \neq 1 \quad (12b)$$

$$Ld_{t} - Lv_{t} \geq \sum_{m \in Tm} Qf_{m,t-1} - \sum_{m \in Tm} Qf_{m,t}, \ \forall t \in T, t \neq 1, (12c)$$

**Production tanks constraints:** Constraints (13a)-(13b) express the mass balance of products tanks and constraint (14) is the capacity limitations of products tanks:

$$\begin{split} Volp_{p,t} &= Volp_p^{Int} + Qp_{p,t} - Qt_{p,t}, \\ \forall m \in Tm, \ p \in P, t = 1 \end{split}$$

$$\begin{aligned} & \text{Vol} p_{\text{p},t} = \text{Vol} p_{\text{p},t-1} + \text{Q} p_{\text{p},t} - \text{Q} t_{\text{p},t}, \\ & \forall m \in \text{Tm}, \ t \in \text{T}, \ p \in \text{P}, t \neq 1 \end{aligned}$$

$$\begin{aligned} & \operatorname{Volp}_p^{\operatorname{Min}} \leq \operatorname{Volp}_{p,t} \leq \operatorname{Volp}_p^{\operatorname{Max}}, \\ & \forall m \in \operatorname{Tm}, \ t \in \operatorname{T}, \ p \in \operatorname{P} \end{aligned}$$

**Border constraints with flexible due date of demands:** Constraint (15) expresses that the delivered amount of products should meet demands. It makes the due dates of products demands becoming flexible:

$$Qt_{p,t} = Dp_{p,b} *Kdp_{p,b,t}, \forall t \in T, \ p \in P, \ b \in Bch$$
(15)

Constraint (16) forces  $Kdp_{p,b,t}$  to be zero when the time interval t is earlier than the expected due dates of products demands. That means if demands cannot be met on time the specified due dates can only move backward from the expecting time interval:

$$\begin{aligned} Kdp_{p,b,t} &= 0, \\ \forall t \in T, \ p \in P, \ b \in Bch, TdpE_{p,b} \geq 2, \ t \leq TdpE_{p,b} - 1 \end{aligned} \tag{16}$$

Constraint (17) determines the real delivery date of the products demands:

$$Tdp_{p,b} = \sum_{t} t *Kdp_{p,b,t},$$
  
$$\forall t \in T, \ p \in P, \ b \in Bch$$
 (17)

Constraint (18) forces the due date of the earlier batch of products demands has priority to be satisfied:

$$Tdp_{p,b-1} \le Tdp_{p,b},$$

$$\forall t \in T, \ p \in P, \ b \in Bch, \ b \neq 1$$
(18)

Constraint (19) makes sure that the due date of the each batch of products demands must be satisfied at the scheduling time horizon:

$$\sum_{i \in T} Kdp_{p,b,t} = 1, \forall b \in Bch, \ p \in P$$
(19)

**Objective:** The objective of the proposed scheduling problem is minimizing costs. Given the production requirement, the optimal schedule is considered to be the one with the lowest costs, which is measured by deviations from specified due dates, changes of the production load and shifts of feeding tanks which is associated with production costs. Minimize:

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$$Cost = \alpha * \sum_{i \in T} (Ld_i + Lv_i) + \gamma * \sum_{i \in T} \sum_{m \in Tm} Ch_{m,i}$$
  
+ 
$$\sum_{p \in P} \beta_p * \sum_{b \in Ech} (Tdp_{p,b} - TdpE_{p,b})$$
(20)

where, constants  $\alpha$  and  $\gamma$  are weight factors of production and operation costs. Terms  $Tdp_{p,b}$ - $TdpE_{pb}$  represents deviations of the real delivery date of products from specified due dates. Constants  $\beta_p$  are weight factors for penalty of the delay of due dates of products demands.

### SOLUTION STRATEGE

The MINLP optimization model of the short-term production scheduling is established and sources of nonlinearity in the model are binary terms  $Pm_{m,p,t-1}$ , \*Volm<sub>m,t-1</sub> appeared in constraint (4b) and Qf<sub>m,t</sub> \*Pm<sub>m,p,t</sub> appeared in constraint (11). McCormick has developed an efficient relaxation technique of the bilinear term x\*y (McCormick, 1976). Gounaris *et al.* (2009) has summarized piecewise relaxation schemes with mixed-integer representations and conducted computational comparison. One of the schemes is adopted to solve the model established in the previous section.

Considering the proposed model, bilinear terms  $Qf_{,mt}$ \* $Pm_{mpt}$  and  $Pm_{mpt-1}$  \* $Volm_{mt-1}$  cannot be relaxed by directly adopting PLR approach because the bounds of  $Pm_{m,p,t}$  are not determined. It is a distinguishing feature of the considered problem comparing with the traditional pooling problem. The bounds of  $Pm_{m,p,t}$  are determined by mixing newly received raw material and stored materials and a novel algorithm is developed to determine hard bounds of variables in next subsection.

Hard bounds of variables in binary terms: From constraints (4a) and (4b), it is observed that  $Pm_{m,p,t}$  is monotonically increasing with  $Qs_{m,t}$  increasing where  $Pm_{m,p,t-1}>Ps_{p,t}$  and decreasing with  $Qs_{m,t}$  increasing where  $Pm_{m,p,t-1}>Ps_{p,t}$  at the each time interval. It's easy deduced that when upper bound  $Pm_{m,p,t}^{Max}$  and lower bound  $Pm_{m,p,t}^{Max}$  are reached, equations  $Qs_{m,t} = Sp_t$  must be established. Let:

$$f(x, y) = \frac{y * x + C * a}{x + a}$$

where, any given variables (x,y) are satisfied following inequalities (UA) and (LA).

If:

If:

$$0 < X^{1} < x < X^{2}, a > 0$$
:  
 $f(x,y) < Min\{(X^{1}, Y^{1}), f(X^{2}, Y^{2}); (LA)\}$ 

**Determination of upper bounds Pm^{Min}\_{m,p,t}:** According to constraint 4(a) and the above deduction, it's easy to determine  $Pm^{Man}_{m,p,t}$  at the first time interval:

$$Pm_{m,p,t}^{Max} = \begin{cases} \frac{Pm_{m,p}^{Init} * Volm_{m}^{Init} + Sp_{t} * Ps_{p,t}}{Volm_{m}^{Init} + Sp_{t}}, & Pm_{m,p}^{Init} \le Ps_{p,t} \end{cases}$$
$$\forall m \in Tm, p \in P, t = 1, (U1)$$

where, if  $Pm_{m,p,t-1}^{Min} < Ps_{p,t}$  and  $t \neq 1$ , Terms  $Pm_{m,p,t}$  satisfy the following inequality. The terms  $Volm_{m,t-1}^{Pm_{m,p,t-1}}$  in the inequality denote the minimum volume of the selected tank at time (t-1) with composition proportion  $Pm_{m,p,t-1}$ .

$$\begin{split} & \mathbf{Pm}_{m,p,t} = \frac{\mathbf{Pm}_{m,p,t-1} * \operatorname{Volm}_{m,t-1} + \operatorname{Ps}_{p,t} * \operatorname{Qs}_{m,t}}{\operatorname{Volm}_{m,t-1} + \operatorname{QS}_{m,t}}, \\ & \leq \frac{\mathbf{Pm}_{m,p,t-1} * \operatorname{Volm}_{m,t-1}^{\operatorname{Pm}_{m,p,t-1}} + \operatorname{Ps}_{p,t} * \operatorname{Sp}_{t}}{\operatorname{Volm}_{m,t-1} + \operatorname{Sp}_{t}} = f(\operatorname{Volm}_{m,t-1}^{\operatorname{Pm}_{m,p,t-1}}, \operatorname{Pm}_{m,p,t-1}) \\ & \forall m \in \operatorname{Tm}, p \in P, \ t \in T, \ t \neq 1 \end{split}$$

According to the (UA), the upper bounds of composition proportion are determined by equation (U2):

$$\begin{split} Pm_{m,p,t}^{Max} = & \begin{cases} Max \left\{ f(Volm_{m}^{lnit}, Pm_{m,p}^{lnit}), ..., \ f(Volm_{m,t-1}^{Pm_{m,p-1}^{Max}}, Pm_{m,p,t-1}^{Max}) \right\}, \quad Pm_{m,p,t-1}^{Max} \leq Ps_{p,t} \\ \\ Pm_{m,p,t-1}^{Max}, \quad Otherwise \\ \forall m \in Tm, p \in P, t \in T, t \geq 2, (U2) \end{cases} \end{split}$$

**Determination of Lower bounds Pm^{Min}\_{m,p,t}:** The algorithm for lower bounds is similar with the one for upper bound, according to constraint (4a),  $Pm^{Min}_{m,p,t}$  of the first time interval is determined by equation (L1):

$$Pm_{m,p,t}^{Min} = \begin{cases} \frac{Pm_{m,p}^{Init} * Volm_{m}^{Init} + Sp_{t} * Ps_{p,t}}{Volm_{m}^{Init} + Sp_{t}}, & Pm_{m,p}^{Init} \ge Ps_{p,t} \\\\ \\Pm_{m,p}^{Init}, & Otherwise \end{cases}$$
  
$$\forall m \in Tm, p \in P, t = 1, (L1)$$

where, if  $\mathtt{Pm}_{m,p,t-1}^{\mathtt{Mm}} \ge \mathtt{Ps}_{p,t}$  and  $t \in T, t \neq 1$ , the terms  $\mathtt{Pm}_{m,p,t}$  satisfy the following inequality. The terms  $\mathtt{Volm}_{m,t-1}^{\mathtt{Pm}}$  in the inequality denote the minimum volume of the selected tank at time (t-1) with composition proportion  $\mathtt{Pm}_{m,p,t-1}$ :

$$\begin{split} & Pm_{m,p,t} = \frac{Pm_{m,p,t-l} \times Volm_{m,t-l} + Ps_{p,t} \times Qs_{m,t}}{Volm_{m,t-l} + QS_{m,t}} \succeq \\ & \frac{Pm_{m,p,t-l} \times Volm_{m,t-l}^{Pm_{m,p,t-l}} + Ps_{p,t} \times Sp_{t}}{Volm_{m,t-l} + Sp_{t}} = f(Volm_{m,t-l}^{Pm_{m,p,t-l}}, Pm_{m,p,t-l}) \\ & \forall m \in Tm, p \in P, t \in T, t \neq 1 \end{split}$$

According to the inequality (LA), the Lower bounds of  $Pm_{m,p,t}$  are determined by equation (L2):

**Determination of Bounds Volm**<sub>m,t-1</sub>: In the binary terms  $Pm_{m,p,t-1} * Volm_{m,t-1}$ , bounds of  $Volm_{m,t-1}$  are:

$$\begin{split} & \operatorname{Volm}_{m,t}^{U} = \begin{cases} \operatorname{Volm}_{m}^{\operatorname{Init}} + \sum_{t} \operatorname{Sp}_{t}, \quad \operatorname{Volm}_{m}^{\operatorname{Init}} + \sum_{t} \operatorname{Sp}_{t} \leq \operatorname{Volm}_{m}^{\operatorname{Max}} \\ & \operatorname{Volm}_{m}^{\operatorname{Max}}, \quad \operatorname{Otherwise} \end{cases} \\ & \forall m \in Tm, t \in T \quad (V1) \\ & \operatorname{Volm}_{m,t}^{L} = \begin{cases} \operatorname{Volm}_{m}^{\operatorname{Init}} - t * \operatorname{Qf}^{\operatorname{Max}}, \quad \operatorname{Volm}_{m}^{\operatorname{Init}} - t * \operatorname{Qf}^{\operatorname{Max}} \geq \operatorname{Volm}_{m}^{\operatorname{Max}} \\ & \operatorname{Volm}_{m}^{\operatorname{Max}}, \quad \operatorname{Otherwise} \end{cases} \end{split}$$

 $\forall m \in Tm, t \in T \qquad (V2)$ 

**Piecewise linear relaxing:** The bounds of variables  $Pm_{mt1}$  and  $Volm_{mt}$  are determined by equations (U1)-(U2), (L1)-(L2) and (V1)-(V2), then the bilinear terms can be relaxed by adopting piecewise linear relaxing approach, which make the original MINLP model transformed into the MILP model.

Constraint (4b) can be transformed into aggregation form as following:

$$\begin{split} & \operatorname{Pm}_{m,p,t} * \operatorname{Volm}_{m,t} = \operatorname{Volm}_{m}^{\operatorname{Int}} * \operatorname{Pm}_{m}^{\operatorname{Int}} \\ & + \sum_{t \in T} \operatorname{Qs}_{m,t} * \operatorname{Ps}_{p,t} - \sum_{t \in T} \operatorname{Pm}_{m,p,t} * \operatorname{Qf}_{m,t} \\ & \forall m \in \mathsf{Tm}, p \in \mathsf{P} \end{split}$$
(4c)

Replace all terms of  $Pm_{m,p,t} \times Volm_{m,t}$  with  $Z^{vol}_{m,p,t}$  and  $Pm_{m,p,t} \times Qf_{m,t}$  with  $Z^{Qf}_{m,p,t}$  of constraint (4c):

$$\begin{split} & \boldsymbol{z}_{m,p,t}^{vol} = \boldsymbol{V} o \boldsymbol{I} \boldsymbol{m}_{m}^{hit} \times \boldsymbol{P} \boldsymbol{m}_{m}^{hit} \\ & + \sum_{t} \boldsymbol{Q} \boldsymbol{s}_{m,t} \times \boldsymbol{P} \boldsymbol{s}_{p,t} - \sum_{t} \boldsymbol{z}_{m,p,t}^{Qt} \left( 4d \right) \\ & \forall \boldsymbol{m} \in \boldsymbol{T} \boldsymbol{m}, t \in \boldsymbol{T}, p \in \boldsymbol{P} \end{split}$$

Constraint (11) can be transformed linear forms as follows:

$$Qp_{p,t} = \sum_{m} z_{m,p,t}^{Q}, \forall m \in Tm, t \in T, p \in P, \qquad (11a)$$

where, there are several formulations of piecewise linear relaxing and they are mathematically equivalent. We adopt "Ch" formulation from (Gounaris *et al.*, 2009) to relaxing the MINLP model and study the following case.

## CASE STUDIES

In this section, a real case is presented to demonstrate the approaches discussed in the paper. A flowchart of petrochemical plant is illustrated in Fig. 2. The plant receives PYGAS as raw material from several refineries to produce toluene and xylene. There are four material tanks, one extraction unit and ten products tanks in the plant. The scheduling horizon is a week with two work shifts a day. Schedules-making mainly depends on experiences and simple spreadsheet calculation. The composition proportions of raw materials are fluctuant with different supply batches, which makes scheduling difficult while considering the mixing of the raw materials.

**Information table:** Information of the plant is presented in Table 1 to 5. And we aggregate products tanks into 4 logical ones according to the categories of the products.

## **RESULT AND ANALYSIS**

The results of scheduling optimization of the presented case are shown in Fig. 3 to 7.

The bounds of  $Pm_{m,p,t}$   $\forall t, m = p = 1$  is shown in Fig. 3,  $Pm_{m,p,t}^{Max}$  is non-decreasing and gradually approach the maximum value of  $Ps_{p,t}$   $\forall t, p = 1$ . Meanwhile  $Pm_{m,p,t}^{Mix}$  remains its initial value in whole horizon because of its value smaller than any of  $Ps_{p,t}$   $\forall t, p = 1$ . Domain determination algorithm is the prerequisite of PLR approach for scheduling optimization of integrated progress.

Reception schedule is shown in Fig. 4. Each of material tanks has been arranged for receiving some of raw material at specified time interval. And a batch of raw materials supply can be piped into different tanks at the same time.

Feeding schedule is shown in Fig. 5. It contains much of production information. The schedule decides the specified feed time of material tanks and production load. According to the schedule, the production process will be stable and smooth in the scheduling horizon.

Figure 6 expresses the arrangement of receiving and feeding tasks of each material tank. The selected tank cannot feed and receive materials at same time. Meanwhile, only one tank could be used as feeding tank at each time interval.

The flexible due date of products demands "loosen" the rigid constraints of the scheduling model. It gives the schedulers a chance to find the best solution under the circumstance of uncertainty occurrence.

Table 6 has showed the first arrived batch of raw materials has different composition proportion from the planned. This situation is always happened in real production scenarios. We adopted the proposed strategy and conducted rescheduling. The results are shown in Fig. 7.

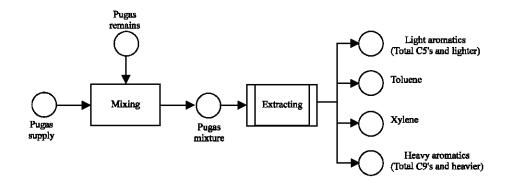
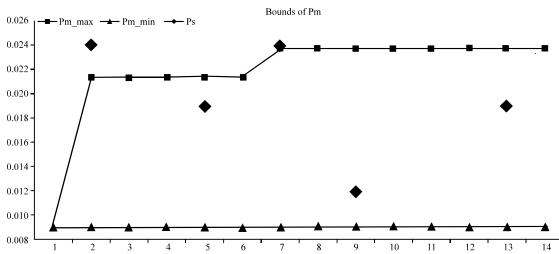


Fig. 2: Aromatics extraction flowchart



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Fig. 3: Illustrative bounds  $Pm_{m,p,t}$   $\forall t, m = p = 1$ 

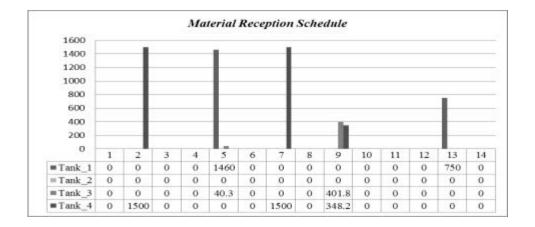


Fig. 4: Reception schedule

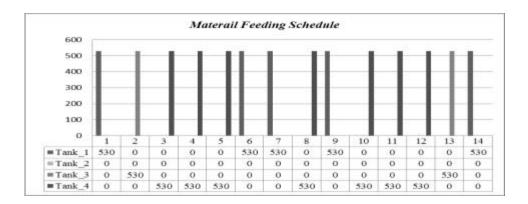
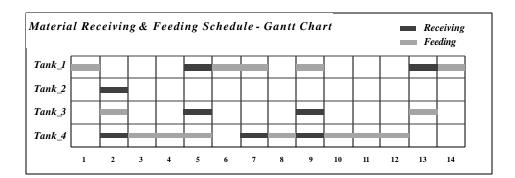


Fig. 5: Feeding schedule

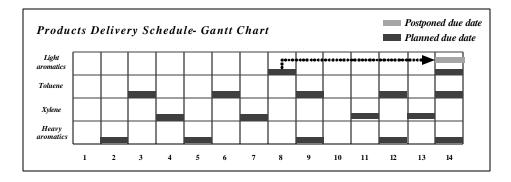
The Fig. 7 shows that the original delivery plan just need a slightly adjustment by moving due

time of light aromatics from the 8th time interval to the 14th time interval. The flexible due time



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Fig. 6: Receiving and Feeding schedule-Gantt chart



## Fig. 7: Production delivery schedules

Material tanks	Maximum Capacity	Minimum capacity	Initial volume	
Tank 1	4500	200	926	
Tank 2	4500	200	473	
Tank_3	4000	200	818	
Tank 4	4000	200	562	

Table 2: Products tanks information						
Product	Maximum	Minimum	Initial			
tanks	capacity	capacity	volume			
Tkp_1	500	30	41			
Tkp_2	2500	200	724			
Tkp_3	2500	200	548			
Tkp 4	800	50	233			

Table 3: Initial PYGAS composition proportion of raw material tanks

Material	Light			Heavy	
tanks	aromatics	Toluene	Xylene	aromatics	Others
Tank_1	0.009	0.5	0.412	0.076	0.003
Tank_2	0.008	0.565	0.345	0.077	0.005
Tank_3	0.012	0.52	0.369	0.097	0.002
Tank 4	0.016	0.485	0.404	0.088	0.007

#### Table 4: PYGAS supplies information

	Arrival		Light			Heavy
Batch	date	Amount	aromatics	Toluene	Xy lene	aromatics
Sp_1	2	1500	0.024	0.464	0.36	0.15
Sp_2	5	1500	0.019	0.51	0.384	0.085
Sp_3	7	1500	0.024	0.464	0.36	0.15
Sp_4	9	750	0.012	0.52	0.376	0.09
Sp 5	13	750	0.019	0.51	0.384	0.085

Table 5 Products demands with specified due date

Due date	Light aromatics	Toluene	Xylene	Heavy aromatics
1				
2				150
3		500		
4			400	
5				150
6		500		
7			400	
8	50			
9		500		150
10				
11			400	
12		500		150
13			400	
14	100	1000		150

Table 6: PYGAS supplies Deviation							
Arrival		Light			Heavy		
Batch	date	Amount	aromatics	Toluene	Xylene	aromatics	
Planed:Sp_1	2	1500	0.024	0.464	0.360	0.15	
Real: Sp 1	2	1500	0.012	0.520	0.376	0.09	

strategy will help scheduler a lot to deal with uncertainty in the real production scenarios.

## CONCLUSIONS

This study presents a whole set of research methodologies from problem definition, modeling and solution strategies for scheduling optimization of integrated petrochemical production process. A complex MINLP model has been established. And an efficient algorithm is developed to determine the bounds of binary variables, which make the model relaxed and converted to a MILP problem by adopting PLR approach. The effect of presented approaches and flexible due dates strategy is demonstrated in the study case.

# ACKNOWLEDGMENTS

The authors would like to thank for the support by the National High Technology Research and Development Program (863 Program) of China (No.2009AA043204, No.2012AA040307).

# NOMENCLATURE

## Sets and indices:

- T: Time horizon
- P: Products/Component
- Tm: Material tanks
- N: Discretization Space
- Bch: Batch of products demand

## **Parameters**:

- Spt: Batch size of raw material supply
- Ps<sub>pt</sub> Composition proportion of the material supply
- Qs<sub>mt</sub>: Reception flowrate of material tanks
- Volm<sub>m+</sub>: Volume of material tanks
- Volm<sup>Init</sup><sub>m</sub>: Initial volume of material tanks
- Volm<sup>Max</sup><sub>m</sub>, Volm<sup>Min</sup><sub>m</sub>: Volume capacity of material tanks
- Pm<sub>m,p,t</sub>: Products proportion in the material tanks
- Pm<sup>Init</sup><sub>mp</sub>: Initial composition proportion in material tanks
- Qf<sub>m,t</sub>: Feeding flowrate of material tanks
- Qf<sup>Max</sup>: Maximum flowrate from material tanks
- Qp<sub>pt</sub>: Products flowrate to production tanks
- Volp<sub>p,i</sub>: Volume of Production tanks
- Volp<sup>Init</sup><sub>p</sub>: Initial volume of Production tanks
- $\operatorname{Volp}_{p}^{\operatorname{Max}}, \operatorname{Volp}_{p}^{\operatorname{Min}}$ : Capacity of production tanks
- Dp<sub>p,b</sub>: Batch size of products demands
- Tdp<sub>Ep,b</sub>: Specified due date of demands
- Ld, Lv, Flowrate fluctuation variables

## **Binary variables:**

- Ki<sub>mt</sub> Activate material from source to tank m
- Ko<sub>mt</sub>: Activate material from tank m to Production unit

- Kdp<sub>nht</sub>: Activate if demand is met at t
- Ch<sub>m,t</sub>: Number of times of material tanks shift

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