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Four Degree of Freedom Manipulator's Kinematics Simulation Analysis Based on MATLAB

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Abstract: The study proposes the process and methods of real-time analyzing the relationship between each joint angle and the terminal position of the mechanical arm. Firstly, each section's club length data of manipulator with degree of freedom is used as the basis, with a 3D modeling tool being used to model and lead the model into the MATLAB environment. Then, a movement solution analysis and inverse solution analysis of the manipulator will be conducted and be used to calculate and solve the problem. On this basis, Robotics Toolbox under the MATLAB software environment is used to conduct simulated analysis to the robot's real-time movement condition and the kinetic characteristic of displacement, velocity, acceleration and the end of the manipulator are obtained. This method can intuitively analyze the motor process of the manipulator and it provides great support to the movement and operation control of robots.

Key words: Manipulator, degree of freedom, MATLAB, motion analysis, computer simulation

INTRODUCTION

With the acceleration of the robot research and the marketalization process, the development tendency of the robot is to more and more serve the social dem and s of people such as some particular task in daily life. Therefore, how to make the robot interact with people is one of the key and difficult points of robot research (Garcia *et al*., 2007). Early in 1996, MATLAB has realized the calculation and analysis capabilities of the manipulator and parameters such as displacement, velocity and acceleration of each joint could be gained (Miller and Allen, 2004). Kuliæ and Croft made use of the MATLAB simulation tool to conduct a real-time analysis to the movement safety during the interaction process of human and robot (Kulic and Croft, 2005). Corke proposed a simulation analysis method which is based on the combination of the robot's vision and the MATLAB tool box (Corke, 2005) and applied this more intuitive way in robot teaching (Corke, 2007). Lin and Meng proposed to conduct simulation analysis to the motion constraints of the manipulator under the MATLAB virtual environment and that the interoperability between human beings and virtual manipulator will be well realized (Qi and Meng, 2009). In order to control the convenience of the manipulator, Dean-Leon *et al.* (2012) developed the user graphical interface of the robot on the basis of MATLAB tool box (Dean-Leon *et al*., 2012). This study came up with a method which is used to conduct real-time analyzing of four degree of freedom manipulator and it can better realize the man-machine interaction between the operator and the virtual manipulator.

KINEMATICS ANALYSIS OF POSITIVE SOLUTIONS

This manipulator arm is a prosthetic robot with degree of freedom and each dimension is revolute pair which belongs to 4R manipulator arm, as shown by Fig. 1.

Fig. 1: Four degree of freedom manipulator arm

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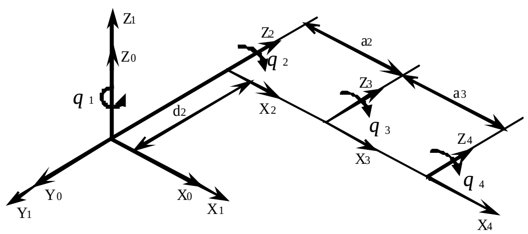

Fig. 2: Connecting rod coordinate system

Table 1: D-H parameters of four degree of freedom manipulator arm

Linkage	ai-1	αi-1	di-1	θi	Anglerange
1	0	0	0	θ1	-160~160
2	0	-90	d2	θ2	-225~45
3	a2	0	0	θ3	-150~150
4	a3	0	0	θ4	-150~150

Fig. 2 for each connecting rod coordinate system and the corresponding link parameters is listed in Table 1.

The essence of building up the manipulator kinematics equation according to D-H's method is to use the second transformation matrix to represent the relative position and movement between two connecting rod coordinate systems. The transformation matrix of manipulator will be gained when these matrix are multiplied, in turn, which means the second transformation matrix of the tool coordinate system relative to the base coordinate system is gained.

Firstly, we can calculate each connecting rod transformation matrix:

$$ {}_1^0T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

Similarly, ${}_2^1T$, ${}_3^2T$ and ${}_4^3T$ can be denoted. Multiply the above connecting rod transformation matrix, in turn and we can get that the transformational matrix ${}_4^0T$, of degree of freedom manipulator arm is:

$$ {}_4^0T = {}_1^0T(\theta_1)\cdot{}_2^1T\cdot(\theta_2)\cdot{}_3^2T(\theta_3)\cdot{}_4^3T(\theta_4) \tag{2} $$

It is the function of the four joint variablesθ_1 (i = 1, 2, 3, 4). For the sake of kinematics reverse solution, we need to calculate the intermediate result:

$$ {}_4^2T = {}_3^2T \bullet {}_4^3T = \begin{bmatrix} c_{34} & -s_{34} & 0 & a_3c_3 + a_2 \\ s_{34} & c_{34} & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

Abbreviation is quoted in the equation, such as $c_3 = c\theta_3 = \cos\theta_3$, $s_3 = s\theta_3 = \sin\theta_3$ and so on. Then:

$$ = \begin{bmatrix} c_{2(34)} & -s_{2(34)} & 0 & a_3c_2c_3 + a_2c_2 - a_3s_2s_3 \\ 0 & 0 & 1 & d_2 \\ -s_{2(34)} & -c_{2(34)} & 0 & -a_3s_2c_3 - a_2s_2 - a_3c_2s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

Of which, $c_{2(34)} = c_2c_{34}-s_2s_{34}$, $s_{2(34)} = s_2c_{34}+c_2s_{34}$ and then:

$$ {}_4^0T = {}_1^0T \bullet {}_4^1T == \begin{bmatrix} c_1c_{2(34)} & -c_1s_{2(34)} & -s_1 & a_3c_1c_2c_3 + a_2c_1c_2 - a_3s_1s_2s_3 - d_2s_1 \\ s_1c_{2(34)} & -s_1s_{2(34)} & c_1 & a_3s_1c_2c_3 + a_2s_1c_2 - a_3s_1s_2s_3 + d_2c_1 \\ -s_{2(34)} & -c_{2(34)} & 0 & -a_3s_2c_3 - a_2s_2 - a_3c_2s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

Equation 5 shows the transformation matrix of degree of freedom manipulator arm. It describes the position and orientation of the end of the connecting rod coordinate system relative to the base coordinate system of the robot and it is the basis of four degree of freedom manipulator arm's motion analysis.

In order to check the accuracy of the results, we made $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 0$ and then the arm matrix ${}_4^0T$ is:

$$ {}_4^0T = \begin{bmatrix} 1 & 0 & 0 & a_2 + a_3 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

ANALYSIS OF THE KINEMATICS INVERSE SOLUTION

Kinematics reverse solution of degree of freedom mechanical arms has a variety of methods and next we will use the inverse transform method. The kinematical equation of four degree of freedom mechanical arms can be written as:

$$ \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}_1^0T{}_2^1T{}_3^2T{}_4^3T \tag{7} $$

The reverse transformation ${}_1^0T^{-1}$ is used to premultiplication the matrix equation:

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1_4T \tag{8}$$

Make the both sides (2, 4) elements equal in the above formula, we obtain that

$$-s_1p_x+c_1p_y = d_2 \tag{9}$$

Use the triangle substitution, make:

$$p_y = \rho\sin\Phi \tag{10}$$

In the equation:

$$\rho = \sqrt{p_x^2 + p_y^2},\ \Phi = A\tan 2(p_x, p_y)$$

Sustituted Eq. 10 into 9 and we get:

$$-\rho\sin\theta_1\cos\Phi + \rho\cos\theta_1\sin\Phi = d_2,\ \rho\sin(\Phi - \theta_1) = d_2$$

Therefore:

$$\sin(\Phi - \theta_1) = \frac{d_2}{\rho},\ \cos(\Phi - \theta_1) = \pm\sqrt{1 - \frac{d_2^2}{\rho^2}}$$

$$\Phi - \theta_1 = A\tan 2(\frac{d_2}{\rho}, \pm\sqrt{1 - \frac{d_2^2}{\rho^2}})$$

$$\theta_1 = A\tan 2(P_y, P_x) - A\tan 2(\frac{d_2}{\rho}, \pm\sqrt{1 - \frac{d_2^2}{\rho^2}})$$

We can work out θ_1:

$$= A\tan 2(P_y, P_x) - A\tan 2(d_2, \pm\sqrt{p_x^2 + p_y^2 - d_2^2}) \tag{11}$$

In the equation, plus and minus sign is corresponding to the two possible solutions of θ_1.

When one of them is selected, make the elements (1, 4) and (3, 4) on both sides of the matrix equation equal and we can get the second equation:

$$\begin{aligned} c_1p_x + s_1p_y &= a_3c_2c_3 + a_2c_2 - a_3s_2s_3 - p_z - p_z \\ &= a_3s_2c_3 + a_2s_2 + a_3c_2s_3 \end{aligned} \tag{12}$$

With the quadratic sum of Eq. 12 and 14, we can get:

$$p_x^2 + p_y^2 + p_z^2 = d_2^2 + a_2^2 + a_3^2 + 2a_2a_3c_3$$

$$s_1^2p_x^2 - 2s_1c_1p_xp_y + c_1^2p_y^2 = d_2^2$$

$$\begin{aligned} &c_1^2p_x^2 + 2s_1c_1p_xp_y + s_1^2p_y^2 \\ &= 2c_3^2c_2^2a_3^2 + 2a_2a_3c_2^2c_3 - 2a_3^2c_2c_3s_2s_3 + a_2^2c_2^2 - 2a_2a_3c_2s_2s_3 \end{aligned}$$

$$p_z^2 = 2c_3^2s_2^2a_3^2 + 2a_2a_3s_2^2c_3 + 2a_3^2c_2c_3s_2s_3 + a_2^2s_2^2 + 2a_2a_3c_2s_2s_3$$

And:

$$\theta_3 = \arccos\left(\frac{p_x^2 + p_y^2 + p_z^2 - d_2^2 - a_2^2 - a_3^2}{2a_2a_3}\right) \tag{13}$$

Of which, θ_3 has two possible solutions in the range of [-150^0, 150^0]

Now we solve θ_2, so we write matrix Eq. 6 as ${}^0_3T^{-1}\,{}^0_6T = {}^3_4T$ and:

$${}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T == \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1s_2c_3 & -s_1 & a_2c_1c_2 - d_2s_1 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1s_2c_3 & c_1 & a_2s_1c_2 + d_2c_1 \\ -s_2c_3 - c_2s_3 & s_2s_3 - c_2c_3 & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

Because:

$${}^A_BT^{-1} = \begin{bmatrix} {}^A_BR^T & -{}^A_BR^T\,{}^AP_{BO} \\ 0 & 1 \end{bmatrix}$$

And:

$${}^0_3T^{-1} = \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & s_1c_2c_3 - s_1s_2s_3 & -s_2c_3 - c_2s_3 & -a_2c_3 \\ -c_1c_2s_3 - c_1s_2c_3 & -s_1c_2s_3 - s_1s_2c_3 & s_2s_3 - c_2c_3 & a_2s_3 \\ -s_1 & c_1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So:

$$\begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & s_1c_2c_3 - s_1s_2s_3 & -s_2c_3 - c_2s_3 & -a_2c_3 \\ -c_1c_2s_3 - c_1s_2c_3 & -s_1c_2s_3 - s_1s_2c_3 & s_2s_3 - c_2c_3 & a_2s_3 \\ -s_1 & c_1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^3_4T \tag{15}$$

We make the Eq. 1, 4 and 2, 4 elements equal, respectively, in the matrix on both sides in the above formula, we can get:

$$c_1c_{23}p_x+s_1c_{23}p_y-s_{23}p_z = -a_2c_3 = a_3$$

$$-c_1s_{23}p_x-s_1s_{23}p_y-c_{23}p_z + a_2s_3 = 0 \tag{16}$$

where, s_{23} and c_{23} will be gained when the above two formulas are simultaneous:

$$s_{23}=\frac{a_2s_3\left(c_1p_x+s_1p_y\right)-\left(a_3+a_2c_3\right)p_z}{p_z^2+\left(c_1p_x+s_1p_y\right)^2}$$

$$c_{23}=\frac{\left(a_3+a_2c_3\right)\left(c_1p_x+s_1p_y\right)+a_2s_3p_z}{p_z^2+\left(c_1p_x+s_1p_y\right)^2} \tag{17}$$

Therefore, the denominator of the expression of s_{23} and c_{23} are equal and are positive, so:

$$\begin{aligned}\theta_{23}=\theta_2+\theta_3=\text{A}\tan 2[a_2s_3\left(c_1p_x+s_1p_y\right)- \\ \left(a_3+a_2c_3\right)p_z,\left(a_3+a_2c_3\right)\left(c_1p_x+s_1p_y\right)+a_2s_3p_z]\end{aligned} \tag{18}$$

According to the four possible combinations of θ_1 and θ_3, we can get the four numerical values of θ_{23}, so we can get the four possible solutions of θ_2:

$$\theta_1 = \theta_{23}-\theta_3 \tag{19}$$

In the formula, take the θ_{23} value of the corresponding to θ_3.

Because the left side of matrix Eq. 15 is known, so make the (1, 2) and (2, 2) elements of both sides of the equation equal, we can get:

$$c_1c_{23}o_x+s_1c_{23}o_y-s_{23}o_z = s_4$$

$$-c_1s_{23}o_x-s_1s_{23}o_y-c_{23}o_z = c_4 \tag{20}$$

and then get:

$$\theta_4=\text{A}\tan 2\begin{pmatrix}c_1c_{23}o_x+s_1c_{23}o_y-s_{23}o_z, \\ c_1s_{23}o_x+s_1s_{23}o_y+c_{23}o_z\end{pmatrix} \tag{21}$$

Because there appears plus and minus sign in the process of solving θ_1 and θ_3, there may be four solutions in the kinematics reverse solution of this degree of freedom manipulator arm.

What should be pointed out is that some of the possible solutions gotten from the above calculation can even not be totally realized because of the structure limitation because each joint of the robot cannot move in a 360° range. Each joint's data range is listed in the table. If there is multiple solutions, the nearest set of solution will be chosen, which means the solution with the shortest trip or solution according to other requirements will be chosen in the trajectory planning.

SIMULATION ENVIRONMENT MODELING OF THE MANIPULATOR

Under the MATLAB environment, the Robotics Toolbox is used to conduct the kinematics simulation. In the process of simulation, not only can the robot's movement be visually observed but we can also get the needed data and display in the form of graphics:

- Assume that we are within the robot reachable place, we can move the h and grasp from the initial position to the target location through control system
- Arrange that the manipulator arm can accomplish a movement from the initial position to the target location in an average time of 2 sec
- Original state of the manipulator arm stretches horizontally and the position is (2, 0, 0). Assuming that the target location is (1, 1, 1.4) and the state is the same with the original state, we can get the stop value of each joint according to the inverse solution formula in chapter 2. The following are the stop value through calculation, so:

$$[\theta_1, \theta_2, \theta_3, \theta_4] = [\text{pi}/4\text{-pi}/400]$$

According to the manipulator model and the given structure size, as well as the requirement of MATLAB simulation environment, we can get the simulation parameter.

SIMULATION ANALYSIS OF THE MANIPULATOR

We establish the simulation model of the manipulator arm. The original position is the extended state when the manipulator is horizontal. The three-dimensional Fig. 3 of this manipulator arm will be gained when this procedure is operated in MATLAB. We drive the slider in the graph to make the manipulator move using the method of manual and it will be just like actual manipulation of the manipulator. We can get the actual effect of three-dimensional figure through controlling each of the joint's angle by the slide in it, Fig. 3.

Based on the above analysis, we conduct the relationship of direction and time along X, Y, Z at the end of the manipulator arm directed at first solution track. In Fig. 4, it is the movement characteristics of displacement with time of rotational joints 1, 2, 3, 4. Among it, No. 1 and No. 4 joints' tracks coincides with each other and No. 2 and No. 3 joints' tracks coincides with each other. Figure 5 is a speed characteristic curve of each joint of the manipulator arm. Figure 6 is an acceleration characteristic

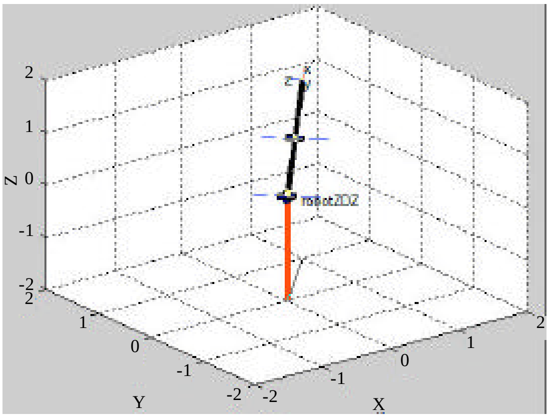

Fig. 3: Manipulator arm's three-dimensional figure and the slider control

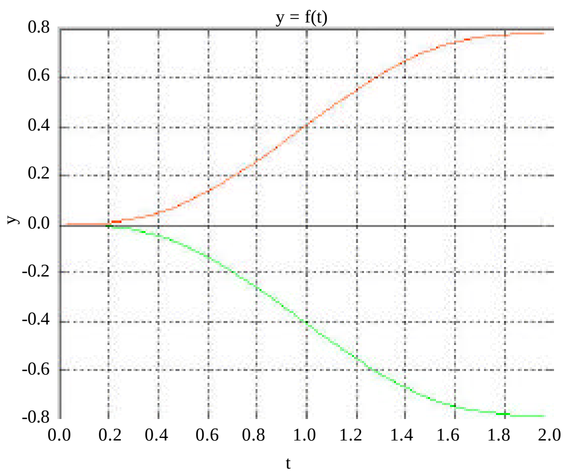

Fig. 4: Movement characteristics of displacement of joints

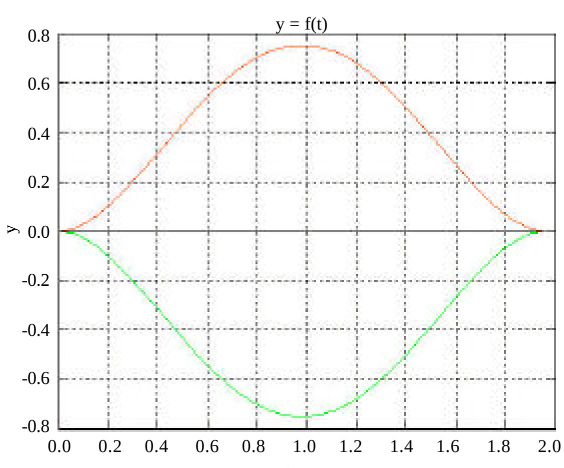

Fig. 5: Speed characteristic curve of joints

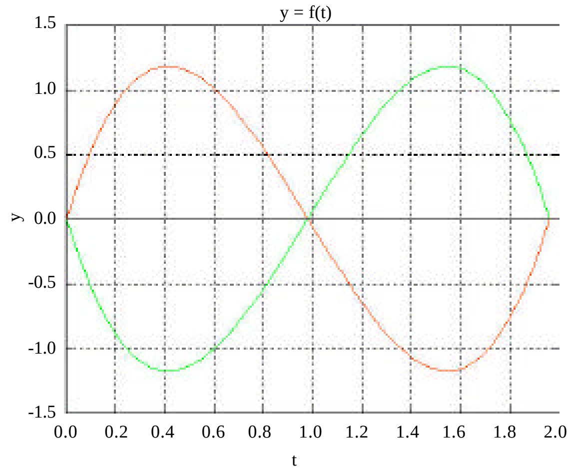

Fig. 6: Acceleration characteristic curve of the joints

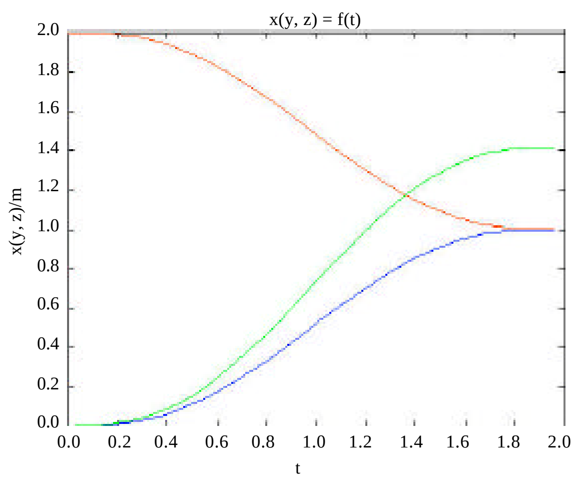

Fig. 7: Motion curve of the manipulator's end

curve of the manipulator arm's end. and Fig. 7 is the end motion curve of the manipulator arm.

CONCLUSION

Through the above simulation, when the manipulator arm move from the original position to the target location, we can observe whether the process and movement of each joint is normal or not, whether there is a dislocation movement phenomenon in each connecting rod. Therefore, the mathematical model and all link parameters' validity are tested and whether the design of each parameter achieves the intended target or not is explained, too. We can see from Fig. 3 that as for a given target location, the manipulator can reach through different paths. Figure and 6 explained the change of each joint wth me in the above process and Fig. 7 further explained that he end of the h and grasp position changes over time in the base coordinate system. However, the real movement process of the manipulator arm will be faced with a lot of limitations, so there really is a practical significance of studying the trajectory planning of manipulator and we subject to further research in the future.

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