http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (3): 506-509, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.506.509

On Covering Multi-granulation Fuzzy Rough Sets

[1,2]Caihui Liu and [1]Duoqian Miao
[1]Department of Computer Science and Technology, Tongji University, Shanghai, 201804, China
[2]Department of Mathematics and Computer Science, Gannan Normal University, Ganzhou, 341000, China

Abstract: Multi-Granulation Rough Set (MGRS) is a new and interesting topic in rough set theory. In this study, three kinds of Covering Multi-Granulation Fuzzy Rough Sets (CMGFRS) have been first proposed, which extend the covering rough sets from single-granulation to multi-granulation. Firstly, the basic properties of these covering multi-granulation fuzzy rough sets are discussed, it is shown that basic properties of MGRS are special cases of those of CMGFRS. Then, the conditions for two CMGFRS generate identical lower and upper approximations of a set in a covering approximation space are studied. Finally, this study explored the relationship between the three types of CMGFRS.

Key words: Fuzzy rough sets, multi-granulation, covering approximation, reduct

INTRODUCTION

Covering rough set is an interesting and meaningful extension direction of Pawlak's rough sets. Since, Zakowski (1983) first proposed the concept of covering-based rough sets, extensive research works have been developed. Based on the mutual correspondence of the concepts of extension and intension, Bonikowski *et al.* (1998) have formulated the necessary and sufficient conditions for the existence of operations on rough sets, which are analogous to classical operations on sets. Pomykala (1987) studied the covering rough set model from the point view of topology. Mordeson (2001) studied algebraic structural properties of certain subsets for a type of covering-based rough sets. Tsang *et al.* (2004) put forward with a new type of upper approximations based on covering. Zhu and Wang (2003, 2007) and Zhu (2007a,b, 2009a, b) did systematic research on six types of covering-based rough sets. Yao and Yao (2012) proposed a framework for the study of covering based rough set approximations.

When a problem involves incomplete, uncertain, or vague information, it may be difficult to differentiate distinct elements and one is forced to consider granules. In the view of granular computing (Yao, 2005), an equivalence relation on a universe can be regarded as a granulation and a partition on the universe as a granulation space. Based on these studies, Qian and Liang (2006) and Qian *et al.* (2010) introduced the multi-granulation rough set, where the approximations are defined using multi-equivalence relations which should be used because of user requirements or problem specification. When two attribute sets in information systems possesses a contradiction or inconsistent relationship, MGRS will display its advantages for knowledge discovery. We find that the binary relations in the MGRS are definitely induced by partitions. By weakening the requirement of equivalence relations, we can have more general granulation and approximation structures based on coverings of the universe. To make the MGRS suitable for larger scopes, this paper proposed three kinds of Covering Multi-Granulation Fuzzy Rough Sets (CMGFRS), that is, the relations in there are induced by coverings.

PRELIMINARIES

In this section, we review some basic notions and notations related in the sequel of our work.

In the following, T stands for FC or SC or TC if there is no extra explanation.

Let U be a universe of discourse and C a family of nonempty subsets of U. If $\cup C = U$, C is called a covering of U. There is no doubt that a partition of U is certainly a covering of U.

Definition 1 Bonikowski *et al.* (1998): The pair (U, C) is called a covering approximation space, where U is a universe of discourse and C a covering of U. For any $x \in U$, the set md(x) is called the minimal description of x, where:

$$md(x) = \{K \in C | x \in K \wedge (\forall S \in C \wedge x \in S \wedge S \subseteq K \Rightarrow K = S)\}$$

Definition 2 Zhu (2009a): Let (U, C) be a covering approximation space, for any $X \subseteq U$, the covering lower and upper approximations of X can be defined as following:

Corresponding Author: Caihui Liu, Department of Computer Science and Technology, Tongji University, Shanghai, 201804, China

$$\underline{FC}(X) = \{x \in U \mid \cap\, md(x) \subseteq X\}$$

$$\overline{FC}(X) = \{x \in U \mid \cap\, md(x) \cap X \neq \varnothing\}$$

Definition 3: Let (U, C) be a covering approximation space, for any $X \subseteq U$, the covering lower and upper approximations of X can be defined as following:

$$\underline{SC}(X) = \{x \in U \mid \cup\, md(x) \subseteq X\}$$

$$\overline{SC}(X) = \{x \in U \mid \cup\, md(x) \cap X \neq \varnothing\}$$

Definition 4: Let (U, C) be a covering approximation space, for any $X \subseteq U$, the covering lower and upper approximations of X can also be defined as following:

$$\underline{TC}(X) = \sup_{K \in md(x)} \{\inf_{y \in K}\{X(y)\}\} \quad \overline{TC}(X) = \inf_{K \in md(x)} \{\sup_{y \in K}\{X(y)\}\}$$

Theorem 1 Zhu and Wang (2003): Let C be a covering of U and $K \in C$. If K is a union of some sets in C-{K}, we say K is a reducible element of C. If all the reducible elements have been discarded, then the remainder of C is called the reduction of C on U, denoted by reduct(C). For any $x \in U$, C and reduct(C) have the same md(x).

Definition 5 Qian and Liang (2006): Let K = (U, C) is a knowledge base, C a family of equivalence relations on U, for any $X \subseteq U$, P, $Q \in C$, the lower and upper approximations of X can be defined by the following:

$$\underline{P+Q}X = \{x \in U \mid [x]_P \subseteq X \text{ or } [x]_Q \subseteq X\} \quad \overline{P+Q}X = C\ \underline{P+Q}(C\ X)$$

where, C X is the complement of X in U.

COVERING MULTI-GRANULATION FUZZY ROUGH SETS AND THEIR PROPERTIES

Here, we study three types of covering multi-granulation fuzzy rough sets. Firstly, we shall discuss the basic properties of CMGFRS, and then we present the similarities and differences between CMGFRS and MGRS. Secondly, for the same type CMGFRS we explore the conditions under which two coverings generate identical CMGFRS lower and upper approximations.

Definition 6 Liu and Wang (2011): Let (U, C) be a covering approximation space, C a family of coverings on U and C_1, $C_2 \in C$. For any $X \in F(U)$, the lower and upper approximations of X in F(U) with respect to C_1, C_2 can be defined as follows: for any $x \in U$,

$$\underline{FC_1 + FC_2}(X)(x) = \inf\{X(y) \mid y \in \cap md_{C_1}(x)\} \vee \inf\{X(y) \mid y \in \cap md_{C_2}(x)\}$$

$$\overline{FC_1 + FC_2}(X)(x) = \sup\{X(y) \mid y \in \cap md_{C_1}(x)\} \wedge \sup\{X(y) \mid y \in \cap md_{C_2}(x)\}$$

If $\underline{FC_1 + FC_2}(X) = \overline{FC_1 + FC_2}(X)$, then X is definable. Otherwise X is called type I covering based multi-granulation fuzzy rough sets with respect to C_1, C_2. The pair $(\underline{FC_1 + FC_2}(X), \overline{FC_1 + FC_2}(X))$ is called a type I CMGFRS.

Definition 7: Let (U, C) be a covering approximation space, C a family of coverings on U and C_1, $C_2 \in C$. For any $X \in F(U)$, the lower and upper approximations of X in F(U) with respect to C_1, C_2 can be defined as follows: for any $x \in U$,

$$\underline{SC_1 + SC_2}(X)(x) = \inf\{X(y) \mid y \in \cup md_{C_1}(x)\} \vee \inf\{X(y) \mid y \in \cup md_{C_2}(x)\}$$

$$\overline{SC_1 + SC_2}(X)(x) = \sup\{X(y) \mid y \in \cup md_{C_1}(x)\} \wedge \sup\{X(y) \mid y \in \cup md_{C_2}(x)\}$$

If $\underline{SC_1 + SC_2}(X) = \overline{SC_1 + SC_2}(X)$, then X is definable. Otherwise X is called type II covering based multi-granulation fuzzy rough sets with respect to C_1, C_2. The pair $(\underline{SC_1 + SC_2}(X), \overline{SC_1 + SC_2}(X))$ is called a type II CMGFRS.

Definition 8: Let (U, C) be a covering approximation space, C a family of coverings on U and C_1, $C_2 \in C$. For any $X \in F(U)$, the lower and upper approximations of X in F(U) with respect to C_1, C_2 can be defined as follows: for any $X \in U$,

$$\underline{TC_1 + TC_2}(X)(x) = \sup_{K \in md_{C_1}(x)} \{\inf_{y \in K}\{X(y)\}\} \vee \sup_{K \in md_{C_2}(x)} \{\inf_{y \in K}\{X(y)\}\}$$

$$\overline{TC_1 + TC_2}(X)(x) = \inf_{K \in md_{C_1}(x)} \{\sup_{y \in K}\{X(y)\}\} \wedge \inf_{K \in md_{C_2}(x)} \{\sup_{y \in K}\{X(y)\}\}$$

If $\underline{TC_1 + TC_2}(X) = \overline{TC_1 + TC_2}(X)$, then X is definable. Otherwise X is called type III covering based multi-granulation fuzzy rough sets with respect to C_1, C_2. The pair $(\underline{TC_1 + TC_2}(X), \overline{TC_1 + TC_2}(X))$ is called a type III CMGFRS.

Proposition 1: Let (U, C) be a covering approximation space, C_1, $C_2 \in C$. For any $X \in F(U)$, the following properties hold:

- (1L) $\underline{T_1 + T_2}(U) = U$, (1U) $\overline{T_1 + T_2}(U) = U$
- (2L) $\underline{T_1 + T_2}(\varnothing) = \varnothing$, (2U) $\overline{T_1 + T_2}(\varnothing) = \varnothing$
- (3L) $\underline{T_1 + T_2}(X) \subseteq X$, (3U) $X \subseteq \overline{T_1 + T_2}(X)$

- (4L) $\underline{T_1+T_2}(X)=\underline{T_1}(X)\cup\underline{T_2}(X)$
- (4U) $\overline{T_1+T_2}(X)=\underline{T_1}(X)\cap\underline{T_2}(X)$
- (5L) $\underline{T_1+T_2}(CX)=C\,\overline{T_1+T_2}(X)$
- (5U) $\overline{T_1+T_2}(CX)=C\,\underline{T_1+T_2}(X)$
- (6L) $\underline{T_1+T_2}(\underline{T_1+T_2}(X))=\underline{T_1+T_2}(X)$
- (6U) $\overline{T_1+T_2}(X)=\overline{T_1+T_2}(\overline{T_1+T_2}(X))$

Proof: It is straightforward according the definitions, we will omit them here.

Remark 1: In general, the following two properties do not hold:

- $\underline{T_1+T_2}(\underline{T_1+T_2}(CX))=C\,\underline{T_1+T_2}(X)$
- $\overline{T_1+T_2}(\overline{T_1+T_2}(CX))=C\,\overline{T_1+T_2}(X)$

Proposition 2: Let K = (U, C) be acovering approximation space and C_1, $C_2 \in C$. Then, for and X, $Y \in F(U)$, the following properties hold:

- $\underline{T_1+T_2}(X\cap Y)=(\underline{T_1}(X)\cap\underline{T_1}(Y))\cup(\underline{T_2}(X)\cap\underline{T_2}(Y))$
- $\overline{T_1+T_2}(X\cup Y)=(\overline{T_1}(X)\cup\overline{T_1}(Y))\cap(\overline{T_2}(X)\cup\overline{T_2}(Y))$
- $\underline{T_1+T_2}(X\cap Y)\subseteq\underline{T_1+T_2}(X)\cap\underline{T_1+T_2}(Y)$
- $\overline{T_1+T_2}(X\cup Y)\supseteq\overline{T_1+T_2}(X)\cup\overline{T_1+T_2}(Y)$
- If $X\subseteq Y$, then $\underline{T_1+T_2}(X)\subseteq\underline{T_1+T_2}(Y)$
- If $X\subseteq Y$, then $\overline{T_1+T_2}(X)\subseteq\overline{T_1+T_2}(Y)$
- $\underline{T_1+T_2}(X\cup Y)\supseteq\underline{T_1+T_2}(X)\cup\underline{T_1+T_2}(Y)$
- $\overline{T_1+T_2}(X\cap Y)\subseteq\overline{T_1+T_2}(X)\cap\overline{T_1+T_2}(Y)$

Definition 9: Let (U, C) be a covering approximation space, $C_1, C_2 \in C$. Reduct (C_1) = $\{C_{11}, C_{12}, ..., C_{1p}\}$, reduct ($C_2$) = $\{C_{21}, C_{22}, ..., C_{2q}\}$. For any $x \in U$, $x \in C_{1i}$ $(1 \le i \le p)$ and $x \in C_{2j}$ $(1 \le j \le q)$ there is $C_{1i} \subseteq C_{2j}$ holds that is reduct (C_1) $\subseteq$ reduct (C_2), then we say that C_1 is finer than C_2, denoted by $C_1 \lhd C_2$.

Theorem 2: Let (U, C) be a covering approximation space, C_1, $C_2 \in C$. Then, for any $X \in F$ (U), the following properties hold.

- If $C_1 \lhd C_2$, then $\underline{T_1+T_2}(X)=\underline{T_1}(X)$
- If $C_1 \lhd C_2$, then $\overline{T_1+T_2}(X)=\overline{T_1}(X)$

Theorem 3: Let (U, C) be a covering approximation space, C_1, C_2, $\in C$ and reduct (C_1), reduct (C_2) are the reductions of C_1, C_2 respectively, then we say that T_1+T_2 and Tredcut (C_1)+Tredcut (C_2) generate identical CMGFRS lower and upper approximation operators.

Definition 10 (Zhu and Wang, 2003): Let C be a covering of U and K an element of C. If there exists another element K′ of C such that $K \subset K$, we say that K is an immured element of covering C.

Definition 11 (Zhu and Wang, 2003): Let C be a covering of U. When we remove all immured elements from C, the set of all remaining elements is still a covering of U and this new covering has no immured element. We call this new covering an exclusion of C, denoted by exclusion (C).

Theorem 4: Let (U, C) be a covering approximation space, $C_1, C_2, C_3, C_4 \in C$ and reduct (C_2), reduct (C_1), reduct (C_3), reduct (C_4) are the reductions of C_1, C_2, C_3, C_4, respectively, if at least one of the following is satisfied:

- Reduct (C_1) = reduct (C_3) and exclusion (C_1) = exclusion (C_3)
- Reduct (C_1) = reduct (C_4) and exclusion = (C_1) = exclusion (C_4)
- Reduct (C_2) = reduct (C_3) and exclusion (C_2) = exclusion (C_3)
- Reduct (C_2) = reduct (C_4) and exclusion (C_2) = exclusion (C_4)

then T_1+T_2 and T_3+T_4 generate identical covering multi-granulation fuzzy rough lower approximation operator.

Remark 2: The reverse of theorem 4 does not hold, i.e., T_1+T_2 and T_3+T_4 may generate identical covering multi-granulation fuzzy rough lower approximation operator but their reductions and exclusions may not the same.

Theorem 5: Let (U, C) be a covering approximation space, C_1, C_2, C_3, C_4 and reduct (C_2), reduct (C_1), reduct (C_3), reduct (C_4), are the reductions of C_1, C_2, C_3, C_4, respectively if at least one of the following is satisfied:

- Reduct (C_1) = reduct = (C_3) and reduct (C_2) = reduct (C_4)
- Reduct (C_1) = reduct (C_4) and reduct (C_2) = reduct (C_3) then T_1+T_2 and T_3+T_4 generate the same first type of covering multi-granulation rough lower and upper approximation operators

Remark 3: The reverse of theorem 5 does not hold, i.e., T_1+T_2 and T_3+T_4 may generate identical covering multi-granulation fuzzy rough upper approximation operator, but their reductions are not the same.

RELATIONSHIP AMONG THREE TYPES OF CMGFRS

We have proposed three types of covering multi-granulation fuzzy rough sets in last section. In this section, we will study the interrelations among them.

According to Definition 6, 7 and 8, one can easily get the following theorem.

Theorem 6: Let (U, C) be a covering approximation space, C_1, $C_2 \in C$. For any $X \in F(U)$, the following formulas hold:

- $\underline{SC_1 + SC_2}(X) \subseteq \underline{TC_1 + TC_2}(X) \subseteq \underline{FC_1 + FC_2}(X) \subseteq X$
- $X \subseteq \overline{FC_1 + FC_2}(X) \subseteq \overline{TC_1 + TC_2}(X) \subseteq \overline{SC_1 + SC_2}(X)$

Definition 12 (Zhu, 2007a): Let C be a covering of a set U. C is called unary if for any $x \in U$, $|md(x)| = 1$, where $|.|$ is cardinality of a set.

Theorem 7: Let (U, C) be a covering approximation space, C_1, $C_2 \in C$. For any $X \subseteq U$, if C_1 and C_2 are both unary, then three kinds of lower (upper) approximation of CMGFRS are identical.

Remark 4: The reverse of theorem 7 does not hold.

CONCLUSION

This study has proposed three types of covering multi-granulation fuzzy rough sets. To compare with MGRS, it is shown that the properties of MGRS are special cases of those of CMGFRS. Then, it studied the sufficient conditions for C_1, C_2 and C_3, C_4 to generate the identical covering multi-granulation lower or upper approximations. Finally, the relationships among the three types of covering multi-granulation rough sets have been explored. Of course, there are several issues are worthy of further investigation. For example, topological properties of CMGFRS may be a potential topic for future research.

ACKNOWLEDGMENTS

The research is supported by the National Natural Science Foundation of China under grant Nos: 60970061, 61075056, China National Natural Science Foundation of Youth Science Foundation under Grant No.: 61103067, China Postdoctoral Science Foundation under Grant Nos.: 2011M500626, 2011M500815.

REFERENCES

Bonikowski, Z., E. Bryniarski and U. Wybraniec-Skardowska, 1998. Extensions and intentions in the roughsettheory. Inform. Sci., 107: 149-167.

Liu, C. and M. Wang, 2011. Covering fuzzy rough set model based on Multi-granulations. Proceedings of 2011 International Conference on Uncertainty Reasoning and Knowledge Engineering, Volume 2, August 04-07, 2011, Bali, Indonesia, pp: 146-149.

Mordeson J.N., 2001. Rough set theory applied to (fuzzy) ideal theory. Fuzzy Set Syst., 121: 315-324.

Pomykala, J.A., 1987. Approximation operations in approximation space. Bull. Polish Acad. Sci., 35: 653-662.

Qian, Y.H. and J.Y. Liang, 2006. Rough set method based on Multi-granulations. Proceedings of the 5th IEEE International Conference on Cognitive Informatics, Volume 1, July 17-19, 2006, Beijing, China, pp: 297-304.

Qian, Y., J. Liang, Y. Yao and C. Dang, 2010. MGRS: A Multi-granulation rough set. Inform. Sci., 180: 949-970.

Tsang, E.C.C., D. Chen, J.W.T. Lee and D.S. Yeung, 2004. On the upper approximations of covering generalized rough sets. Proceedings of 2004 International Conference on Machine Learning and Cybernetics, Volume 7, August 26-29, 2004, Hong Kong, China, pp: 4200-4203.

Yao Y. and B. Yao, 2012. Covering based rough set approximations. Inform. Sci., 200: 91-107.

Yao, Y., 2005. Perspectives of granular computing. Proc. IEEE Int. Conf. Granular Comput., 1: 85-90.

Zakowski, W., 1983. Approximations in the space (U, C). Demonstratio Math., 16: 761-769.

Zhu, W. and F.Y. Wang, 2003. Reduction and axiomatization of covering generalized rough sets. Inform. Sci., 152: 217-230.

Zhu, W. and F.Y. Wang, 2007. On three types of Covering-based rough sets. IEEE Trans. Knowl. Data Eng., 19: 1131-1144.

Zhu, W., 2007a. Topological approaches to covering rough sets. Inform. Sci., 177: 1499-1508.

Zhu, W., 2007b. Generalized rough sets based on relations. Inform. Sci., 177: 4997-5011.

Zhu, W., 2009a. Relationship between generalized rough sets based on binary relation and covering. Inform. Sci., 179: 210-225.

Zhu, W., 2009b. Relationship among basic concepts in covering-based rough sets. Inform. Sci., 179: 2478-2486.