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A New Signal De-noising Method Based on Energy Difference Spectrum of Singular Value

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Abstract: The order of effective rank is difficult to determine for noise reduction based on singular value decomposition. In order to improve the signal to noise ratio of practical sample data, a novel de-noising method was proposed by energy difference spectrum of singular value to solve this problem. According to the energy difference between useful signal and noise, the energy difference spectrum of singular value was constructed and then the reconstruction order number was determined according to the peak position of the energy difference spectrum. The effectiveness of the method was proved by simulation and practical results. And the results of comparing the performances of the proposed method to the morphological filter and the Ensemble Empirical Mode Decomposition (EEMD) show the proposed method can retain the original signal characteristic effectively and eliminate noise as much as possible. It's very important for signal feature extraction and analysis next step.

Key words: Energy difference spectrum of singular value, de-noising, singular value decomposition, vibration signal processing

INTRODUCTION

Signal processing technology is the core for condition monitoring and fault diagnosis of rotating machinery. In the practical measurement, vibration data obtained in the test field always contain all kinds of interferences; how to decrease noise and increase the signal-to-noise ratio is the key for fore fault diagnosis of machinery (Zhang *et al.*, 2010). The common de-noising methods include time-domain average method, adaptive filtering, wavelet filtering and frequency-domain feature extraction. However, some disadvantages of these technologies impact the de-noising effect, such as the application of time-domain average method must have the support of time scale and it needs enough data sets; the de-noising effectiveness of adaptive filtering and wavelet filtering largely depend on the performance of the filter, different filter selection will influence the de-noising performance while the frequency-domain feature extraction technology will be influenced by signal amplitude, frequency and phase information, therefore, these technologies could not realize signal de-noising successfully (Wenxian *et al.*, 2001).

Mathematical Morphology is a subject concerning with the shape of an object based on set theory and integral geometry. Its filtering idea is based on the geometrical structure of the filtered signals and realized through moving predefined structure element to match

and adjust the singular parts of the signals. It has been used in signal de-noising of rotating machinery (Zhang *et al.*, 2009). Due to the selective random of structure element, its de-noising performance could not achieve the best result (Shen *et al.*, 2009). The Ensemble Empirical Mode Decomposition method (EEMD) (Chen *et al.*, 2012) is a new technology in signal de-noising field but the algorithm of EEMD has two parameters are selected artificially (Wu and Huang, 2009). So these methods can not achieve better performance by randomly selective parameters.

Singular value decomposition (Qian *et al.*, 2011) is a common used method in signal de-noising. But there are two problems existing in practical application, i.e., the number of array for reconstructed matrix and the effective rank order are difficult to determine (Zhao *et al.*, 2010). For the former problem, the effective methods have been introduced (Zhao *et al.*, 2012). For the latter one, the singular value entropy increment (Wenxian *et al.*, 2001) and threshold method are put forward but these methods still depend on the experiences and the de-noising result is not very good. In this study, a novel algorithm is proposed based on energy difference spectrum of singular value. This method could determine the effective rank order by different singular value energy between useful signal and noises. Then the noise interferences will be eliminated successfully.

PRINCIPLE OF SINGULAR VALUE DECOMPOSITION

Singular Value Decomposition (SVD) is a powerful numerical technique for solving systems of linear equations and is easy to implement for order matrix calculations. It also has the advantage that it can deal with sets of equations that are close to singular which would happen if the measured angular data were redundant because of co-linearity of vectors (Losonczi *et al.*, 1999).

It is well known from linear algebra that any matrix A with M rows and N columns can be written in terms of its singular value decomposition, i.e., as the product of an M×M column-orthogonal matrix U and an M×N diagonal matrix D with nonnegative diagonal elements and the transpose of an N×N orthogonal matrix V. That is,

$$A = UDV^T \quad (1)$$

Here, $D = (\text{diag}(\sigma_1, \sigma_2, \dots, \sigma_q), 0)$ or its transposition which depends on $M < N$ or $M > N$. And 0 refers to zero matrix. Moreover, $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_q > 0$ and the numbers σ_q are called the singular values of A. If $M = N$, D is a M×M diagonal matrix with the singular values in decreasing order on its main diagonal. If $M < N$, D consists of a M×M diagonal matrix with the M singular values on its main diagonal (in decreasing order) extended on the right-hand side with a M×(N-M) matrix of zeros. If $M > N$, D consists of a N×N diagonal matrix with the N singular values on its main diagonal (in decreasing order) on top of a (M-N) × N matrix of zeros. The de-noising principle of singular value decomposition is using the energy difference between useful signal and noises, the matrix constructed by noise interrupted signal is decomposed, then the useful singular values are obtained in reconstructed procedure and the singular values corresponding with the noise are replaced by zero (Xu and Liu, 2011).

Let $x(i)$ as sample data, here $i = 1, 2, \dots, N$. The matrix A is constructed by phase space reconstruction theory. That is:

$$A = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ x_2 & x_3 & \dots & x_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_m & x_{m+1} & \dots & x_{m+n-1} \end{bmatrix} \quad (2)$$

Here, $1 < n < N$ and $m+n-1 = N$ this matrix is called Hankel matrix.

The difficulty of using singular value decomposition to process the interrupted signal is how to determine the number of array for reconstructed matrix and the effective

rank order. According to the conclusion from Zhao *et al.* (2012) and simulation experiments, if N is even number, the column $n = N/2$ and the row $m = N/2+1$. While N is odd number, the column and the row may be the same as $(N+1)/2$.

In signal reconstruction procedure, determination of the effective rank order is very important. When the less singular values are selected, due to the low de-noising rank order, useful information may be lost; while the more singular values are selected, due to the high de-noising rank order, some noises will be included in reconstructed signal. Therefore, in this study, the energy difference spectrum of singular value is defined and the effective rank order is determined by the singular value energy between useful signal and noise interference.

Wang *et al.* (2012) said that signal energy could be defined by singular value as follows:

$$E = \sum_{i=1}^q \sigma_i^2 \quad (3)$$

Then, define the energy difference spectrum of singular value and normalized it as follows:

$$p(i) = \frac{\sigma_i^2 - \sigma_{i+1}^2}{E} \quad (4)$$

The numbers of $p(i)$ are called energy difference spectrum of singular value. Due to energy of the useful signal is bigger than that of the noises, in the boundary of useful signal and noises, the energy difference spectrum must have a peak. So we can search the peak position as the effective rank order for the reconstructed matrix. Zhao *et al.* (2010) divided signal into signal with direct-current component and signal without direct-current. When signal with direct-current component, the peak of energy difference spectrum will be gotten in the first position. If we use this peak position as the effective rank order, we will only get the direct-current component. So, in this study, we only study the vibration signal without direct-current component.

SIMULATION

In order to test the good de-noising effectiveness of singular value decomposition, let the simulation signal as follows:

$$x(t) = \sin(2\pi \times 25 \times t) + \sin(2\pi \times 50 \times t) + i(t) \quad (5)$$

Here, $i(t)$ contain periodical spike pulse interference and white noises. Set the sample frequency is 2048Hz, sample number is 1024. The waveform in time-domain can be seen in Fig. 1.

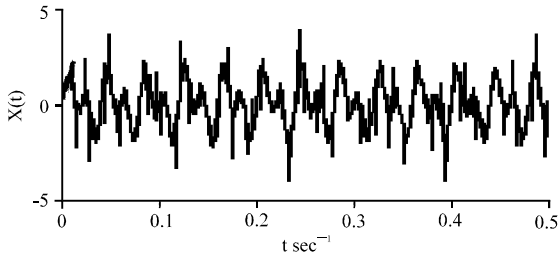


Fig. 1: Simulation signal

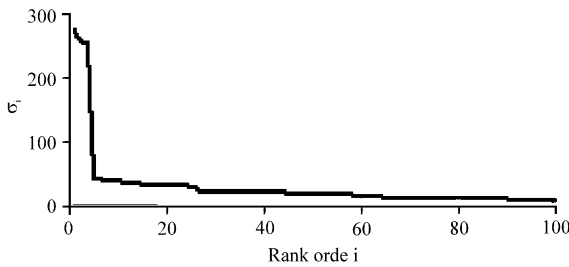


Fig. 2: Curve of singular value

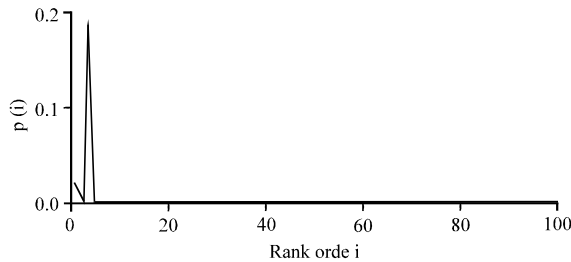


Fig. 3: Curve of energy difference spectrum of singular value

Now use singular value decomposition to de-noise. Fig. 2 gives the curve of singular values. Fig. 3 shows the energy difference spectrum of singular value, here we select the former 100 singular values to plot the figure. From the figure, we can see that the useful singular value is in the former position and its value is big and the latter values are small and refer to the noises. According to the definition of energy difference spectrum of singular value, the dividing point is $i = 4$, so we select 4 as the rank order of reconstructed matrix. Fig. 4 shows the de-noising result. We can see that the noises are eliminated successfully.

In order to prove the good effectiveness of the proposed method, Fig. 5 shows the de-noising results of other two filtering methods. Fig. 5a uses the EEMD method to process the interrupted signal while Fig. 5b uses the morphological filter. We can see that these two filtering methods can not eliminate the noises successfully.

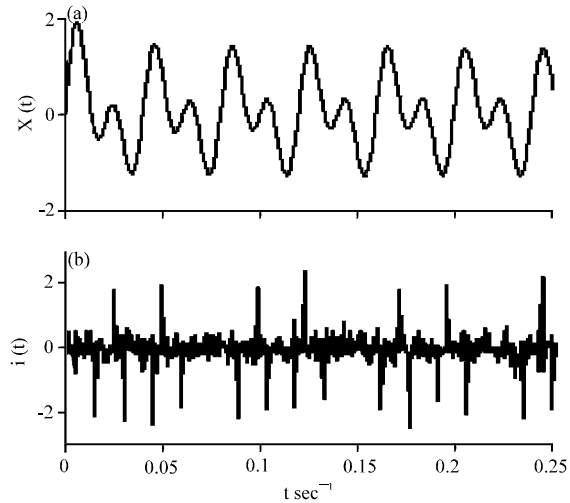


Fig. 4(a-b): De-noising results (a) simulation signal without noises (b) eliminated noises

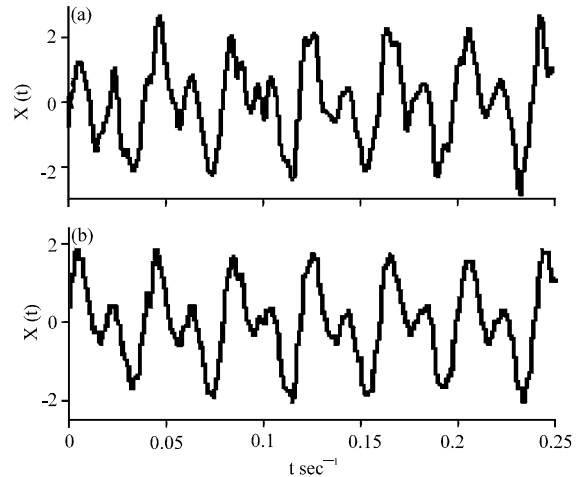


Fig. 5(a-b): De-noising results comparison (a) with EEMD method (b) with morphological filter

PRACTICAL APPLICATION

Figure 6 shows a practical vibration signal of the turbine generator units in the power plant. The speed of the rotor is near 3000 rpm. Let the sampling frequency equal to 6400 Hz. Due to the serious noise interference, the original signal is too disordered to get any fault information.

Now we use the proposed method to remove noise interference. According to the above definition, the effective rank order is determined automatically in the decomposition procedure. Fig. 7 shows the de-noising result. Fig. 7a shows the useful vibration signal while Fig. 7b shows the noise interference obtained in the

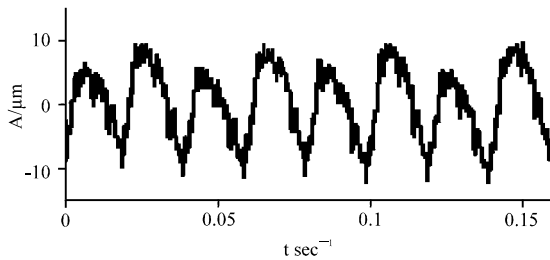


Fig. 6: Practical signal

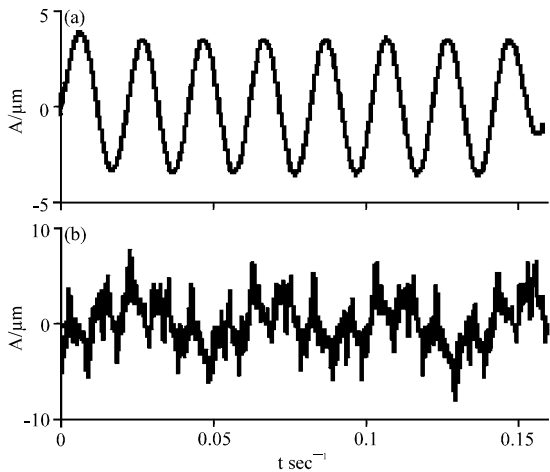


Fig. 7(a-b): De-noising results for practical signal (a) practical signal without noises (b) eliminated noises

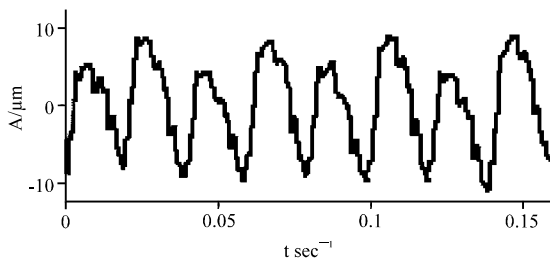


Fig. 8: De-noising results by morphological filter

original sample data. Fig. 8 shows the de-noising result by morphological filter. We can see that the morphological filter can not eliminate the noises completely while peak pulse and noise interference are removed clearly by the proposed novel approach.

CONCLUSION

In this study, a novel de-noising method is proposed by using singular value decomposition. The definition of energy difference spectrum of singular value is introduced

and it has been used to determine the effective rank order of reconstructed matrix. When using the singular value decomposition to process the vibration signal, there is no need to consider the frequency-domain feature for the sample data and it only needs to process the signal in time-domain. Simulation and practical results show the good performance of the proposed method in noise de-noising. It will supply a new method for fault diagnosis of rotating machinery.

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