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An Improved Music Algorithm Based on Signal Conversion for Real Modulation Domains

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Abstract: Multiple Signal Classification (MUSIC) algorithm is an excelent algorithm which has high resolution and low computational complexity for estimating Direction of Arrival (DOA) signals. It performs well when the number of array antennas M is much larger than the sources D, otherwise its performance severely degrades and DOA of the signal sources even can not be estimated. Based on the idea of Euler's formula, this study introduces an improved MUSIC algorithm utilizing signal conversion to reconstruct a new received signal matrix for real constellation signals. Derivation process of the improved algorithm considers that the new matrix does not change original one's characteristic and still retains the same rank. Simulation results show that estimating performances of improved algorithm outperform the traditional ones for the case of D = M and it is also observed that the improved MUSIC algorithm is able to estimate the number of antennas up to 2(M-1) in lower SNRs.

Key words: DOA estimation, MUSIC algorithm, array antennas, Multi-sources

INTRODUCTION

The subject of smart antennas is beginning to enjoy immense popularity due to the current rapidly growth in all forms of wireless communications. In the realm of mobile wireless applications, ranging from mobile cellular (Winters, 1998; Anderson *et al.*, 1999; Alexiou and Haardt, 2004) to Personal Communications Services (PCS) to radar (Gross, 2005) are hoping to utilize smart antennas to boost capacities, expand bandwidths, mitigate mutipath fading and increase Signal-to-noise Ratios (SNRs) and improve MIMO communications. Smart antennasgenerally refers to any antenna arrays with a sophisticated and smart signal processing algorithm used to direct narrow beams toward the users of interest while nulling other users not of interest. Therefore it is an important prerequisite for antenna arrays to distinguish the directions of interesting signals or interfering signals. In other words, Direction of Arrival (DOA) estimation is a pivotal signal processing technology by structuring the space spectral function according to the characteristics of the received signal to estimate the azimuth angles for smart antennas.

There has been considerable work on DOA estimation algorithms that can be divided into three categories: Linear prediction methods (Capon, 1969; Pisarenko, 1973) subspace decomposition methods (Schmidt, 1986; Zoltowski *et al.*, 1993; Roy *et al.*, 1986) and subspace fitting methods (Viberg and Ottersten, 1991). In all of DOA estimation algorithms, multiple signal classification (MUSIC) algorithm put forward by (Schmidt,1986) has a wide range of applications and extensively studied for offering a good trade-off between estimation performances and computation costs. Under the ideal conditions, the algorithm has high-resolution and estimation accuracy. However, the performance of MUSIC algorithm will become weak in some conditions such as correlated sources, multipath channels, non-Gaussian white noise, low signal-to-noise ratios (SNRs) which usually encountered in the practical applications.

Therefore, (Pillai and Byung, 1989; Debasis, 1996) respectively proposed forward/backward spatial smoothing method and modified MUSIC algorithm for coherent signal identification. Later (Amina *et al.*, 2012) proposed a Support Vector Machine (SVM) MUSIC algorithm which combines the benefits of subspace methods with those of SVM having better performance with uncorrelated and coherent signals and in small sample size situations. In the recent years with the developed FPGA technology the research objective of academia is more emphasis on reducing computational complexity of MUSIC algorithm for hardware implementation. For example, (Bouri, 2012) proposed an approximation of MUSIC algorithm and (Wang *et al.*, 2013) proposed a new mixed-order MUSIC algorithm to reduce the computation which make a good foundation for the practical application of the MUSIC algorithm. (Majid *et al.*, 2013) made some improvement of MUSIC algorithm and tested on the hardware platform.

However, the performance will deteriorate in practice, for example when the number of sources are equal to or greater than the number of sensors, or simply low SNRs (Ioannopoulos *et al.*, 2012; Yang *et al.*, 2011). In order to study the performance in this context (Wang and Wang,

2012) proposed to an improved algorithm using the Euler's complex number formula and proved that the proposed method outperform the traditional MUSIC in the same conditions. The literature also mentioned the proposed method can estimate the number of sources 2(M-1) but had no specific theoretical explanation and gave no discussion in the case of low SNRs or small number of snapshots in practice. On the other hand, (Zhang *et al.,* 2009) proposed to reconstruct the noise subspace whose performance had been greatly improved relative to the traditional algorithm in low SNRs. Later, (Wang *et al.*, 2011) proposed a self-adapting root-MUSIC algorithm for vector hydrophone array, compared with MUSIC algorithm which also had better performance for low SNRs and it had almost the same estimation performance for high SNRs with MUSIC algorithm. But neither one of these studys is able to deal with the number of sources much larger than sensors'.

This study proposed an improved MUSIC algorithm using the idea of Euler's formula to estimate real signals in low SNRs. The remainder of this study is organized as follows. Models of signals and the MUSIC algorithm are described in section 2. In section 3, we introduce all the reasoning process of the improved algorithm. In section 4, we present some numerical examples that the proposed approach compared with the traditional MUSIC and MMUSIC algorithm. Section 5 gives some conclusions about the MUSIC algorithms.

MODELS OF SIGNALS AND THE MUSIC ALGORITHM

Model of received signals: Considering D narrow-band source signals are received by an array of M sensors which are Uniform Linear Array (ULA). We assume that the users are located in the far-field region. As Fig. 1, each received signal $x_m(k)$ includes additive, zero mean, Gaussian noise. Time is represented by the kth time sample. Thus, the mth element of the received signal at time k can be given in the following form:

$$x_m(k) = [a_m(\theta_1)a_m(\theta_2)\cdots a_m(\theta_D)]\cdot\begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_D(k) \end{bmatrix} + n_m(k)$$

Where:

$$a_m(\theta_i) = e^{j(m-1)\frac{2\pi}{\lambda}d\sin\theta_i}$$

is a steering function associated with the ith sources with DOA θ_i, element spacing d and the wavelength of the

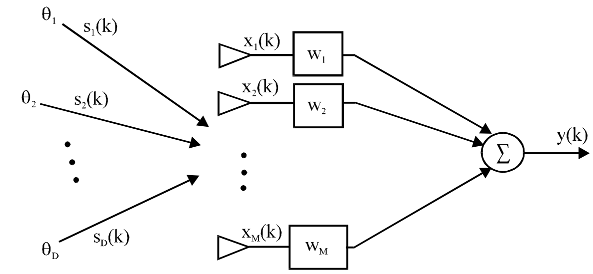

Fig. 1: M-element array with arriving signals

incident radiation λ. $s_i(k)$ represents the *i*th incident complex signal at time k and $n_m(k)$ represents noise signal at the mth array element, zero mean, variance σ^2. Thus, the received signal of M-elements array at time k can be written as:

$$X(k) = \begin{bmatrix} a_1(\theta_1) & a_1(\theta_2) & \cdots & a_1(\theta_D) \\ a_2(\theta_1) & a_2(\theta_2) & \cdots & a_2(\theta_D) \\ \vdots & \vdots & \cdots & \vdots \\ a_M(\theta_1) & a_M(\theta_2) & \cdots & a_M(\theta_D) \end{bmatrix}\cdot\begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_D(k) \end{bmatrix} + \begin{bmatrix} n_1(k) \\ n_2(k) \\ \vdots \\ n_D(k) \end{bmatrix}$$

that

$$X(k) = A(\theta)S(k)+N(k) \quad (1)$$

where, $A(\theta)$ is M×D matrix of the steering vectors, S(k) represents vector of incident complex signals and N(k) is noise vector at the mth array element.

Music algorithml It is assumed that S(k) and N(k) are independent of each other, source signals uncorrelated and noise with equal variances. Then MUSIC algorithm steps are as follows:

- Calculate the covariance matrix of the received signals data samples X(k) (k = 1,2,…,K), then:

$$R_{xx} = E[XX^H] = AR_{ss}A^H + \sigma^2 I$$

or:

$$R_{xx} = \frac{1}{K}\sum_{i=1}^{K} X(i)\cdot X^H(i)$$

Where:

$$R_{ss} = E[S(k)S^H(k)]$$

is the covariance matrix of source signals and I represents M-order unit matrix

- Find the eigenvalues and eigenvectors for R_{xx}, we have $R_{xx}V = V\Lambda$, where $\Lambda = \text{diag}\{\lambda_0, \lambda_1, \ldots, \lambda_{M-1}\}$ represents the eigen-values from small to large □□ arranged and $V = [q_0\ q_1\ \ldots\ q_{M-1}]$ are the eigenvectors associated with the eigenvalues
- Utilize the multiple number of the smallest eigenvalues N to estimate the number of sources $\hat{D}$, namely:

$$\hat{D} = M - N$$

- Construct the M×N dimensional subspace spanned by the noise eigenvectors such that:

$$E_N = [q_0 q_1 \cdots q_{N-1}]$$

- According to the orthogonal of the noise subspace eigenvectors to the array steering vectors at the angles of arrival $\theta_1, \theta_2, \ldots, \theta_D$, the MUSIC pseudospectrum is given as:

$$P_{MUSIC} = \frac{1}{a^H(\theta)E_N E_N^H a(\theta)}$$

then finding out the maximum point is the angle of incident signals.

AN IMPROVED ALGORITHM BASED ON SIGNAL CONVERSION

In modern communications systems, BPSK and MASK modulation signals having characteristics of the real signal are widely used. Therefore, the new method makes use of the characteristic that $S(k) = S^*(k)$, we apply Euler's equation:

$$e^{\pm j\psi} = \cos\psi \pm j\sin\psi$$

to preprocess the received matrix in Equ. (1), then:

$$X(k) = \left(A_C(\theta) + jA_S(\theta)\right)S(k) + N_C(k) + jN_S(k)$$

We define:

$$X_C(k) = \frac{X(k) + X^*(k)}{2} = A_C(\theta)S(k) + N_C(k)$$

$$X_S(k) = \frac{X(k) - X^*(k)}{2j} = A_S(\theta)S(k) + N_S(k)$$

The data received signal matrix $X_r(k)$ can be reconstructed as:

$$X_r(k) = \begin{pmatrix} X_C(k) \\ X_S(k) \end{pmatrix} = \begin{pmatrix} A_C(\theta) \\ A_S(\theta) \end{pmatrix} \cdot S(k) + \begin{pmatrix} N_C(k) \\ N_S(k) \end{pmatrix}$$
$$= A_r(\theta) \cdot S(k) + N_r(k)$$

Thus the covariance matrix $\hat{R}_{xx}$ of $X_r(k)$ is:

$$\hat{R}_{xx} = E[X_r X_r^H] = A_r R_{ss} A_r^H + \sigma_r^2 I \quad (2)$$

According to the definition of eigenvalues, □□ the eigenvalues of $\hat{R}_{xx}\{\hat{\lambda}_0, \cdots \hat{\lambda}_{2M-1}\}$ (from small to large □□ arranged) meet the condition:

$$\left|\hat{R}_{xx} - \hat{\lambda}_i I\right| = 0$$

From (2), we can rewrite it as:

$$\left|A_r R_{ss} A_r^H - (\hat{\lambda}_i - \sigma_r^2)I\right| = 0 \quad (3)$$

Because of matrix *A* is a full column rank for composed of independent linear steering vectors, the pretreated A_r does not change the matrix characteristic and is still a full column rank matrix. R_{ss} is a non-singular matrix when the sources are uncorrelated. Therefore, when the number of incident signals D is less than 2M, the dimension 2×2M matrix $A_r R_{ss} A_r^H$ is a □□ positive semi-definite matrix as follow:

$$\text{rank}(A_r R_{ss} A_r^H) = D \quad (4)$$

As the basic knowledge of linear algebra shown, the matrix $A_r R_{ss} A_r^H$ has 2M-D zero eigenvalues. Therefore equation (3) can be inferred that $\hat{R}_{xx}$ has 2M-D eigenvalues σ_r^2. To arbitrary eigenvalues exist the following relationships:

$$\hat{R}_{xx}\hat{q}_i = \hat{\lambda}_i \hat{q}$$

$$(\hat{R}_{xx} - \hat{\lambda}_i I)\hat{q}_i = 0 \quad (5)$$

To 2M-D eigenvalues σ_r^2 and the eigenvectors associated with the eigenvalues, we have:

$$(\hat{R}_{xx} - \sigma_r^2 I)\hat{q}_i = (A_r R_{ss} A_r^H + \sigma_r^2 I - \sigma_r^2 I)\hat{q}_i = 0 A_r R_{ss} A_r^H \hat{q}_i = 0$$

As A_r is a full column rank matrix and R_{ss} is a □□ positive semi-definite matrix, so:

$$A_r^H \hat{q}_i =$$

We can also infer:

$$A_r^H \lambda_i^n \hat{q}_i = 0 \quad (6)$$

where, n is weighted factor that greater than or equal to 0. We define the noise subspace as follows:

$$\hat{E}_N = \left(\lambda_0^n q_0, \cdots, \lambda_{N-1}^n q_{N-1}\right)$$

As shown in Eq. 6, the steering vectors corresponding to the signal component are orthogonal to the noise subspace eigenvector, so:

$$a_r^H(\theta)\hat{E}_N\hat{E}_N^H a_r(\theta) = 0$$

Therefore the incident DOAs can be estimated by determining the spatial spectrum peak:

$$P_{MUSIC} = \frac{1}{a_r^H(\theta)\hat{E}_N\hat{E}_N^H a_r(\theta)}$$

From the above reasoning process known, the reconstructed signal matrix $X_r(k)$ has been turned into real number matrix to make the computation complexity less than the traditional MUSIC algorithm. The matrix $X_r(k)$ with 2M×D is equivalent to double the number of available array elements. Therefore, the improved algorithm can deal with source signals up to 2(M-1). Utilizing the noise eigenvalues to weight eigenvectors and selecting an appropriate weighting factor n can improve the performance of the algorithm in low SNRs.

DISCUSSIONS AND NUMERICAL RESULTS

In this section, simulation results are presented to illustrate the performance of the improved algorithm and to compare it to the traditional MUSIC method, modified MUSIC (Debasis, 1996) method. The additive background noise is assumed to be white complex Gaussian with zero-mean, having the 0.1 variance value.

Relations between *n* and resolution: Consider 4-elements uniform linear array with the inter-element spacing d = λ/2 (λ is the carrier wavelength). Two narrow-band uncorrelated sources are received from -5 and 10°. The number of snapshots is K = 100 and the sources SNR is 15dB. In Fig. 2, we show the relations between weighted index n and the resolution of improved algorithm. It is evident that n is not the bigger the performance better. When n beyond a certain value, there will be some false peaks. We find the simulation result is relatively good at n = 5, so the following simulations take n = 5.

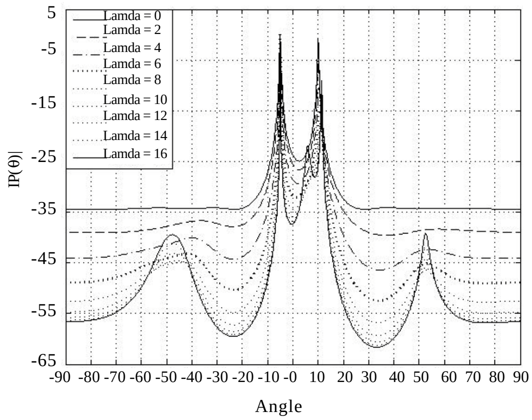

Fig. 2: Relations between n and resolution

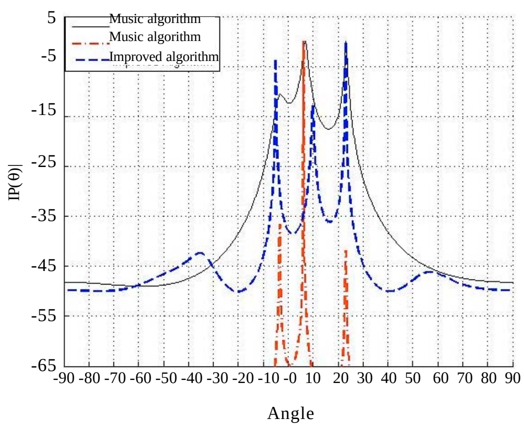

Fig. 3: SNR=15dB

Estimation performances analysis with SNRs: Consider 4-elements uniform linear array with the inter-element spacing d = λ/2. Three narrow-band uncorrelated sources are received from -5, 10 and 23°. The number of snapshots is K = 100. Fig. 3 and 4 depict the performance of the improved algorithm, MUSIC algorithm modified MUSIC algorithm in the case the SNRs is 5dB, 15dB and weighted index n is 5.

As Fig. 3 shown the MUSIC algorithm and MMUSIC algorithm cannot be able to give an accurate estimating with two uncorrelated signal sources when SNR is 15dB. Especially there is a clear deviation when the direction of arrival θ is 10°. However, the improved algorithm can clearly distinction the three angles. Fig. 4 shows that the MUSIC and MMUSIC algorithm have not been completely distinguish the DOAs except θ = 23° when SNR is 5dB. But the improved algorithm still can accurately distinguish the three angles DOAs. So it can be drawn that the improved algorithm has good estimation performance not only in high SNRs but also in low SNRs.

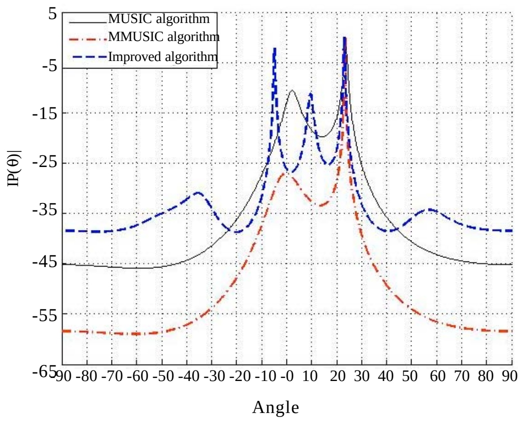

Fig. 4: SNR=5dB

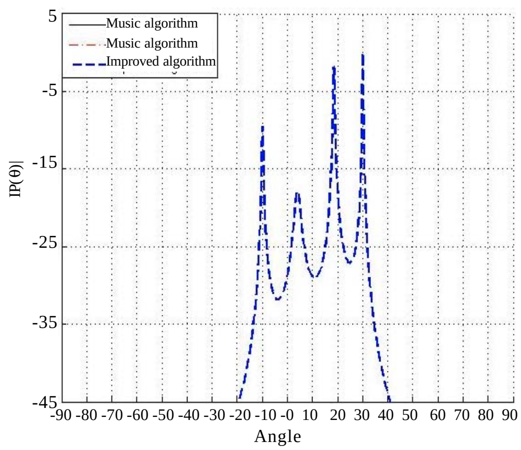

Fig.5: D = M = 4, θ = -10, 23, 5, 18, 30°

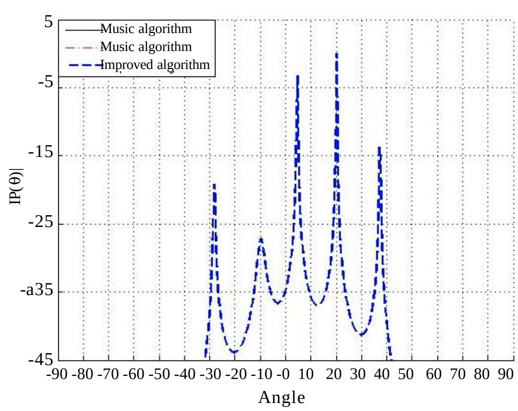

Fig. 6: D = 5£¾M, θ = -28, -10, 23, 5, 20, 37°

Estimation performances analysis in multi-sources: Consider 4-elements uniform linear array with the inter-element spacing d = λ/2. The number of snapshots is K = 100 and the sources SNR is 15 dB. Fig. 5 and 6, respectively estimate four sources, five sources at n = 5 and depict the performance of three algorithms.

As Fig.5 and 6 shown the MUSIC algorithm and MMUSIC algorithm has been completely ineffective to estimate DOAs when 4 or 5 uncorrelated sources incident in 4-elements antenna array. As the literature [24] said the two algorithms can estimate the number of signal sources up to M-1. However, the improved algorithm still can give a estimation when the number of signal sources is equal to or greater than the number of array elements.

CONCLUSION

The improved algorithm is a method which gives a pretreatment of the received signal and reconstructing the noise subspace based on the traditional MUSIC algorithm. It not only increases the number of sources which is twice more than the MUSIC algorithm but also can clearly distinguish the DOAs regardless of high or low SNRs. However, the improved algorithm which cannot be widely used is only suitable for uncorrelated sources and real modulation constellations. Estimating DOAs in the smart antenna systems will often encounter some problems such as multipath channels, the coherent sources estimating in practice and high efficiency complex modulations. In the next stages, we will make the improved algorithm perfect to estimate the coherent sources of multipath channels.

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