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## Dynamical Bayesian Testing for Feature Information of Time Series with Poor Information using Phase-space Reconstruction Theory

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**Abstract:** A dynamical Bayesian testing method is proposed to examine feature information on performance variation of time series with poor information in advance. Sub-series of time series are obtained via a regularly sampling, a multidimensional information space is formed by phase-space reconstruction method, probability density functions of phase trajectories are acquired with bootstrap and maximum entropy theory, a referenced sequence from phase trajectories is found by minimum variance principle, the posterior probability density function is established according to Bayesian theory and the mutation probability is defined in the light of fuzzy set theory. At the given significance level, dynamical Bayesian testing for feature information on performance variation of the poor information process is put into effect with the help of the mutation probability. Experimental investigation on vibration acceleration of a rolling bearing for space applications presents that the method proposed can effectively detect feature information on performance variation of time series with the unknown probability distribution and trend for the early detection of the hidden danger, thus avoiding serious accident.

**Key words:** Information process, information-poor theory, Bayesian theory, performance variation, vibration acceleration, rolling bearing

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### INTRODUCTION

Feature information is a series of data that can depict performance variation of time series. In fields of science and technology, feature information of many systems must be tested in time for the early detection of the hidden danger and safe system operation, therefore the theory of feature information process has attracted much attention, with many new findings.

For example, Wang *et al.* (2013) proposed a distance feature information method for recognizing digital images by the conservative smoothing, mean filtering, Gaussian sharpening and binarization; Jukic (2013) proposed a method for supervised feature extraction for tensor objects based on maximization of an approximation of mutual information; Wang *et al.* (2013a) designed the bag-of-words model of local feature information gain for the problem of indoor home scene classification; Raytchev *et al.* (2013) established a new general framework to obtain more distinctive local invariant features by projecting the original feature descriptors into low-dimensional feature space while simultaneously incorporating also class information; Lee *et al.* (2012) put forward a new multi-label feature selection method

that captures relationships between features and labels without transforming the problem into single-label classification; Bai *et al.* (2012) presented segmentation results by using multi-feature similarity measure under information based clustering framework, compared to pair wise similarity measure and Kamandar and Ghassemian (2013) structured a new supervised linear feature extractor for hyperspectral image classification.

In many fields of science and technology at present, studies of feature information mainly rely on a known probability distribution and trend in advance. For example, the probability distribution is considered as a normal distribution, a Weibull distribution, or a Poisson distribution and the trend is regarded as a given potential function and kernel function and wavelet basis function and a piecewise linearized function (Ahmad *et al.*, 2009). If the probability distribution and trend is unknown in advance, feature information of performance variation of time series can be extracted hardly. Thus, information analysis theory relied on prior information of probability distributions and trends encounters serious challenges. This, in fact, belongs to the category of the information-poor theory (Xia, 2012, 2012a).

Poor information means incomplete and insufficient information, such as, in system analysis, a small sample, an unknown probability distribution and trends without any prior information and so forth. Time series with poor information is a poor information process.

Based on the information-poor theory, a method for dynamical Bayesian testing is proposed to examine feature information on performance variation of time series with poor information for the early detection of the hidden danger, thus avoiding serious accident. Experimental investigation on vibration acceleration of a rolling bearing for space applications is conducted for corroboration of the method.

### MATHEMATICAL MODEL

Suppose performance data of a poor information process in service are sampled  $R$  times and  $R$  time series of performance data are obtained. Let  $X_r$  stand for the  $r$ th time series that is given by:

$$X_r = (x_r(1), x_r(2), \dots, x_r(h), \dots, x_r(H)); r = 1, 2, \dots, R \quad (1)$$

where  $x_r(h)$  is the  $h$ th datum in  $X_r$ ;  $h$  is a sequence number,  $h = 1, 2, \dots, H$  and  $H$  is the number of data in  $X_r$ .

The  $r$ th time series  $X_r$  is divided into  $D$  sub-series and the  $d$ th sub-series is given by:

$$X_{rd} = (x_{rd}(1), x_{rd}(2), \dots, x_{rd}(i), \dots, x_{rd}(I)); d = 1, 2, \dots, D \quad (2)$$

where  $x_{rd}(i)$  stands for the  $i$ th datum in  $X_{rd}$ ;  $i$  for a sequence number,  $i = 1, 2, \dots, I$  and  $I$  for the number of data which is expressed as:

$$I = \frac{H}{D} \quad (3)$$

Applying the phase-space reconstruction method to divide the  $d$ th sub-series  $X_{rd}$  into segments for essential revelation of original dynamics of the poor information process in service, a multidimensional information space that consists of  $N$  phase trajectories can be obtained.

Based on chaos theory (Lv *et al.*, 2002; Ionita, 2000), the  $j$ th phase trajectory in the multidimensional information space is expressed as:

$$X_{rd}(j) = (x_{rd1}, x_{rd2}, \dots, x_{rdq}, \dots, x_{rdt}); j = 1, 2, \dots, N \quad (4)$$

with:

$$x_{rdq} = x_{rd}(j + (l-1)\tau); l = 1, 2, \dots, ? \quad (5)$$

and:

$$N = I - (v-1)\tau \quad (6)$$

where,  $N$  is the number of phase trajectories,  $v$  is the delay time which can be solved by the autocorrelation function method (Lv *et al.*, 2002) and  $\tilde{o}$  is the embedded dimension which is given by:

$$v = \frac{I}{\tau} \quad (7)$$

The phase trajectory  $X_{rd}(j)$  in Eq. 4 is one eigen trajectory. It can be employed for essential revelation of original dynamics of the poor information process in service.

According to bootstrap (Xia, 2012; Efron, 1979), an equiprobable resampling with replacement from  $X_{rd}(j)$  is implemented by following steps:

- Step 1:** Let the constant  $B$  be equal to 500000 and let the variable  $b$  take a value 1, where  $B$  is the number of the resampling samples and  $b$  is the  $b$ th equiprobable resampling
- Step 2:** Let one datum be drawn by an equiprobable resampling with replacement from  $X_{rd}(j)$
- Step 3:** Let the step 2 be repeated  $\tilde{o}$  times, so that  $\tilde{o}$  data can be sampled
- Step 4:** Calculate the mean  $y_{rd}(b)$  of  $\tilde{o}$  data which is considered as one of the data in the generated data series  $Y_{rd}$
- Step 5:** Add 1 to  $b$
- Step 6:** If  $b > B$ , go to the step 7; otherwise go to the step 2
- Step 7:** Let the generated data series be of size  $B = 500000$ , so that many generated data are obtained

Via steps 1-7, the generated data series  $Y_{rd}$  is gained, as follows:

$$Y_{rd} = (y_{rd}(1), y_{rd}(2), \dots, y_{rd}(b), \dots, y_{rd}(B)) \quad (8)$$

with:

$$y_{rd}(b) = \frac{1}{v} \sum_{i=1}^{\tilde{o}} \theta_b(i); b = 1, 2, \dots, B \quad (9)$$

where,  $\theta_b(i)$  is the  $i$ th data obtained and  $y_{rd}(b)$  is the mean of  $v$  data in the  $b$ th sampling.

The origin moment of  $X_{rd}(j)$  is as follows:

$$M_{rdm} = \frac{1}{B} \sum_{b=1}^B (y_{rd}(b))^m; m = 1, 2, \dots, M_{rd} \quad (10)$$

where  $M_{rd}$  is the highest order of the origin moments and  $M_{rdm}$  is the  $m$ th order origin moment.

Assume  $x$  is a random variable for describing performance data of the poor information process. According to maximum entropy theory, a probability density function  $f_{rd}(x)$  is obtained by:

$$f_{rd}(x) = \exp \left( \sum_{k=0}^{M_{rd}} c_{rdk} x^k \right) \quad (11)$$

where,  $c_{rdk}$  is the  $k$ th Lagrangian multiplier about  $X_{rd}$  and  $k = 0, 1, \dots, M_{rd}$ .

In Eq. 11, the Lagrangian multiplier  $c_{rdk}$  ( $k = 1, 2, \dots, M_{rd}$ ) can be solved by:

$$M_{rdm} = \frac{\int_{R_{rd}} x^m \exp \left( \sum_{k=1}^{M_{rd}} c_{rdk} x^k \right) dx}{\int_{R_{rd}} \exp \left( \sum_{k=1}^{M_{rd}} c_{rdk} x^k \right) dx}; m = 1, 2, \dots, M_{rd} \quad (12)$$

The first Lagrangian multiplier  $c_{rd0}$  can be obtained by:

$$c_{rd0} = -\ln \left( \int_{R_{rd}} \exp \left( \sum_{m=1}^{M_{rd}} c_{rdm} x^m \right) dx \right) \quad (13)$$

where,  $R_{rd}$  is the integrating range of  $x$  about  $X_{rd}(j)$ .

Let  $r = 1$  in Eq. 11, then the probability density function of  $X_{1d}(j)$  about the first time series  $X_1$  is obtained as:

$$f_{1d}(x) = \exp \left( c_{1d0} + \sum_{m=1}^{M_{1d}} c_{1dm} x^m \right) \quad (14)$$

For the first time series  $X_1$ , let  $X_{1d}(j)$  be both a prior sample and a current sample and  $f_{1d}(x)$  be both a prior distribution and a current sample distribution. According to Bayesian statistics, the posterior probability density function of  $X_{1d}(j)$  is obtained as:

$$\phi_{1d}(x) = \frac{f_{1d}(x)f_{1d}(x)}{\int_{R_{1d}} f_{1d}(x)f_{1d}(x)dx} \quad (15)$$

According to statistics, the mathematical expectation  $E_{1d}$  of  $X_{1d}(j)$  is defined as:

$$E_{1d} = \int_{R_{1d}} x \phi_{1d}(x) dx \quad (16)$$

and the variance  $D_{1d}$  of  $X_{1d}(j)$  is defined as:

$$D_{1d} = \int_{R_{1d}} (x - E_{1d})^2 \phi_{1d}(x) dx \quad (17)$$

According to the minimum variance principle, the minimum variance  $D_{1min}$  is given by:

$$D_{1min} = \min(D_{1,1}, D_{1,2}, \dots, D_{1,d}, \dots, D_{1,D}) \quad (18)$$

For the first data series, suppose the phase trajectory with the minimum variance  $D_{1min}$  is marked by  $X_{1min}$  and the posterior probability density function of  $X_{1min}$  is marked by  $\phi_{1min}(x)$ . Define  $X_{1min}$  and  $f_{1min}(x)$  as the referenced sequence and the referenced distribution, respectively.

For the  $r$ th time series ( $r = 2, 3, \dots, R$ ), let  $X_{rd}(j)$  and  $f_{rd}(x)$  be the current sample and current sample distribution, respectively, then according to Bayesian statistics the posterior probability density function  $\phi_{rd}(x)$  of  $X_{rd}(j)$  is as follows:

$$\phi_{rd}(x) = \frac{f_{1min}(x)f_{rd}(x)}{\int_{R_0} f_{1min}(x)f_{rd}(x)dx}; r = 2, 3, \dots, R \quad (19)$$

where,  $R_0$  is the integrating range of  $x$ .

According to statistics, the mathematical expectation  $E_{rd}$  of  $X_{rd}(j)$  is defined as:

$$E_{rd} = \int_{R_0} x \phi_{rd}(x) dx; r = 2, 3, \dots, R \quad (20)$$

and the variance  $D_{rd}$  is defined as:

$$D_{rd} = \int_{R_0} (x - E_{rd})^2 \phi_{rd}(x) dx; r = 2, 3, \dots, R \quad (21)$$

Variance ratio of  $X_{rd}(j)$  to  $X_{1min}$  is defined as:

$$\lambda_{1,rd} = \frac{D_{rd}}{D_{1min}}; r = 2, 3, \dots, R \quad (22)$$

In the light of concept of intersection of fuzzy sets, a mutation probability  $\alpha_{1,rd}$  is defined as:

$$\alpha_{1,rd} = 1 - A(\phi_{rd}(x) \cap \phi_{1min}(x)) \quad (23)$$

where  $A(\phi_{rd}(x) \cap \phi_{1min}(x))$  stands for the area of the intersection of  $\phi_{rd}(x)$  and  $\phi_{1min}(x)$ .

The mutation probability  $\alpha_{1,rd}$  can take values in  $[0, 1]$ . Let significance level be  $\alpha = 0.1$ , then significance testing for performance variation of the poor information process in service can be conducted.

If:

$$\alpha_{1,rd} > \alpha \quad (24)$$

then variation of the  $d$ th sub-series  $X_{rd}$  is of significance; otherwise, variation of the  $d$ th sub-series  $X_{rd}$  is of no significance.

The mutation probability can be used to test feature information on performance variation of time series with poor information for the early detection of the hidden danger, thus avoiding serious accident.

### CASE STUDIES

This case involves with an experiment on vibration acceleration of a rolling bearing for space applications. The rolling bearing that was installed on a specialized performance rig worked for 46 days (time interval: 8 November 2010 to 23 December 2010, running conditions of axial load of 49N and of rotational speed of 1000 r/min) and test data, in dB, were sampled 10 times (viz.,  $R = 10$ ), one time every 5 days and 4000 data (viz.,  $H = 4000$ ) every time, as shown in Fig. 1 and 2.

It is easy to see from Fig. 1 and 2 that as time series, information of rolling bearing vibration acceleration presents a complex and variational status, with an unknown probability distribution and trend.

From Fig. 1 and 2, every 400 data are considered as a sub-series, viz.,  $I = 400$  and 4000 data in the first sub-series  $X_1$  are regarded as prior information that includes ten sub-series,  $X_{1,1}, X_{1,2}, \dots, X_{1,d}, \dots, X_{1,10}$  (including 400 data in every sub-series).

From Eq. 1-7,  $\tau = 4$  and  $v = 100$  and the  $j$ th phase trajectory  $X_{1d}(j)$  in the multidimensional information space can hence be obtained. Using Eq. 16 and 17, the mathematical expectation  $E_{1d}$  and the variance  $D_{1d}$  of  $X_{1d}(j)$  are calculated for selection of the referenced sequence  $X_{lmin}$  and results show that the fourth sub-series, viz.,  $X_{lmin} = X_{1,4}$ , can be selected as the referenced sequence due to its minimum variance ( $D_{1,4} = D_{lmin} = 7.0140 \times 10^{-5}$ ). Based on this, with the help of Eq. 20, 21 and 23, the mathematical expectation, the variance ratio and the mutation probability are obtained as shown in Fig. 3, 4 and 5, respectively.

From Fig. 5, overall, from beginning of the second sub-series (corresponding to abscissa values 1 to 10 in Fig. 5) to end of the third sub-series (corresponding to

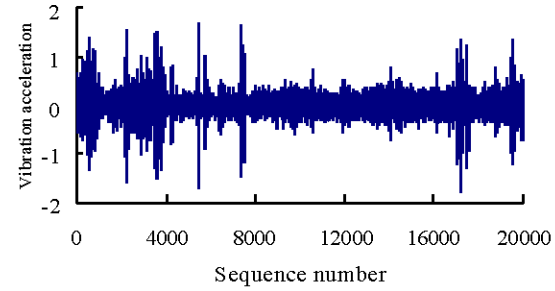


Fig. 1: Experimental data of time series from  $X_1$  to  $X_5$

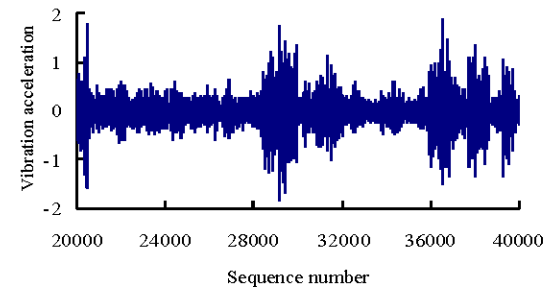


Fig. 2: Experimental data of time series from  $X_6$  to  $X_{10}$

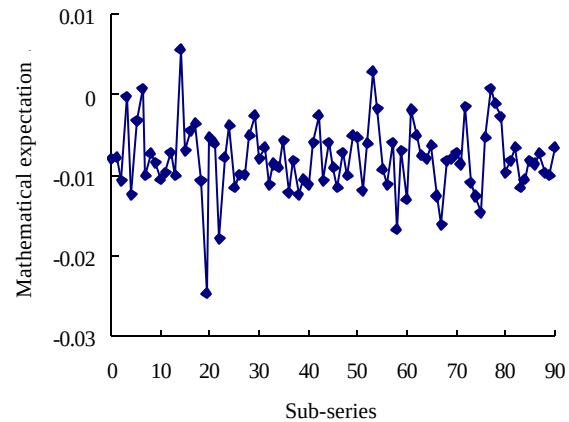


Fig.3: Mathematical expectation

abscissa values 11 to 20 in Fig. 5), the mutation probability is in a rising trend; from beginning of the fourth sub-series (corresponding to abscissa values 21 to 30 in Fig. 5) to end of the ninth sub-series (corresponding to abscissa values 71 to 80 in Fig. 5), the mutation probability that takes values in the range from 0.013 to 0.528 is in a large fluctuation and from beginning of the tenth sub-series (corresponding to abscissa values 80 to 90 in Fig. 5), the mutation probability that takes values about 0.1 is in a new stability. As a result, variation information of rolling bearing vibration acceleration is tested as follows:

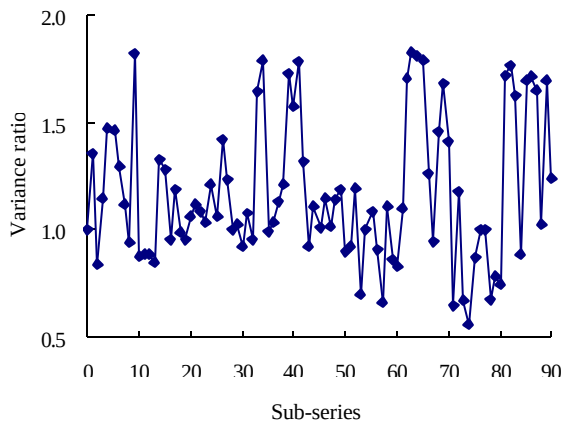


Fig. 4: Variance ratio

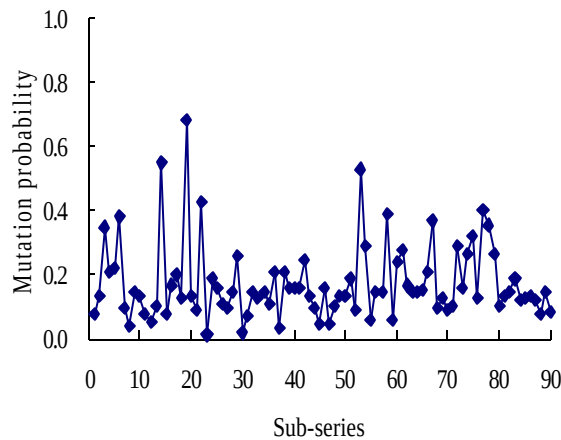


Fig. 5: Mutation probability

- From 8 November to 18 November, vibration performance variation becomes gradually significant, showing an early degradation phase
- From 23 November to 18 December, vibration performance variation is complex and variable, alternating significance and no significance and revealing a transitional period from early degradation phase to gradual degradation phase
- On 23 December, vibration performance variation is not significant, meaning a start of gradual degradation phase

It can be seen from the above that the method proposed is able to test feature information on rolling bearing performance variation.

## CONCLUSION

The dynamical Bayesian significance testing method, under the condition of unknown probability

distributions and trends in advance, can examine feature information on performance variation of time series with poor information for the early detection of the hidden danger, thus avoiding serious accident. Experimental investigation on vibration acceleration of the rolling bearing for space applications shows correctness of the method.

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