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## A New Coefficient of Evidential Conflict

Jian Ling Xu

Center for Food Security and Strategic Studies, Nanjing University of Finance and Economics,  
 Jiangsu, Nanjing, 210003, China

**Abstract:** Since, the widely-used applications of D-S theory in multi-sensor data fusion, various effective methods to overcome the inconsistent combination result of different Bodies Of Evidence (BOE), due to evidential conflict, have been proposed. In view of some of their deficiencies, this study, however, introduces a new evidential conflict coefficient, drawing on the theory of evidential distance, can effectively represent the evidential conflict, with numerical examples eventually demonstrated.

**Key words:** Data fusion, evidential theory, evidential conflict, conflict representation

### INTRODUCTION

D-S evidence theory (Dempster, 1967; Shafer, 1976), powerful in processing information of imprecision and uncertainty, has acquired extensive applications in areas of object recognition and tracking in multi-sensor systems. Powerful as it may be, applications of this theory will still be limited if problems of evidential conflict remain unsettled. Previous efforts have been made to present the degree of evidential conflict and attenuate its deficiency in combination. However, its effectiveness can be further improved with new algorithms proposed.

This study is organized as follows. In the first part, we review the basic knowledge of the theory of evidence and formerly presented methods addressing the degree of evidential conflict. In the second part, we propound a new coefficient based on evidential distance and its characteristic features. In the third part, we demonstrate the advancement of this new coefficient by comparing it with previously introduced solutions and finally come to our conclusion.

### PRELIMINARIES

**Dempster shafer evidence theory:** Evidence theory first supposes the definition of a set of hypotheses  $T$  called the frame of discernment, defined as follows:

$$T = \{t_1, t_2, \dots, t_N\} \quad (1)$$

It is composed of  $N$  exhaustive and exclusive hypotheses. From the frame of discernment  $T$ , let us denote  $P(T)$ , the power set composed with the  $2^N$  propositions  $A$  of  $T$ :

$$P(T) = \{\emptyset, \{t_1\}, \{t_2\}, \dots, \{t_N\}, \{t_1 \cup t_2\}, \{t_1 \cup t_3\}, \dots, T\} \quad (2)$$

where,  $\emptyset$  denotes the empty set. The  $N$  subsets containing only one element are called singletons. A key point of evidence theory is the Basic Probability Assignment (BPA). A BPA is a function from  $P(T)$  to  $[0, 1]$  defined by:

$$m: P(T) \rightarrow [0, 1] \quad (3)$$

and which satisfies the following conditions:

$$\sum_{A \in P(T)} m(A) = 1 \quad (4)$$

$$0 \leq m(A) \leq 1 \quad (5)$$

$$m(\emptyset) = 0 \quad (6)$$

BPA signifies the degree of support of  $A$  in the midst of discernment frame by one evidence, ie  $m(A)$ . As for  $\forall A \subset T$ , if  $m(A) \geq 0$ , then we call it a focus element. A set with all focus elements within, we call it core.

Dempster's rule of combination (also called orthogonal sum), noted by  $m = m_1 \oplus m_2$ , is the first one within the framework of evidence theory which can combine two BPAs to yield a new BPA:

$$m(A) = \frac{\sum_{B \cap C \cap D \dots = A} m_1(B)m_2(C)m_3(D) \dots}{1 - k} \quad (7)$$

with:

$$k = \sum_{B \cap C \cap D \dots = \emptyset} m_1(B)m_2(C)m_3(D) \dots \quad (8)$$

In (8)  $k$  reflects the degree of evidential conflict. Closer the  $k$  value towards 1, higher the conflict of evidence and more contradictory the combination result with our common sense judgment.

Previous methods to present evidential conflict. We now come to an example. Supposing a discernment frame  $T = \{x, y, z\}$ , with two evidence listed as follows:

$$\begin{aligned} E_1 : m_1(x) &= 0.9, m_1(y) = 0.1, \\ E_2 : m_2(z) &= 0.9, m_2(y) = 0.1 \end{aligned}$$

According to the combination algorithms already stated,  $k = 0.99$ ; (indicating highly conflict),  $m\{x\} = 0$ ,  $m\{y\} = 1$ ,  $m\{z\} = 0$ .

Clearly at a glance, a low belief proposition  $\{y\}$  gains the highest belief, whereas high belief value of  $\{x\}$  and  $\{z\}$  vanishes into zero. Further more, when completely conflicted, ie.,  $k = 1$  and denominator of (8) is zero, D-S combination is invalid.

To better solve such problem, Yager considered to remove  $1/1-k$  and attribute  $k$  value utterly to  $m(\Omega)$  (Yager, 1987), but unideal when dealing with more than 2 evidences. Fabio and Sergio (Dubois and Prade, 1992) introduced a evidential conflict function  $1 + \log(1/k)$ , proportionally assigning  $k$  value to  $m(\Omega)$  so as to make conflicted evidences partially available (Campos and Cavalcante, 2003). Dubois and Prade put forward Disjunctive Consensus Rule (Dubois and Prade, 1992), although denying no evidences, yet enlarging the uncertainty of proposition meanwhile lowering the decision precision.

## PROPOSED METHOD TO CREATE A NEW COEFFICIENT OF EVIDENTIAL CONFLICT

In order to further allow for the discrepancy between evidences, in other words, the impact BPAs have on the conflict, we adopt the idea of evidential distance proposed by Jousselme *et al.* (2001).

Let  $T$  be a complete discernment frame of mutually exclusive elements, containing  $N$  elements and  $S_{P(T)}$  is a space spanned by all the subsets of  $T$ . Then define a BPA vector  $m$  in  $S_{P(T)}$  with  $m(A_i)$  as its coordinates, which satisfies the following conditions:

$$\sum_{i=1}^{2^N} m(A_i) = 1, m(A_i) \geq 0, A_i \in P(T) \quad (9)$$

where,  $i = 1, 2^N$ .

$m_1$  and  $m_2$  are two BPAs based on discernment frame  $T$ , thus we define the distance between  $m_1$  and  $m_2$ :

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(m_1 - m_2)^T \hat{D}(m_1 - m_2)} \quad (10)$$

where,  $\hat{D}$  is a matrix of  $2^N \times 2^N$ , in which one element can be expressed as:

$$d(X, Y) = \frac{\sqrt{2}|X \cap Y|}{\sqrt{|X|^2 + |Y|^2}}, X, Y \in P(T) \quad (11)$$

$d_{BPA}$  can effectively indicate the overall influence of different subsets and BPAs, thus characterizing the discrepancy.

Combining the reasonableness of previously defined conflict and the effect of discrepancy between two BOEs (Body of Evidences), this study brings forward a new evidential conflict expression:

$$k_{ij}^* = \sqrt{\frac{k_{ij}^2 + d_{BPA}^2(m_i, m_j)}{2}} \quad (12)$$

where,  $k_{ij}$  represents the classical conflict coefficient in the D-S theory, resulting from (8).  $d_{BPA}(m_i, m_j)$  symbolizes the newly defined distance between  $E_i$  and  $E_j$ , available via (10) and (11).

## EXPERIMENTS

In this section, we'll give examine the effectiveness of  $k_{ij}^*$ .

An example of comparison

Let  $T = \{1, 2, 3, \dots, 15\}$  be a discernment frame and two BPAs be assigned as follows:  $m_1(7) = 0.05$ ,  $m_1(L) = 0.9$ ,  $m_2\{1, 2, 3, 4\} = 1$ .

Note that  $L$  successively stands for the rest subsets, ie.,  $\{1\}$ ,  $\{1, 2\}$ ,  $\{1, 2, 3\}$ ,  $\{1, 2, 3, 4\}$ , ...,  $\{1, 2, 3, \dots, 15\}$  for comparisons in the table below.

Obviously, we can draw the conclusion from Table 1 and Fig. 1, that:

- Classical conflict coefficient  $k$  equals to 0.1, regardless of the variations of subset  $L$ , which is totally contradictory to our common sense judgment
- The new coefficient  $k_{ij}^*$  varies with the variation of  $L$ . Clearly, when  $L = \{1, 2, 3, 4\}$ , the coefficient reaches its minimum

Classical coefficient  $k = 0.1$ , indicating a low conflict between evidences, inconsistent with reality. With the variation of  $L$ , the new coefficient  $k_{ij}^*$  reaches its maximum of 0.5642, which effectively signifies the rather high conflict between evidences.

Table 1: Comparison between new conflict coefficient and classical representation

L	$d_{BPA}$	$k_{ij}^*$	k
{1}	0.7916	0.5642	0.1
{1,2}	0.5930	0.4252	0.1
{1,2,3}	0.3826	0.2796	0.1
{1,2,3,4}	0.0500	0.0791	0.1
{1,...,5}	0.3365	0.2482	0.1
{1,...,6}	0.4553	0.3296	0.1
{1,...,7}	0.5435	0.3908	0.1
{1,...,8}	0.6000	0.4301	0.1
{1,...,9}	0.6436	0.4606	0.1
{1,...,10}	0.6784	0.4849	0.1
{1,...,11}	0.7067	0.5047	0.1
{1,...,12}	0.7302	0.5212	0.1
{1,...,13}	0.7500	0.5350	0.1
{1,...,14}	0.7669	0.5469	0.1
{1,...,15}	0.7815	0.5571	0.1

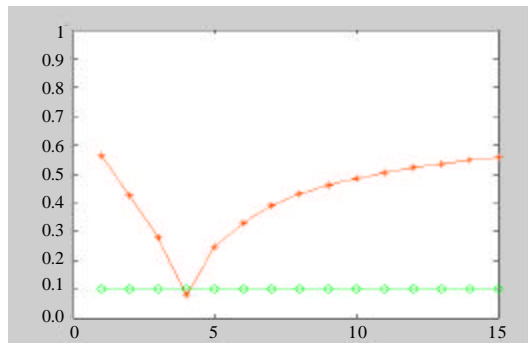


Fig. 1: Comparison between two conflict coefficients

Such result may attribute to the fact that the classical coefficient merely reflects the exclusiveness among evidences. In the example above, the focal element L always intersects with {1, 2, 3, 4}, which therefore determines the k to be a rather small value. Whereas the newly introduced coefficient takes both exclusiveness and discrepancy between evidences into allowance and get a more logical result. Similar to the expression of classical coefficient  $k_{ij}^*$ ,  $k_{ij}^*$  varies between [0, 1]. Larger the value, higher the conflict, vice versa.

## CONCLUSION

In dealing with combination problems with high degree of conflict, this study comes up with another new idea involved in the research of conflict itself, besides the clue of combination rules research. This new method gives consideration to both exclusiveness and discrepancy of evidences and effectively represents the degree of evidential conflict. In future, some modified

models and more advanced methods, which may consider other appropriate influencing factors to give a more comprehensive expression, will be expected.

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