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Fuzzy Simulation on the Vehicle Routing Problem

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Abstract: In this study, the vehicle routing problem with fuzzy demands is considered, and a fuzzy chance constrained programming mathematical model is established based on fuzzy possibility theory. Then fuzzy simulation and differential evolution algorithm are integrated to design a hybrid intelligent algorithm to solve the fuzzy vehicle routing model. Moreover, under the target that the total driving distance of vehicles is the shortest, the influence of the decision-maker's preference on the final objective of the problem is discussed using the method of stochastic simulation, and the rational range of the preference number is obtained.

Key words: Vehicle routing problem, fuzzy possibility, fuzzy simulation, differential evolution, optimization

INTRODUCTION

The vehicle Routing Problem (VRP) belonging to the classic complex combinatorial optimization problem is proposed by Dantzig and Ramser (1959). Teodorovic and Pavkovic (1996) used the fuzzy reasoning algorithm by introducing decision-makers preferences to solve the problem of fuzzy customer demands (Teodorovic and Pavkovic, 1996). Chen and Gen used genetic algorithm to solve the fuzzy appointment vehicle routing problem (Zheng and Liu, 2006). Zheng and Liu (2006) used a hybrid genetic algorithm to study the fuzzy time travel vehicle routing problem (Cheng and Gen, 1995). Based on fuzzy possibility theories, Zhang Jianyong studied the vehicle routing problem of fuzzy demands by hybrid genetic algorithm (Storn,1996).

In recent years, the fields of evolutionary computation, differential evolution algorithm as a performance optimization algorithm are being increasingly concerned. And their application areas are becoming more and more widely (Liu, 2004). Compared with genetic algorithm, the characteristic of Differential Evolution Algorithm (DEA) is that individuals of each new generation use the process of the parent linear combination of multiple individuals, rather than the traditional genetic algorithm for single-parent chromosome cross-technology. At the same time, DEA keeps down global search strategy that used real-coded, based on differential mutation and one to one simple survival competition strategy, which reduce the genetic complexity of the operation. But there are no differential evolution algorithms used for vehicle routing problem now.

In this article, based on the fuzzy possibility theory, we advance the fuzzy chance constrained programming model of the Fuzzy Vehicle Routing Problem (VRPFD) and use the hybrid differential evolution algorithm to solve thevehicle routing problem. At last, the results show that the algorithm can effectively solve the fuzzy vehicle routing problem and the improved differential evolution algorithm also canbe used to solve other deterministic vehicle routing problems.

PROBLEM'S DESCRIPTION AND MODEL

A vehicle routing problem with fuzzy demands can be described as: A service station and n customers that respective by 0, 1, …, n ina transportation network, the vehicles start from the depot, and come back yard after serving a certain number of customers. Each vehicle has the same capacity (C) and the maximum traveled distance (L). Each customer can only be done once by a car service. Each vehicle can only be used once. Each customer demands for the fuzzy triangular number $d = (d_1, d_2, d_3)$, the distance between customers i and j is c_{ij}. k is the maximum number of vehicles for the distribution center. We can find the shortest running route after vehicles complete all the customer service. We consider the loading of vehicles service to each customer's (goods filled with the car in the parking when delivery. If next customer demand is greater than the cargo vehicle, the vehicle come back yard, similarly considerations). Since, each customer's needs is fuzzy numbers, for agiven customer i the demand fuzzy numbers is $d_i = (d_{1i}, d_{2i}, d_{3i})$. A vehicle's total transport load is:

$$d'_k = \sum_{i=1}^{k} d_i$$

after services the number of kcustomers. Vehicle carrying capacity of the remaining:

$$Q_k = C - \sum_{i=1}^{k} d_i$$

is also a triangle fuzzy number and:

$$Q_k = (C - \sum_{i=1}^{k} d_{3i}, C - \sum_{i=1}^{k} d_{2i}, C - \sum_{i=1}^{k} d_{1i})$$

That demanding for transport capacity of the next node is less than the possibility p =pos ($d_{k+1} \leq Q_k$) of the remaining vehicles can be expressed as:

$$\begin{aligned} p &= pos\{d_{k+1} \leq Q_k\} \\ &= \sup\{\min\{ud_{k+1}(x), uQ_k(y)\} x \leq y\} \\ &= \begin{cases} 1 & d_{2,k+1} \leq q_{2k} \\ \dfrac{q_{3k} - d_{1,k+1}}{(q_{3k} - q_{2k}) + d_{2,k+1} - d_{1,k+1})}, & d_{2,k+1} \geq q_{2k}, d_{1,k+1} \leq q_{3k} \\ 0 & d_{1,k+1} \geq q_{3k} \end{cases} \end{aligned}$$

For a given value α, possibility that demand of the next customer is less than vehicle transport capacity is p. During the arrangement of the vehicle path, when $p \leq \alpha$, we can send a car to continue transporting the customer to complete the next task, if $p < \alpha$, then the car go back yard, and the new transportation is sent a car to complete the remaining tasks. Repeat the process until the customer permutation is arranged end to all customers, so the feasible vehicle arrangement can be produced. But in the actual delivery process, because of the ambiguity of the demand, when the vehicle reaches a feasible path to customers by plan, there may be that residual transport capacity of the vehicle cannot meet the customer demands and then lead the mission fail. The vehicle can only return to the depot after failing to complete the remaining transport tasks, resulting in extra traveling distance. Suppose:

$$x_{ijk} = \begin{cases} 1, \text{vehicle k directly to customer j from customer i} \\ 0, \text{else} \end{cases}$$

$$y_{ik} = \begin{cases} 1, \text{ the task of customer i completed by vehicle k} \\ 0, \text{else} \end{cases}$$

Based on possibility theory fuzzy chance constrained programming model of VRPFD is:

$$\min \sum_{k=1}^{\bar{k}} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ijk} \tag{2}$$

$$\min c' \tag{3}$$

$$\text{s.t. } pos(\sum_{i=1}^{n} d_i y_{ik} \leq C) \geq \alpha, k = 0,1,\cdots,\bar{k} \tag{4}$$

$$\sum_{i=0}^{n} \sum_{k=1}^{\bar{k}} x_{ijk} = 1, j = 1,2,\cdots,n \tag{5}$$

$$\sum_{i=0}^{n} x_{ijk} - \sum_{i=0}^{n} x_{jik} = 0, j = 0,1,\cdots,n; k = 0,1,\cdots k \tag{6}$$

$$\sum_{j=1}^{n} x_{ojk} \leq 1, k = 1,2,\cdots,\bar{k} \tag{7}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{jk}, j = 0,1,\cdots,n; k = 0,1,\cdots \bar{k} \tag{8}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{ik}, i = 0,1,\cdots,n; k = 0,1,\cdots \bar{k} \tag{9}$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} d_{ij} x_{ijk} \leq L, k = 0,1,2,\cdots,\bar{k} \tag{10}$$

$$x_{ijk} \in \{0,1\}, y_{ik} \in \{0,1\}, i,j = 0,1,\cdots,n; k = 0,1,\cdots,\bar{k} \tag{11}$$

In the above expression, the objective function (2) is the minimized planed traveling distance. The objective function (3) is the minimized additional traveling distance because of "path to failure", and the value generated by the fuzzy simulation. Equation 4 describes that each vehicle does not exceed the carrying capacity of the possibility which should be withinthe confidence interval. Equation 5 describes that each client completes only once by a car service. Equation 6 describes that the vehicles reached and left are same. Equation 7 shows that only the k vehicles can be used. Equation 8 and 9 shows the relationship between the two decision variables. Equation (10) is the traveling distance bound of per vehicle, L is the maximum driving distance of per vehicle. Equation 11 is the properties of the decision variable.

HYBRID ALGORITHM OF VRPFD

For each given value of subjective preference α, within the transport capacity of the vehicle n triangular fuzzy numbers are randomly generated as per customer's demand. First we can use the differential evolution algorithm to obtain expected driving distance under fuzzy demand of customers, and then use fuzzy simulation algorithm to computer the additional driving distance returning yards when the occurrence of "path failure" as customers' demands are more than the remaining capacity of the vehicle, lastly the sum of both will become the total traveling distance, then use the difference evolutionary algorithm to obtain theminimum total traveling distance under the subjective preference value. In the following, we will study the effect of different subjective preference values on the traveling distance of vehicles.

Fuzzy simulation algorithm: For any "feasible" path arrangement (with the chromosome denotation), using fuzzy simulation to estimate the extra driving distance (c') created by its "path failed", the basic steps are as follows:

- **Step 1:** For each customer, the additional traveling distance can be generated from "real" demand, which is produced by the simulation method, the steps are:
  - Generate a random number x in the range of a customer's fuzzy demand numbers and calculate the membership u
  - Generate a random number α within [0,1]
  - Compare α to u, If α is less than u, then x is the actual demands of customers. Otherwise, repeat the above steps
  - Repeat the above steps until the generated simulation of all of customers' "real" demands
- **Step 2:** Calculate the additional traveling distance caused by failure of path arrangement under the "real" demand conditions
- **Step 3:** Repeat step 1, step 2 M times
- **Step 4:** Calculate the average of M simulation values, which can be as the estimated value of extra traveling distance caused by failure of path arrangement

Improved differential evolution algorithm: DEA is a optimization algorithm proposed by Storn (1996) using floating-point vector coding in the continuous space for random search to solve Chebyshev polynomials [6,7]. Specific steps are as follows:

Generate initial population: Chrom (I, :) = randperm (n); i = 1, 2, ..., NP, where n is the number of customers served, NP is the population size. The initial feasible population can be generated as follows:

- **Step 1:** Generate random customer order
- **Step 2:** Remove the most left one of the customer order, according to the customer demands and the remaining transport capacity of the current vehicles, calculated p by equation (1). For a given α, if $p\geq\alpha$, then assign the customer to the current vehicle; Otherwise, assign the customer to a new vehicle
- **Step 3:** Remove the customer from the customer order
- **Step 4:** Repeat step 2, step 3, until the completion of arrangements for all customers, resulting in a feasible chromosome
- **Step 5:** Repeat the process until producing given NP feasible chromosome

Mutation: Let the gene value of the resulting chromosomal Chrom (i, :) (I = 1, 2,..., n) the real number in (min, max), where min is the smallest gene value, max is the largest gene value. First sort each chromosome by the gene value in descending order, the largest gene value becomes the largest customer n, the second largest gene value corresponding n-1 and so on, until the smallest gene value to 1.

Crossover operation: Crossover operation is to increase the diversity of population, for the target vector individual Chrom (i, :) in population, made crossover operation with the variation vector v, resulting in detectional individual trial. The equation for the crossover operation is as following:

$$\text{trial}(j)^{G+1} = \begin{cases} v(j)^{G+1}, & \text{rand}(j) \leq CR \text{ or } j = \text{randn}(i) \\ \text{Chrom}(i,:)^{G}, & \text{rand}(j) > CR \text{ and } j \neq \text{randn}(i) \end{cases}$$

Where CR is the cross rate. Obviously the larger CR is, the contribution of v to trial is more. When CR = 1, trial = v, which is conducive to local searching and speeding up the convergence rate; CR is smaller, the more the contribution of the Chrom (i, :) to trial. When CR = 0 , trial = Chrom (i, :), which is conducive to maintaining species' diversity and global searching. In order to improve the performance of differential evolution algorithm, let:

$$CR = CR_{min} + g\frac{CR_{max} - CR_{min}}{G}$$

and CR_{max} is the smallest crossover probability, CR_{max} is the maximum crossover probability; g is current evolutionary generation; G is the total evolution of algebra.

Selection operation: Expected traveling distance (c) can be obtained according to Eq. 2 and extra traveling distance (c') can be obtained according to Eq. 3, the sum of both is the total distance, and make the inverse of the value as the value of fitness , that is f = 1/(c+c').

NUMERICAL EXPERIMENTS AND RESULTS ANALYSIS

Random experiment consists of 30 customers, each customer's location coordinates are generated randomly in the range of [100 100], fuzzy demands are triangular fuzzy numbers randomly generated within the transport capacity of the vehicles, all the cars' capacity is 8 tons, relevant parameters are supposed as following: n = 3, C = 8, k = 30, G = 200, M = 100, L = 2000, NP = 40, CR_{max} = 0.3CR_{max} = 0.9, F = 0.5. The hybrid differential evolution algorithm is implemented in MATLAB, In the above basic setting, the value of subjective preference parameter α varies between 0 and 1. Table 1 shows the average of 10 running results under each setting.

From Table 1, we can see that as subjectivethreshold α increases, expected traveling distance is strictly monotone increasing, moreover the extra traveling distance is strictly decreasing. But when $\alpha \le 0.6$, increment of expected traveling distance is less than decrement of extra traveling distance, so with α increasing, the total traveling distance decreases. When α>0.6, increment of expected traveling distance is more than decrement of extra traveling distance, so with α increasing, the total traveling distance increases and when α = 0.6, the total traveling distance is the smallest. Thesmaller the value of α, it is indicated that the subjective desire of full use of vehicles' capacity is more intense, expected traveling distance is much shorter. But at the same time,smaller α make the chance of failure performing task increases, and leads to more extradriving distance. On the contrary, larger α leads to expected driving distance increasing, extra traveling distance decresing. Therefore, during the actual fuzzy vehicle scheduling,under the target of total traveling distance minimized, decision-makers' subjective critical value should be choosed about 0.6.

VRPFD problems are randomly generated, and experimented by DEA with different parameters. After several experiments the following experience rules are concluded:

Table 1: Results under different values of α

α	Expected distance traveled	Additional driving distance	Total distance traveled
0.0	1396.7	2920.8	4317.5
0.1	2004.5	1396.6	3401.1
0.2	2061.1	1191.1	3252.2
0.3	2207.3	767.80	2975.1
0.4	2296.1	550.20	2816.3
0.5	2637.2	335.20	2972.4
0.6	2661.8	117.00	2778.8
0.7	2919.0	9.8000	2928.8
0.8	2959.2	3.5000	2962.7
0.9	3019.6	0	3019.6
1.0	3044.2	0	3044.2

- For the vehicle routing problem using sequence coding, operating results by DEA after mutation must be improved, if we use rounded down method or rounded up method or rounded method, which are not adapted to solve the problem
- The process crossover probability automatically adapted to the evolutionary make algorithm maintaining the population diversity in the initial stage of searching. After global search,we can obtain as much as possible global optimal solution, while in the later period of searching strengthen the local search capabilities to improve the algorithm accuracy
- In the process of parameters setting, the fuzzy simulation algorithm's computational times (M) , the number of evolutionary generation by DEA, the average computing number of hybrid algorithm affectrunning time of algorithm more deeply

CONCLUSION

In this study, fuzzy chance programming model for vehicle routing problem with fuzzy demands is established, and we propose a hybrid differential evolution algorithm to solve the problem. Meanwhile, as the selection of decision makers on the subjective preferences has a huge impact on the final decision, by random tests we study the effect of decision makers' subjective preferences on the final decision goal and make a conclusion that under the target of total traveling distance minimized, decision-makers' subjective critical value should be choosed about 0.6. Meanwhile because of the connection between fuzzy vehicle routing problem and other certain vehicle routing problem, we can see that the proposed modified differential evolution algorithm can be used to solve the certain vehicle routing problem.

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REFERENCES

Cheng, R. and M. Gen, 1995. Vehicle routing problem with fuzzy due-time by generic a lgorithm. Japanese J. Fuzzy Theory Syst., 7: 1050-1057.

Dantzig, G.B. and J.H. Ramser, 1959. The truck dispatching problem. Manage. Sci., 6: 80-91.

Liu, B., 2004. Uncertainty Theory: An Introduction to Its Axiomatic Foundations. Springer, Berlin, ISBN-13: 9783540213338, Pages: 411.

Storn, R., 1996. Differential evolution design of an IIR-filter. Proceedings of the IEEE International Conference on Evolutionary Computation, May 20-22, 1996, Nagoya, pp: 268-274.

Teodorovic, D. and G. Pavkovic, 1996. The fuzzy set theory approach to the vehicle routing problem when demand at nodes is uncertain. Fuzzy Sets Syst., 82: 307-311.

Zheng, Y. and B. Liu, 2006. Fuzzy vehicle routing model with credibility measure and its hybrid intelligent algorithm. Applied Mathe. Comput., 176: 673-683.