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Research on Characteristic Model for Multivariable Systems Based on Linear Superposition

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Abstract: The characteristic modeling method is a practical model reduction method by considering the system dynamics and the control purpose. In this study, the characteristic model of multi-variable linear time invariant systems is theoretically deduced based on linear superposition. The proposed characteristic models are second-order difference equations, all the dynamic information are compressed into the coefficients of the characteristic model. The coefficients of the characteristic model are analyzed in detail. The method on characteristic model provides a theoretical foundation for design of intelligent controllers and the control of higher order plants using lower order controllers.

Key words: Multivariable system, characteristic model, linear superposition

INTRODUCTION

Model reduction is very important in the field of control theory and control engineering. Plenty of results on this topic are reported in the literature, e.g., (Ghafoor and Sreeram, 2008; Knockaert and De Zutter, 2003; Obinata and Anderson, 2000; Phillips *et al*., 2003; Safonov and Chiang, 1989), just to name a few. In control engineering projects, such as in the fields of astronautics and industry, the systems to be controlled are often complex and high order systems (Wu *et al*., 2001, 2002, 2007). The control methods based on accurate models of the dynamic characteristics of the control plant is not ideal to solve the control problems of such kind of systems, because the accurate dynamic models of this kind of systems are very difficult to establish. Actually, it is often impossible to obtain the accurate dynamic model. In order to overcome these problems in the control engineering, Academician of Chinese Academy of Sciences, Wu, H.X. proposed and developed a so-called characteristic modeling method to reduce a high order model of linear systems (Wu *et al*., 2009; Meng and Wu, 2007). The characteristic modeling is based on the dynamics characteristics and control performance requirements of the systems, rather than only based on accurate dynamics analysis. In this study, the characteristic modeling for multi-variable linear time invariant linear system is deduced based on the characteristic modeling for the single-input and single-output systems.

CHARACTERISITC MODEL

Characteristic modeling establishes a mathematical model by the dynamic characteristics of the plant to be controlled and the control performance requirements (Wu *et al*., 2001). It includes the following four aspects:

- Under the same input, the characteristic model is equivalent to the practical plant
- The structure of the characteristic model is mainly selected by the requirements of control performance
- The structure of the characteristic model should be simpler than the original dynamics equation of the plant, it is easier and more convenient be to realize in engineering
- The characteristic model is different from the reduced order model, it compresses all the information of the high order model into several characteristic parameters, in general, the characteristic model is a timing-varying difference equation

Consider the multivariable linear time-invariant system:

$$\begin{cases} y_1(s) = G_{11}(s)u_1(s) + \cdots G_{1p}(s)u_p(s) \\ y_2(s) = G_{21}(s)u_1(s) + \cdots G_{2p}(s)u_p(s) \\ . \\ . \\ y_q(s) = G_{q1}(s)u_1(s) + \cdots G_{qp}(s)u_p(s) \end{cases} \tag{1}$$

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where, $u(s)\epsilon R^p$ is the control input and $y(s)\epsilon R^q$ is the control output.

Assumption: All the input and output of the system 1 are bounded and the derivatives of the input and output of the system 1 are also bounded.

It is clear that the system 1 can be written as:

$$\begin{cases} y_1(s) = y_{11}(s) + \cdots y_{1p}(s) \\ y_2(s) = y_{21}(s) + \cdots y_{2p}(s) \\ . \\ . \\ y_q(s) = y_{q1}(s) + \cdots y_{qp}(s) \end{cases} \tag{2}$$

Considering the first output:

$$y_1(s) = y_{11}(s) + \cdots y_{1p}(s) \tag{3}$$

$y_1(s)$ can be seen as the linear superposition results of p single-input and single-output systems:

$$\begin{cases} y_{11}(s) = G_{11}(s)u_1(s) \\ . \\ . \\ y_{1p}(s) = G_{1p}(s)u_p(s) \end{cases} \tag{4}$$

Each equation of the Eq. 4 can be seen as a single-input and single-output system and each G_{1j} denotes the transformation function of each single-input and single-output system. Similarly, the other outputs of the system 1 can also be seen as the linear superposition results of p single-input and single-output systems.

Lemma: Wu *et al.* (2001) in certain sampling period, the SISO linear time invariant system need to achieve the control requirements of position- keeping or position-tracking, its characteristic model can be expressed by a second-order timing-varying difference equation:

$$\begin{aligned} &y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) \\ &+g_0(k)u(k) + g_1(k)u(k-1) \end{aligned} \tag{5}$$

It compresses all the information of the high order model into the coefficients.

So, the characteristic model of each SISO system of Eq. 4 can be expressed by a second-order timing-varying difference equation and the characteristic model of Eq. 4 can be expressed as:

$$\begin{cases} y_{11}(k+1) = f_{11.1}(k)y_{11}(k) + f_{11.2}(k)y_{11}(k-1) \\ +g_{11.1}(k)u_1(k) + g_{11.2}(k)u_1(k-1) \\ . \\ . \\ y_{1p}(k+1) = f_{1p.1}(k)y_{1p}(k) + f_{1p.2}(k)y_{1p}(k-1) \\ +g_{1p.1}(k)u_p(k) + g_{1p.2}(k)u_p(k-1) \end{cases} \tag{6}$$

The characteristic model of the system 3 can be obtained by summing all difference Eq. 6 and it can be expressed as:

$$\begin{aligned} &y_1(k+1) = f_{11}(k)y_1(k) + f_{12}(k)y_1(k-1) \\ &+\sum_{i=1}^{p} g(k)_{1i\bullet 1}(k)u_i(k) + \sum_{j=1}^{p} g(k)_{1j\bullet 2}(k)u_j(k-1) \end{aligned} \tag{7}$$

Where:

$$y_1(k+1) = y_{11}(k+1) + \cdots y_{1p}(k+1) \tag{8}$$

$$y_1(k) = y_{11}(k) + \cdots y_{1p}(k) \tag{9}$$

$$y_1(k-1) = y_{11}(k-1) + \cdots y_{1p}(k-1) \tag{10}$$

$$f_{11}(k) = \frac{f_{11.1}(k)y_{11}(k) + \cdots + f_{1p.1}(k)y_{1p}(k)}{y_{11}(k) + \cdots + y_{1P}(k)} \tag{11}$$

$$f_{12}(k) = \frac{f_{11.2}(k)y_{11}(k-1) + \cdots + f_{1p.2}(k)y_{1p}(k-1)}{y_{11}(k-1) + \cdots + y_{1P}(k-1)} \tag{12}$$

Denote the maximum of the coefficients of Eq. 6 $f_{11.1}(k)\ldots f_{1p.1}(k)$ as $f_{1p.1}(k)_{max}$, the minimum of the coefficients of Eq. 6 $f_{11.1}(k)\ldots f_{1p.1}(k)$ as $f_{1p.1}(k)_{min}$, by Eq. 11, we have:

$$\begin{aligned} &\frac{f_{1p.1min}(k)((y_{11}(k) + \cdots + y_{1p}(k))}{y_{11}(k) + \cdots + y_{1P}(k)} \le f_{11}(k) \\ &\le \frac{f_{1p.1max}(k)((y_{11}(k) + \cdots + y_{1p}(k))}{y_{11}(k) + \cdots + y_{1P}(k)} \end{aligned} \tag{13}$$

Thus, we have:

$$f_{1p.1}(k)_{min} \le f_{11}(k) \le f_{1p.1}(k)_{max} \tag{14}$$

Let maximum of the coefficients of Eq. 6 $f_{11.2}(k)\ldots f_{1p.2}(k)$ is $f_{1p.2}(k)_{max}$, the minimum of the coefficients of Eq. 6 $f_{11.2}(k)\ldots f_{1p.2}(k)$ is $f_{1p.2}(k)_{min}$. By Eq. 12, we have:

$$\begin{aligned} &\frac{f_{1p.2min}(k)((y_{11}(k-1) + \cdots + y_{1p}(k-1))}{y_{11}(k-1) + \cdots + y_{1P}(k-1)} \le f_{12}(k) \\ &\le \frac{f_{1p.2max}(k)((y_{11}(k-1) + \cdots + y_{1p}(k-1))}{y_{11}(k-1) + \cdots + y_{1P}(k-1)} \end{aligned} \tag{15}$$

That is:

$$f_{1p.2}(k)_{min} \le f_{12}(k) \le f_{1p.2}(k)_{max} \tag{16}$$

The coefficients of Eq. 7 $g_{11.1}(k) \ldots g_{1p.1}(k)$ are introduced in detail in (Wu *et al*., 2001), they are functions of the sampling period. Given any $\varepsilon \succ 0$, as long as the sampling period is little enough, we have:

$$|g_{1i.1}(k)| \prec \varepsilon \tag{17}$$

The coefficients of Eq. 7 $g_{11.1}(k) \ldots g_{1p.2}(k)$ are also introduced in detail by Wu *et al.* (2001). Given any $\varepsilon \succ 0$, as long as the sampling period is little enough, we have:

$$|g_{1i.2}(k)| \prec \varepsilon \tag{18}$$

According to the deduction of the characteristic model of the system 3, it is not difficult to obtain the characteristic model of the Eq. 1 which is given by the following second order difference equations:

$$\begin{cases} y_1(k+1) = f_{11}(k)y_1(k) + f_{12}(k)y_1(k-1) \\ +\sum_{i=1}^{p} g(k)_{1i\bullet1}(k)u_i(k) + \sum_{j=1}^{p} g(k)_{1j\bullet2}(k)u_j(k-1) \\ . \\ . \\ y_q(k+1) = f_{q1}(k)y_q(k) + f_{q2}(k)y_q(k-1) \\ +\sum_{i=1}^{p} g(k)_{qi\bullet1}(k)u_i(k) + \sum_{j=1}^{p} g(k)_{qj\bullet2}(k)u_j(k-1) \end{cases} \tag{19}$$

CONCLUSION

This study proposed a characteristic modeling method for multivariable linear time-invariant systems based on linear superposition, it is proved that the characteristic model of the multi-variable linear time invariant system can be expressed by a set of time varying difference equations of order 2, the coefficients of characteristic model have been discussed. And the problem of characteristic modeling of time varying systems and nonlinear systems needs to be further investigated.

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