http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (21): 6184-6190, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.6184.6190

Research on Risk Measure of Electricity Market Based on Armax-garch Model with Conditional Skewed-t Distribution and Extreme Value Theory

Ruiqing Wang
School of Computer and Information Engineering, Anyang Normal University,
Anyang, Henan 455000, China

Abstract: How to effectively evaluate price of volatility risk is the basis of risk management in electricity market. An ARMAX-GARCH model imposing a skewedt-t distribution with time-varying skewness and degree of freedom over the error terms (ARMAX-GARCH-ST) is proposed and used to filter electricity price series in order to capture the dependencies, seasonalities, heteroscedasticities, skewnesses, leptokurtosises, volatility-clustering and relationship to system loads. In this way, an approximately independently and identically distributed residual series with better statistical properties is acquired. Then Extreme Value Theory (EVT) is adopted to explicitly model the tails of the normalized residuals of ARMAX-GARCH-ST model and accurate estimates of electricity market Value-at-Risk (VaR) can be produced. The empirical analysis shows that the ARMAX-GARCH-EVT models can be rapidly reflect the most recent and relevant changes of spot electricity prices and can produce accurate forecasts of VaR at all confidence levels, showing better dynamic characteristics. These results present several potential implications for electricity markets risk quantifications and hedging strategies.

Key words: Value-at-risk, extreme value theory, skewed student-t distribution, probability distribution assumption, ARMAX-GARCH model

INTRODUCTION

The introduction of competitive mechanism has provided more lucrative opportunities for electricity market participants but also brought the price of volatility risk hitherto unknown at the same time. If price of volatility risk in electricity market cannot be effectively identified and managed, it is possible to cause disastrous consequences for electricity market participants (Bushnell, 2004).

With Value-at-risk (VaR) as the risk measure, the purchasing risk of electric utility is calculated using a normal distribution based Delta model (Zhang and Zhou, 2004.). Via capacity sufficient and must-run rate as exogenous variables to depict supply-demand conditions and generators' market power, a GARCH model with normal distribution innovations (N-GARCH) has been used to assess the price of volatility risk in electricity markets (Huang *et al*., 2009). Considering that N-GARCH models cannot effectively address the leptokurtosis and heavy-tailed phenomenon in the data of profit and loss, a resampling method based on bias-correction and bootstrap has been developed, further improving the VaR forecasting accuracy of the N-GARCH models (Hartz *et al*., 2006). Via GARCH-based model, the impacts of probability distribution assumption for innovations on VaR estimation accuracy are analyzed for three distributions: Normal, student-t and General Error Distribution (GED), showing that the GED performs better in predicting VaR but the mutil-seasonalities, higher moments and relationship with loads cannot be effectively addressed (Wang *et al*., 2012).

The above-mentioned models of estimating VaRs are all based on the probability distribution assumption for the whole sample. The accuracy and stability of estimated values are heavily dependent on the selection of probability distribution for innovations (Li and Sun, 2010). Extreme Value Theory (EVT) provides a firm theoretical foundation to study the asymptotical distribution of extreme value for order statistics, without assuming the probability distribution for the sample data. Bystrom (2005) extended the classic unconditional EVT approach by first filtering the data via GARCH specification to capture some of the dependencies in electricity return series and thereafter applying ordinary EVT techniques. In this way the Independently and Identically Distributed (IID) assumption behind the EVT-based tail-quantile estimator is less likely to be violated and the better tail estimates in-sample and better predictions of future extreme price changes can be acquired. To describe the leverage effects of volatility of electric power price, an EGARCH specification (Chan and Gray, 2006; Gong *et al*., 2009) is used to filter the return series to obtain nearly IID residuals, showing that EGARCH-EVT model can reflect the most recent changes of electricity prices and produce accurate forecasts of VaR in the more volatile markets where the distribution of returns is characterized by higher levels of skewness and excess kurtosis.

With careful consideration of the basic features of electricity prices, a time series analysis and EVT based two-stage model to estimate VaR (ARMAX-GARCH-EVT) is proposed. In the first stage, a GARCH model with exogenous variables and skewed-t innovations (ARMAX-GARCH-ST) is used to pre-filter the raw data to capture the dependences of electricity price series. In this way, an approximately IID residual series with better statistical properties is acquired. In the second stage, EVT is adopted to explicitly model the tails of the normalized residuals of ARMAX-GARCH-ST model and accurate estimates of VaR in electricity market can be produced. There are several contributions. First, the study proposes a model that has the potential to generate more accurate quantile estimates for electricity market. The seasonalities, heteroscedasticities, skewnesses, kurtosises and relationship to system loads of electricity prices are accommodated via an ARMAX-GARCH-ST specification. In forecasting VaR, EVT is applied to the standardized residuals from this model. Clearly, the proposed ARMAX-GARCH-EVT combination is a sophisticated approach to forecasting VaR. The second contribution is to acquire an approximately IID residual series with better statistical properties by using a conditional skewed-t distribution over the error terms which can more accurately depict the leptokurtosis and heavy-tail of electricity price series. The effectiveness of the VaR estimates via peaks over thresholds model (POT) can be further improved. The third contribution of this study is to compare the accuracy of VaR forecasts under the proposed model with a number of conventional approaches (respectively ARMAX-GARCH with Gaussian, student-t, skewed student-t and GED distribution). Tail quantiles are estimated under each competing model and the frequency with which realized returns violate these estimates provides an initial measure of model success. The empirical analysis based on the Pennsylvania-New Jersey-Maryland (PJM) historical data indicates that the ARMAX-GARCH-ST model can produce accurate forecasts of VaR at all confidence levels but the ARMAX-GARCH-EVT model can be more rapidly reflect the most recent and relevant changes of electricity prices and performs more strongly, showing better dynamic characteristics. These results suggest that the proposed approach is robust and therefore useful.

VAR ESTIMATION MODEL

Risk measures: Value-at-risk is one of the most intuitive and comprehensible risk measures. Assuming normal market conditions and no trading in a given portfolio, VaR is defined as a threshold value such that the probability that the worst loss on the portfolio over a target horizon exceeds this value is the given level of probability. Mathematically, the VaR of the portfolio with a confidence interval c, VaR_c, is defined as (Huang, R.H., *et al*., 2009):

$$VaR_c = \inf\{x \in R \mid Prob(\Delta P \geq x) \leq 1 - c\} \quad (1)$$

where, Prob (•) denotes the portfolio probability distribution and Δ P the portfolio losses over the given holding period.

For a given time horizon t, suppose that the system demand for electricity is Q_t, the retail price to ultimate customers is P_0, the spot price is $p_t = E(p_t|I_{t-1})+\varepsilon_t$, where E (•) is the conditional expectation operator, I_{t-1} the information set available at time t-1 and ε_t the random shock such that $E(\varepsilon_t) = 0$ and $E(\varepsilon_t\varepsilon_s) = 0, \forall_t \neq s$. Given that h_t denotes the conditional standard deviation and z_t denotes a white nose process with zero mean and constant variance equal to 1, ε_t can be defined as $h_t z_t$. The trading losses of an electric utility over the target horizon t is:

$$\Delta P_t = Q_t\left(E(p_t \mid I_{t-1}) + h_t z_t - P_0\right) \quad (2)$$

Because P_0 is a regulated price approved by the regulators and Q_t can be accurately forecasted (usually error is below 3%), Q_t and P_0 can be regarded as constant, let $F_z(x|I_{t-1})$ denote the cumulative distribution function of z_t conditional on the information set I_{t-1} available at time t-1. The VaR of an electric utility in the specified period t with the pre-assigned probability level c, denoted by $VaR_{c,t}$, is:

$$\begin{aligned} 1 - c &= Prob(\Delta P_t \geq VaR_{c,t}) \\ &= 1 - F_z\left(\frac{VaR_{c,t} - Q_t(E(p_t \mid I_{t-1}) - P_0)}{Q_t h_t}\middle| I_{t-1}\right). \end{aligned} \quad (3)$$

Now inverting Eq. 3 for the given probability c, we obtain:

$$VaR_{c,t} = Q_t\left(E(p_t \mid I_{t-1}) - P_0 + h_t F_z^{-1}(c \mid I_{t-1})\right), \quad (4)$$

where, F_z^{-1} is the quantile function defined as the inverse of the distribution function F_z. Therefore, calculating VaR does require some knowledge of the underlying asset distribution.

ARMAX-GARCH-ST model: In order to effectively pre-filter the electricity price series, we choose a combined ARMAX and GARCH model due to the strong seasonality, heteroscedasticity, skewness, kurtosis pattern and the significant volatility clustering in the electricity market. The seasonality in the spot market is particularly obvious over the day and over the week. We therefore include a general formulation for sinusoidal function in the model to capture the possibility of having many cycles per year. Assuming that p_t, d_t, ε_t and z_t denote the electricity spot price, the system load, the random shock and the normalized innovation at time t, h_t denotes the conditional standard deviation of ε_t, then the ARMAX-GARCH model depicting the changing rule of electricity spot price at time t can be formulated as follows:

$$
\begin{aligned}
& p_t = E(p_t \mid I_{t-1}) + \varepsilon_t \\
& E(p_t \mid I_{t-1}) = \alpha_0 + \alpha_1 d_{wkd} + \gamma(\beta) d_t^2 + \varphi(B) p_t \\
& + \kappa(\beta)\varepsilon_t + \sum_{i=1}^{m} \alpha_{1i} \sin\left(\left(\frac{2i\pi}{365} t + \alpha_{2i}\right) \right) \\
& \gamma(\beta) = \gamma_0 + \gamma_1\beta + \gamma_2\beta^2 + \cdots + \gamma_u\beta^u \\
& \varphi(\beta) = \varphi_1\beta + \varphi_1\beta^2 + \varphi_3\beta^3 + \cdots + \varphi_v\beta^v \\
& \kappa(\beta) = \kappa_1\beta + \kappa_2\beta^2 + \kappa_3\beta^3 + \cdots + \kappa_q\beta^q \\
& \varepsilon_t = h_t z_t, z_t \mid I_{t-1} \sim D(0,1) \\
& h_t^2 = \beta_0 + \sum_{i=1}^{r_h} \beta_{1i} h_{t-i}^2 + \sum_{i=1}^{s_h} \beta_{2i} \varepsilon_{t-i}^2 \\
& \beta_0 > 0, \beta_{1i}, \beta_{2j} \ge 0, \forall i \in [1, r_h], j \in [1, s_h]
\end{aligned}
\tag{5}
$$

where, B is the backshift operator, d_{wkd} is a dummy variable that takes a value of 1 if the observation is in weekday and zero otherwise; u, v and q, respectively denote the lagged orders of d_t^2, p_t and ε_t in the mean equation; r_h and s_h denote the lagged orders of h_t^2 and ε_t^2 in the conditional variance equation; m is the number of changing cycles of electricity price series per year, the amplitude and location of the peak can be respectively captured by:

$$\alpha_1^* = (\alpha_{11}, \cdots, \alpha_{1m})$$

And:

$$\alpha_2^* = (\alpha_{21}, \cdots, \alpha_{2m})$$

$$\alpha = (\alpha_0, \alpha_1, \alpha_1^*, \alpha_2^*)$$

$$\beta = (\beta_0, \beta_{11}, \cdots, \beta_{1r_h}, \beta_{21}, \cdots \beta_{2s_h})$$

$y = (y_0, \ldots, y_u)$, $\varphi = (\varphi_1, \ldots, \varphi_p)$, $k = (k_1, \ldots, k_q)$ and $\theta = (\theta_0, \ldots, \theta_v)$ are the parameters to be estimated.

Before parameters calibration, assumption on the distribution of random errors needs to be made. In order to effectively capture the skewness and kurtosis of electricity price series, we assume that the Probability Density Function (PDF) for the standardized innovations z_t is consistent with a skewed student-t distribution with time-varying skewness and degree of freedom. The PDF of z_t can be expressed as:

$$
\begin{aligned}
& f_{z_t}(z_t \mid I_{t-1}) = b_t c_t \left(1 + \frac{1}{(\eta_t - 2)} \left(\frac{b_t z_t + a_t}{1 \pm \lambda_t} \right)^2 \right)^{-\frac{\eta_t + 1}{2}} \\
& 1 \pm \lambda_t = \begin{cases} 1 + \lambda_t \, x \ge -a_t / b_t \\ 1 - \lambda_t x < -a_t / b_t \end{cases} \\
& c_t = \frac{\Gamma(\frac{\eta_t + 1}{2})}{\sqrt{\pi(\eta_t - 2)}\,\Gamma(\frac{\eta_t}{2})} \\
& a_t = 4\lambda_t c_t \frac{\eta_t - 2}{\eta_t - 1}, b_t^2 = 1 + 3\lambda_t^2 - a_t^2
\end{aligned}
\tag{6}
$$

where, ↾ (•) is a Gamma function, λ_t and η_t are the conditional skewness and degree of freedom corresponding to the skewed student-t distribution of z_t, respectively. If we denote the upper and lower limits of η_t by $U_ç$ and $L_ç$, the upper and lower limits of λ_t by $U_ė$ and $L_ė$, then λ_t and $ç_t$ can be calculated by:

$$
\begin{aligned}
& \eta_t = L_\eta + \frac{U_\eta - L_\eta}{1 + \exp(-\omega_t)} \\
& \omega_t = \delta_0 + \sum_{i=1}^{r_\eta} \delta_{1i} \varepsilon_{t-i} + \sum_{i=1}^{s_\eta} \delta_{2i} \varepsilon_{t-i}^2 + \sum_{i=1}^{v\eta} \delta_{3i} \omega_{t-i} \\
& \lambda_t = L_\lambda + \frac{U_\lambda - L_\lambda}{1 + \exp(-\mu_t)} \\
& \mu_t = \tau_0 + \sum_{i=1}^{r\lambda} \tau_{1i} \varepsilon_{t-i} + \sum_{i=1}^{s\lambda} \tau_{2i} \varepsilon_{t-i}^3 + \sum_{i=1}^{v_\lambda} \tau_{3i} \mu_{t-i}
\end{aligned}
\tag{7}
$$

where, $r_ç$, $s_ç$ and $v_ç$ are the lagged orders of ε_t, ε_t^2-i and ω_t in equation of conditional degree of freedom, $r_ė$, $s_ė$ and $v_ė$ are the lagged orders of ε_t, ε_t^3-i_t and $ì_t$ in the conditional skewness Eq.:

$$d = (d_0, d_{11}, \cdots, d_{1r_\gamma}, d_{21}, \cdots, d_{2s_\gamma}, d_{31}, \cdots, d_{3v_\gamma})$$

And:

$$t = (t_0, t_{11}, \cdots, t_{1r_\gamma}, t_{21}, \cdots, t_{2s_\gamma}, t_{31}, \cdots, t_{3v_\gamma})$$

Are the parameters to be estimated.

There are currently numerous techniques for estimating the parameters of ARMAX-GARCH model and no standard is set on how it should be done. Gebizlioglu *et al.* (2011) have shown that the Maximum Likelihood Estimator (MLE) performs better for large samples. Along this line, we estimate the parameters of the proposed models by maximizing conditional log-likelihood function under different probability distribution. Let $\zeta = (\alpha, \gamma, \varphi, k, \beta, \delta, \tau)$, the log-likelihood function for all observations corresponding to ε_t is given by:

$$L(\zeta)=\sum_{t=1}^{T} l_t(\zeta)=\sum_{t=1}^{T}\left(\ln\left(\frac{b_t c_t}{h_t}\right)-\frac{\eta_t+1}{2}\ln\left(1+\frac{1}{(\eta_t-2)}\left(\frac{b_t z_t+a_t}{1\pm\lambda_t}\right)^2\right)\right) \quad (8)$$

where, T is the sample volume, I_t is the log-likelihood function for the t-th observation. By maximizing L (ζ), ζ, the estimated values of parameters ζ can be obtained. It is important to note that the log-likelihood function L (ζ) is highly nonlinear. The starting values of parameters ζ must be selected with care.

ARMAX-GARCH-EVT model: There exists strong temporal dependence in the sequence of electricity prices due to the specific characteristics of electric power. It violates the underlying assumption that the data sequence to which EVT models are applied should be a sequence of IID random variables. In this study, a two-stage approach, provided by McNeil and Frey (2000), is used to this problem. Firstly, the heteroscedasticities, skewnesses, lepkurtosises and seasonalities of electricity price series are filtered by the proposed ARMAX-GARCH-ST model to obtain a nearly IID normalized residual series. In stage two, the EVT framework is applied to the standardized residuals to better capture the heavy-tails and improve the accuracy of VaR estimation.

For POT method uses data more efficiently, it has become the commonly used method in recent applications (Gilli and Kellezi, 2006). POT method is to model the excess distribution for the IID sample data that exceed a high threshold. Given the distribution function F_z (z) of a random variable Z, the distribution function of values of z above a certain threshold u, F_u (y), is called the conditional excess distribution function and is defined as:

$$F_u(y)=\text{Prob}(Z-u\le y\mid Z>u),\forall 0\le y\le z_F-u \quad (9)$$

where, Z is a random variable, u is a given threshold, y = z-u are the excesses and $Z_F \le 00$ is the right endpoint of F_z (z). We verify F_u (y) that can be written in terms of F_z (z), i.e.:

$$F_u(y)=\frac{F_z(u+y)-F_z(u)}{1-F_z(u)}=\frac{F_z(z)-F_z(u)}{1-F_z(u)}. \quad (10)$$

The theorem of Balkema-De Haan-Pickands states that for sufficiently large u, the conditional excess distribution function F_u (y) is well approximated by the Generalized Pareto Distribution (GPD) $G_{\zeta,\sigma}$ (y) which is defined as:

$$G_{\xi,\sigma}(y)=\begin{cases}1-\left(1+\frac{\xi}{\sigma}y\right)^{-1/\xi} & \xi\neq 0\\ 1-e^{-y/\sigma} & \xi=0\end{cases} \quad (11)$$

for y ∈ (0, z_F-u) if ξ≥ = 0 and y ∈ [0,-σ/ξ] if ξ<0. ξ is the shape parameter or tail index and σ>0 is the scaling parameter. In general, we cannot fix an upper bound for financial losses, so only distributions with shape parameter ξ>0 are suited to model fat-tailed distributions. If T is the total number of observations and T_u the number of observations above the threshold u, the value of F_z(u) can be well approximated by the estimate (T-T_u)/T for sufficiently high threshold u. Replacing F_u (y) by the GPD for ξ>0 and F_z u) by (T-T_u)/T, we obtain the estimate of F_z (z) from Eq. 11

$$\hat{F}_z(z)=1-\frac{T_u}{T}\left(1+\frac{\xi}{\sigma}(z-u)\right)^{-1/\xi},\forall\xi>0 \quad (12)$$

For z>u. A reasonable threshold u must be chosen to effectively estimate the values of parameters ξ and σ. So far, no automatic algorithm with satisfactory performance f or the choice of the threshold u is available. A graphical tool for visually selecting the threshold u is the sample mean excess plot defined by the points (u,e (u)). Let $z_{(1)}>z_{(2)}>\ldots>z_{(T)}$ represent the IID order random variables, e (u) can be estimated by (Coles, 2001):

$$e_n(u)=\sum_{i=k}^{n}(z_{(i)}-u)\Big/(n-k+1), \quad (13)$$

where, k = min {i|$z_{(i)}$>u}, n-k+1 is the number of observations exceeding the threshold u. Because the mean excess function will be linear with respect to the threshold u. Hence a plot of the sample mean excess function against threshold u should be approximately linear in u if the GPD provides a good description of the data. So we can select the value that locates at the beginning of the sample mean excess plot which is roughly linear as the suitable threshold.

Having determined a threshold, the estimates of the shape parameter ξ and scale parameter σ of the GPD, ξ and σ, can be obtained by applying maximum likelihood estimation for the excesses of a threshold u. Replacing the values of parameters by their estimates and inverting Eq. 12 for a given probability c, the estimates of the c-th tail quantile for the sample distribution can be gotte:

$$\hat{F}_z^{-1}(c)=u+\frac{\hat{\sigma}}{\hat{\xi}}\left((Tc/T_u)^{-\hat{\xi}}-1\right) \quad (14)$$

which is valid for positive excesses, that is $z > u$. Substituting the estimates of the c-th quantile (by POT model), the conditional mean and standard deviation (by ARMAX-GARCH-ST model) into Eq. 4, the VaR of electric utility in a specified period t with a pre-assigned probability level c can be obtained:

$$\widehat{VaR}_{c,t} = Q_t\left(\hat{p}_t - P_0 + \hat{h}_t\hat{F}_z^{-1}(c)\right). \tag{15}$$

Backtesting for VaR estimates: It is of crucial importance to assess the accuracy of VaR estimates, as they are only useful insofar as they accurately characterize risk. Backtesting or verification testing is the way that we verify whether forecasted losses are in line with actual losses. The most widely known backtesting method based on failure rates has been suggested by Kupiec, (1995). Kupiec's test measures whether the number of violation exceptions (losses larger than estimated VaR) is in line with the expected number for the chosen confidence interval. Under the null hypothesis that the VaR estimated model is correct at a pre-assigned confidence interval, the observed failure rate should act as an unbiased measure of the level of significance as sample size is increased. Denoting the number of times that the actual portfolio returns fall outside the VaR estimate as N and the total number of observations as T, the following Likelihood Ratio (LR):

$$LR = -2\log\frac{(1-c)^N c^{T-N}}{\left(\frac{N}{T}\right)^N\left(1-\frac{N}{T}\right)^{T-N}} \tag{16}$$

is asymptotically X^2 (chi-squared) distributed with one degree of freedom. If the value of LR exceeds the critical value of the X^2 distribution, the null hypothesis will be rejected and the model is deemed as inaccurate. On the contrary, the null hypothesis will be accepted and the model should be considered correct.

EMPIEICAL RESULTS

Data description: The PJM is organized as a day-ahead market. Participants submit their buying and selling bid curves for each of the next 24 h. Then the market operator aggregates bids for each hour and determines market clearing prices and volumes for each h of the following day. In this study, a total of 1197 observations of average daily electricity spot prices in \$/MWh and average daily loads in Gw are employed to validate the performance of the VaR calculating model. The sample period begins on 1st June 2007 and ends on 9th September 2010. Without loss of generality, in this study we assume that an electric

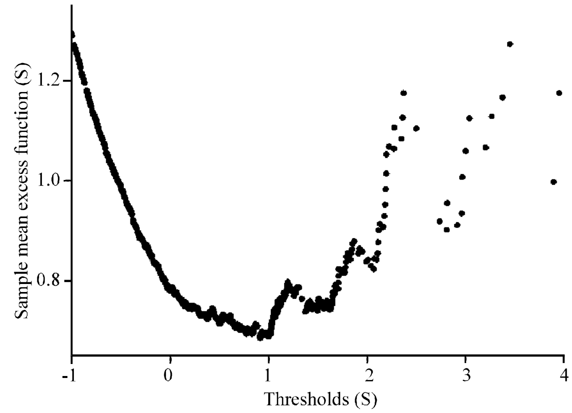

Fig. 1: Mean excess function plots of innovations

Table 1: Estimation of GPD parameters and quantiles

Threshold	Shape	Scale	Confidence	Tail quantile
1.30	0.174	0.6418	95.0%	1.6767
			99.0%	2.9955

utility has the obligation to serve 1MW of load 24 h a day and the retail price has been frozen at a level equivalent to 0\$/MWh.

Estimates of ARMAX-GARCH-EVT: Although the standardized residual series is much closer to being IID than the original series, the Q-statistics of Ljung-Box tests indicate remaining autocorrelation, suggesting that $\{z_t\}$ is a weak dependent stationary series. Therefore EVT can be implemented on the standardized series (Coles, 2001).

Figure 1 shows the sample mean excess function for the standardized errors of ARMAX-GARCH-ST model. We find that the sample mean excess plot is roughly linear when the value of the threshold u is about 1.3. In this case, the number of resulting excesses are 107, accounting for 7.94% of the whole sample which is consistent with the suggestion by McNeil and Frey (2000).

When the residuals above the selected threshold u are determined, the estimates of the shape and scale parameters can be determined by fitting the GPD to the standardized residuals via MLE and the tail quantiles at a given confidence level c can be calculated by Eq. 14. Table 1 reports the estimated results for tail index, scale parameter and tail quantiles. It can be seen that the ξ estimates is positive and statistically significant, indicating that the right tail of the standardized residuals is characterized by the Fréchet distribution

VaR estimates and backtesting: Substituting the calculated results into Eq. 15, the VaR at each confidence level can be estimated. Fig. 2 shows the estimated results of the dynamic VaR during the high volatile period at the

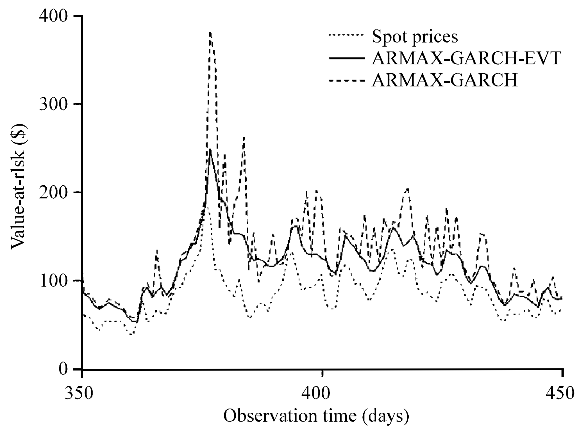

Fig. 2: Dynamic VaR for electric power company at 99% confidence level

Table 2: Backtests of estimated VaRs

	Exception	Normal	Student-t	Skewed-t	GPD
95%	Expected	60	60	60	60
	Real	63	43	54	60
	LR	0.172	5.513**	0.621	0.000
97.5%	Expected	30	30	30	30
	Real	42	20	29	30
	LR	4.449**	3.816*	0.030	0.000
99%	Expected	12	12	12	12
	Real	22	12	9	13
	LR	6.805***	0.001	0.814	0.087

Note:*, **and***, respectively indicate statistical significance of estimated parameters at 90, 95 and 99% confidence intervals

99% confidence level (from 2008-5-15 to 2008-8-23). It can be seen from Fig. 2 that the ARMAX-GARCH-EVT models can be rapidly reflect the most recent and relevant changes of spot electricity prices and can produce accurate forecasts of VaR at all confidence levels, showing better dynamic characteristics.

The Kupiec's test results for actual and forecasted losses are shown in Table 2. It can be seen from Table 2 that the model with normal innovations underestimates the volatility of price risk above 95% confidence level and that the one with student-t innovations overestimates the volatility of price risk below 99% confidence level whereas the null hypothesis cannot be rejected for the models with skewed student-t and GPD distributions in each significance level. Summarizing the results for the Kupiec's tests, our method is able to improve the VaR forecasts so much that VaR predictions are obtained which are insignificantly different from the proposed downfall probability.

CONCLUSION

The distinctive characteristics of electric energy make electricity price present highly volatility and extreme movements of magnitudes rarely seen in markets for regular financial assets, thus volatility of price risk management in electricity market are more important than in financial markets. In this study, a two-stage model for estimating VaR is proposed. In stage one, an ARMAX-GARCH-ST model with load as an exogenous explanatory variable is used to pre-filter the electricity price series in order to acquire the approximately IID standardized residuals. In stage two, an EVT model is employed to explicitly deal with the right tail of the standardized errors of the ARMAX-GARCH-ST model and accurate estimates of VaR in electricity market can be produced. The empirical analysis indicates that the ARMAX-GARCH-EVT models can be rapidly reflect the most recent and relevant changes of spot electricity prices and can produce accurate forecasts of VaR at all confidence levels, showing better dynamic characteristics. These results present several potential implications for electricity market risk quantifications and hedging strategies.

REFERENCES

Bushnell, J., 2004. California's electricity crisis: a market apart? Energy Policy, 32: 1045-1052.

Bystrom, H.N.E., 2005. Extreme value theory and extremely large electricity price changes. Int. Rev. Econ. Finance, 14: 41-55.

Chan, K.F. and P. Gray, 2006. Using extreme value theory to measure value-at-risk for daily electricity spot prices. Int. J. Forecast., 2: 283-300.

Coles, S., 2001. An Introduction to Statistical Modeling of Extreme Values. 1st Edn., Springer, New York, ISBN-13: 978-1852334598.

Gebizlioglu, O.L., B. Senoglu and Y.M. Kantar, 2011. Comparison of certain value-at-risk estimation methods for the two-parameter Weibull loss distribution. J. Comput. Applied Math., 235: 3304-3314.

Gilli, M. and E. Kellezi, 2006. An application of extreme value theory for measuring financial risk. Comput. Econ., 27: 207-228.

Gong, X.S., X. Luo and J.J. Wu, 2009. Electricity auction market risk analysis based on EGARCH-EVT-CVaR model. Proceedings of the International Conference on Industrial Technology, February 10-13, 2009, Gippsland, VIC., pp: 1-5.

Hartz, C., S. Mittnik and M. Paolella, 2006. Accurate value-at-risk forecasting based on the normal-GARCH model. Comput. Stat. Data Anal., 51: 2295-2312.

Huang, R.H., J. Zhang, L.Z. Zhang and Z.L. Li, 2009. Price risk forewarning of electricity market based on GARCH and VaR theory. Proc. Chaines Soc.Elect. Eng., 29: 85-91.

Kupiec, P., 1995. Techniques for verifying the accuracy of risk measurement models. J. Derivatives, 2: 174-184.

Li, L.S. and C.H. Sun, 2010. Risk assumption of probability distribution in VaR estimation and its improvement. Stat. Res., 10: 40-46.

McNeil, A.J. and R. Frey, 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. J. Empirical Finance, 7: 271-300.

Wang, R.Q., F.X. Wang and X.J. Guo, 2012. Research on price risk of electricity market based on GARCH model. J. Hainan Normal Univ. Natural Sci., 1: 36-40.

Zhang, F.Q. and H. Zhou, 2004. Financial risk analysis in electricity market by analytical approach. Proc. Elect. Power Syst. Automat., 16: 23-28.