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Stability of Tubing String in Vertical Well Based on Transfer Matrix Method

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Abstract: The tubing string will be deformed or even broken when subjected to axial compression load. In the study, the stability of tubing string was discussed when the joint was taken into consideration. First, the tubing and the joint was dispersed into various elements and their element matrices were derived separately. Secondly, the whole transfer matrix of the tubing string was derived based on transfer matrix method by sequential matrix multiplication of tubing element matrix and joint element. Then, the critical buckling load was derived by taking into consideration the boundary condition. At last, the influence of the outer diameter and the length of joint on the critical buckling load in vertical well were discussed with partial tubing and joints in API Spec 5CT as the objects. The derived critical buckling load of tubing string considering the joint will be more accurate than that derived by the ordinary method in which the tubing string was simply taken as a thin-wall cylinder of uniform cross-section.

Key words: Transfer matrix method, stability, tubing string, vertical well, joint

INTRODUCTION

The tubing string will be deformed or even broken when subjected to axial compression load. To simplify the model of tubing string, the joint was always ignored and the tubing string was taken as a thin-wall cylinder of uniform cross-section in the stability analysis (Gao, 2006; Li, 2008). In actual, the outer diameter of joint is larger than that of tubing, the ratio is up to 1.5, so there exists error in the critical buckling load inevitably when simply taken the tubing string as a thin-wall cylinder of uniform cross-section (Wu and Juvkam-Wold, 1993; Mitchell, 2000). The formula of critical buckling load of tubing string when the joint and weight were taken into consideration was derived based on transfer matrix method (Zhu and Deng, 2005). And, the influence of the outer diameter and the length of joint on the critical buckling load were discussed with partial tubing and joints in API Spec 5CT as the objects.

ESTABLISHMENT OF ELEMENT TRANSFER MATRIX OF TUBING AND JOINT

The deformation of tubing and joint when subjected to axial compression load was shown in Fig. 1. Where, F is the axial compression load, M is the bending moment, Q is the shear force, x is the axial displacement and y is the deflection. The critical buckling load of tubing string was derived under the assumptions that the tubing string is considered as elastomer, the elastic modulus of tubing and joint are both 2.1×1011 Pa, the axis of the tubing string coincides with the trajectory of vertical well, the deflection is small enough compared with the length of tubing string, the weight of tubing and joint is concentrated in the bottom of each element in the transfer matrix method.

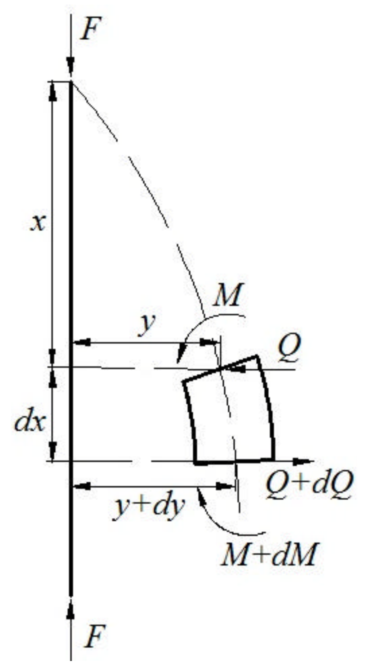

Fig. 1: Schematic representation of a tubing string subjected to axial compression load

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The differential equation of the tubing string subjected to axial compression load in Fig. 1 can be expressed as:

$$\frac{d^4y}{dx^4} + k \cdot \frac{d^2y}{dx^2} = 0 \tag{1}$$

Where:

$$k^2 = \frac{F}{EI}$$

The general solution of Eq. 1 can be written as:

$$y = C_1 + C_2 \cdot x + C_3 \cdot \sin kx + C_4 \cdot \cos kx$$

In terms of material mechanics (Liu, 2011):

$$\begin{aligned} \theta &= C_2 + C_3 \cdot k\cos kx - C_4 \cdot k\sin kx \\ M &= EI(C_3 \cdot k^2 \sin kx + C_4 \cdot k^2 \cos kx \\ Q &= -C_2 \cdot EIk^2 \end{aligned} \tag{2}$$

The state matrix $S = [y, \theta, M, Q]^T$ at arbitrary section is composed of deflection y, angle θ, bending moment M and shear force Q. If the state matrix on the right of arbitrary element is $[y_1, \theta_1, M_1, Q_1]^T$ and state matrix on the left is $[y_0, \theta_0, M_0, Q_0]^T$, so there exists the following relationship:

$$\begin{aligned} \theta_1 &= \theta_0 \cdot \cos kx - M_0 \cdot \frac{\sin kx}{EIk} - Q_0 \cdot \frac{1-\cos kx}{EIk^2} \\ M_1 &= \theta_0 \cdot EIk\sin kx + M_0 \cdot \cos kx + Q_0 \cdot \frac{\sin kx}{k} \\ Q_1 &= Q_0 \end{aligned} \tag{3}$$

The above relationship can be expressed in matrix form as:

$$S_1 = C \times S_0 \tag{4}$$

where, C is the transfer matrix of arbitrary element:

$$C = \begin{bmatrix} 1 & \frac{\sin kx}{k} & -\frac{1-\cos kx}{EIk^2} & -\frac{kx - \sin kx}{EIk^3} \\ 0 & \cos kx & -\frac{\sin kx}{EIk} & -\frac{1-\cos kx}{EIk^2} \\ 0 & EIk\sin kx & \cos kx & \frac{\sin kx}{k} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

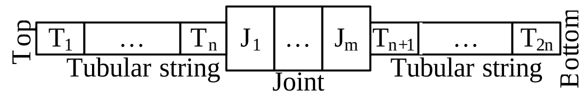

Fig. 2: Schematic representation of tubing string composed of tubing and joint element

The actual axial force in element transfer matrix can be expressed as the followings when the weight of tubing and joint was included:

$$F = F_n + (G_1 + G_2) \times n_0 + G_{01} \times n_1 + G_{02} \times n_2 \tag{5}$$

where, F_n is the axial compression load not including the weight of tubing and joint, G_1 is the weight of single joint, G_2 is the weight of single tubing, G_{01} is the weight of single joint element, G_{02} is the weight of single tubing element, n_0 is the number of the set combined with tubing and joint above the element, n_1 is the number of the tubing above the element and n_2 is the number of the joint above the element.

ESTABLISHMENT OF THE WHOLE TRANSFER MATRIX OF TUBING STRING BASED ON THE ELEMENT TRANSFER MATRIX

The tubing string was composed of tubing and joint element sequentially and the schematic representation of tubing string is shown in Fig. 2 (He *et al*., 2008). In Fig. 2, the element from T_1 to T_{2n} represents tubing while the element from J_1 to J_m represents joint. And, the corresponding transfer matrix is $[C]_1 \ldots [C]_n$, $[C]_{n+1}$, $[C]_{n+m}$, $[C]_{n+m+1} \ldots [C]_{2n+m}$ separately.

From Eq. 4, we can get the following relationship:

$$\{S\}_n = [C]_n \times \{S\}_{n-1} = [C]_{n-2} \times [C]_{n-1} \times \{S\}_{n-2} \cdots \tag{6}$$

i.e.:

$$\{S\}_n = [H] \times \{S\}_0 \tag{7}$$

Where, $[H] = [C]_n \times [C]_{n-1} \times [C]_{n-2} \cdots [C]_2 \times [C]_1$ is the whole transfer element of tubing string, $\{S\}_0$ and $\{S\}_n$ are the boundary element matrices of the tubing string. The Eq. 7 can be rewritten as:

$$\begin{Bmatrix} y_n \\ \theta_n \\ M_n \\ Q_n \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{bmatrix} \begin{Bmatrix} y_0 \\ \theta_0 \\ M_0 \\ Q_0 \end{Bmatrix} \tag{8}$$

CRITICAL BUCKLING LOAD OF TUBING STRING CONSIDERING THE JOINT

The boundary of a pinned-pinned tubing string is:

$$\begin{cases} y_0 = 0, M_0 = 0 \text{ (at upper end)} \\ y_n = 0, M_n = 0 \text{ (at lower end)} \end{cases} \tag{9}$$

Substituting equation 9 into equation8, we get the following characteristic equation.

$$\begin{vmatrix} H_{12} & H_{14} \\ H_{32} & H_{32} \end{vmatrix} = 0 \tag{10}$$

The critical buckling load of the whole tubing string can be derived from the Eq. 10.

FLOW CHART OF THE CRITICAL BUCKLING LOAD

Firstly, the tubing and joint should be dispersed and the tubing element and joint element is gotten correspondingly. Secondly, the element transfer matrix of tubing and joint can be derived by the solving of the governing equations. Thirdly, the whole transfer matrix of the tubing string is derived based on the transfer matrix method. Then, the characteristic matrix of the whole tubing string considering the boundary condition is derived. At last, the critical buckling load can be derived. And, the corresponding flow chart of the process is shown in Fig. 3.

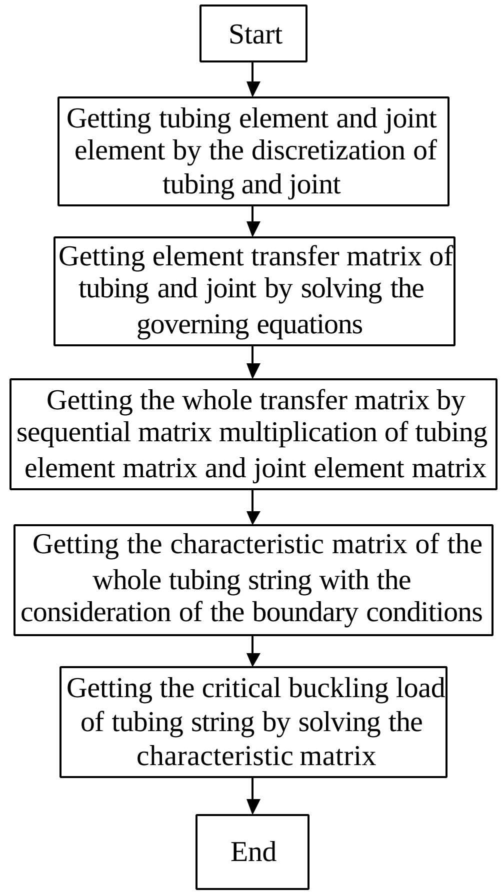

Fig. 3: Flow chart of the calculation of critical buckling load

APPLICATIONS

The influence of the outer diameter and the length of joint on the critical buckling load of tubing string in vertical well based on the transfer matrix method were discussed with partial tubing and joints in API Spec 5CT as the objects.

Influence of the outer diameter of joint on the critical buckling load of tubing string: The outer diameter, length and elastic modulus of tubing and joint are listed in Table 1. The inner diameter of tubing investigated is 77.92, 76, 74.22 and 69.86 mm separately. For different inner diameters of tubing string, MATLAB programs are written to explore the tendency critical buckling load varied with the set number of tubing and joint. And, the critical buckling loads of tubing string with various inner diameters and the outer diameter of joint as 107.95mm are shown in Fig. 4.

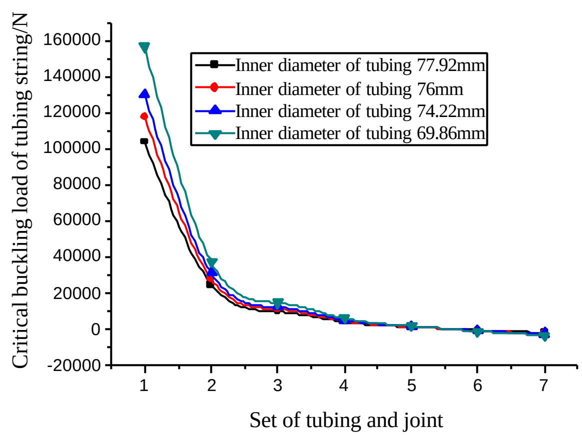

Fig. 4: Critical buckling loads for tubing string when the outer diameter of joint is 107.95mm

Table 1: Parameters of tubing and joint

Outer diameter/mm		Length/m		Elastic modulus/Pa	
Tubing	Joint	Tubing	Joint	Tubing	Joint
88.9	107.95	10	0.14	2.1×10^{11}	2.1×10^{11}

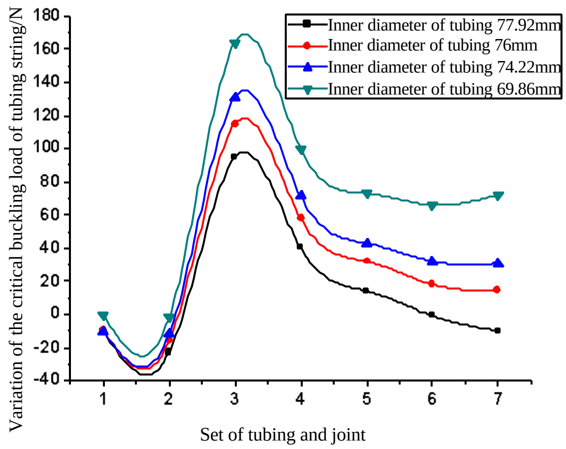

Fig. 5: Variation of critical buckling load

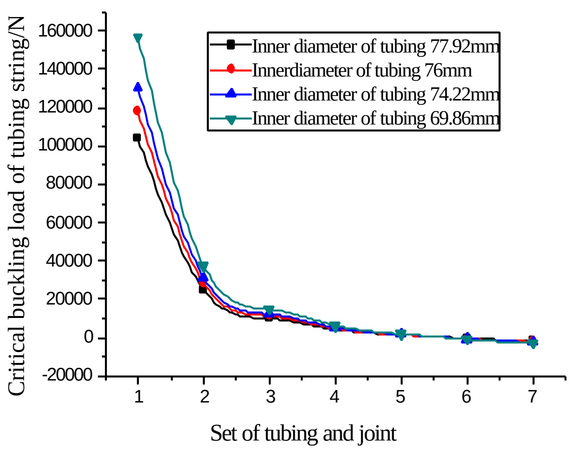

Fig. 6: Critical buckling loads for tubing string when the outer diameter of joint is114.30 mm

If we define the critical buckling load of the tubing string with the joint of which the outer diameter is 107.95 mm as F_{cr1} and the critical load of the tubing string without the joint as F_{cr2}, we can get the variation of when the outer diameter of joint is 107.95 mm the critical buckling load with the set of tubing and joint for the outer diameter of joint as 107.95 mm in Fig. 5.

When the outer diameter of joint is 114.30 mm and other parameters remain the same, we can get the critical buckling loads of tubing string with various inner diameters and the outer diameter of joint as 114.30 mm shown in Fig. 6.

If we define the critical buckling load of the tubing string with the joint of which the outer diameter is 114.30mm as F_{cr3}, we can get the variation of the critical buckling load with the set of tubing and joint for the outer diameter of joint as 114.30 mm in Fig. 7.

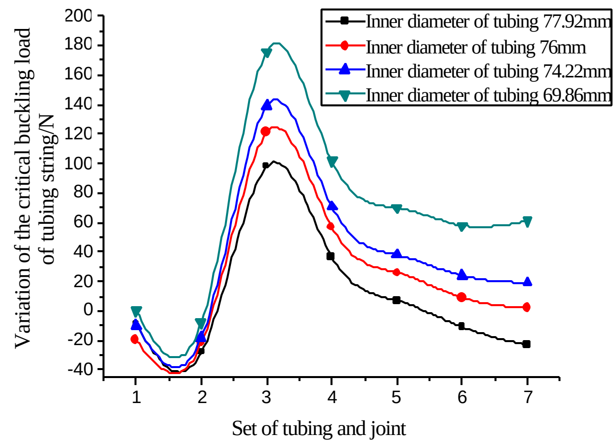

Fig. 7: Variation of critical buckling load when the outer diameter of joint is 114.30 mm

When the critical buckling load increases, the possibility of instability of the tubing string will decrease. From Fig. 4 and 6, we can see that the critical buckling will decline in exponent regularity with the increase of the length of tubing string and the tubing string will be more likely to be instable. From Fig. 5 and 7, we can see that the joint may influence the stability of tubing string in two aspects which is increasing the stability for the increase of the stiffness and decreasing the stability for the increase of the axial compression load caused by the weight. When the length of tubing string is shorter than 20m, i.e. the set number of tubing and joint is smaller than 2, the weight plays the main role, thus the tubing string will be more likely to be instable. When the length of tubing string is larger than 20 m, the stiffness plays the main role, thus the tubing string will be more likely to be stable. When the length of tubing string is larger than 50 m, the variation of critical buckling load tends to be a constant.

Influence of the length of joint on the critical buckling load of tubing string: The inner diameter of tubing is 76mm, the outer diameter of tubing is 88.9mm and the length of tubing string is 40 m. The length of joint investigated is 140, 160, 180, 200, 220, 240 and 260 mm separately. MATLAB programs are written to explore the tendency critical buckling load varied with the length of joint. And, the critical buckling loads of tubing string with various lengths of joint and the outer diameter of joint as 107.95 and 114.30 mm are shown in Fig.8.

The increase of the length of joint may influence the stability of tubing string in two aspects which is increasing the stability for the increase of the stiffness and decreasing the stability for the increase of the axial

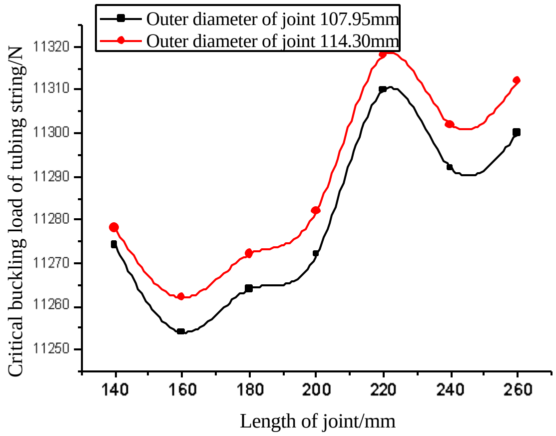

Fig. 8: Critical buckling loads for tubing string with various lengths of tubing string

compression load caused by the weight. From Fig. 8, we can see that the critical buckling will take on a fluctuant increasing process with the increase of the length of joint and the tubing string will be more likely to be stable. The critical buckling load increases with the outer diameter of joint, that is to say, the stability of tubing string will increase with the outer diameter of joint.

CONCLUSION

- To get the accurate critical buckling load, it's necessary to take the joint into consideration rather than ignoring it
- The critical buckling will take on a fluctuant increasing process with the increase of the length of joint and the tubing string will be more likely to be stable
- The critical buckling will decline in exponent regularity with the increase of the length of tubing string and the tubing string will be more likely to be instable
- The joint may influence the stability of tubing string in two aspects which is increasing the stability for the increase of the stiffness and decreasing the stability for the increase of the axial compression load caused by the weight. When the length of tubing string is shorter than 20m, the tubing string will be more likely to be instable. When the length of tubing string is larger than 20m, the tubing string will be more likely to be stable

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REFERENCES

Gao, D.L., 2006. Down-Hole Tubular Mechanics and Engineering. 1st Edn., China University of Petroleum Press, China.

He, B., X.T. Rui and G.P. Wang, 2008. Transfer Matrix Method of Multi-Body System and its Applications. 1st Edn., Science Press, Beijing.

Li, Z.F., 2008. Down-Hole Tubular Mechanics and its Applications. 1st Edn., Petroleum Industry Press, Beijing.

Liu, H.W., 2011. Mechanics of Materials. 5th Edn., Higher Education Press, Beijing.

Mitchell, R.F., 2000. Lateral buckling of pipe with connectors in horizontal wells. Proceedings of the IADC/SPE Drilling Conference, February 23-25, 2000, New Orleans, Louisiana.

Wu, J. and H.C. Juvkam-Wold, 1993. Helical buckling of pipes in extended reach and horizontal wells-part 2: Frictional drag analysis. J. Energy Resources Technol., 115: 196-201.

Zhu, B.K. and D.P. Deng, 2005. Analyzing the stability of horizontal drill string by using matrix transfer method. China Petroleum Machinery, 33: 18-20.