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ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (4): 640-647, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.640.647

Iterative Linear Programming Design of Digital Lowpass FIR Filters with Linear Phase

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Abstract: The minimax optimization is widely used in wireless communications to design the equiripple lowpass filter, such as the Linear Programming (LP) method. However, the conventional LP method suffered from its large computation loads. Hence, this study investigates an iterative LP method, in which constraints are iteratively thrown on the non-uniformly distributed frequency grid to reduce the problem scale as much as possible, resulting in much lower computations. Moreover, since the non-uniform frequency grid allows us to precisely control the ripple, the proposed method also yields a better equiripple result compared to the conventional LP method and the Particle Swarm Optimization (PSO) algorithm.

Key words: Iterative linear programming, FIR filter, linear phase, equiripple, wireless communications.

INTRODUCTION

Digital filters have been widely used in digital signal process, such as the speech coding, the image processing and the matched filter in digital communications (Lyons, 2004; Lee and Miller, 1998; Oppenheim *et al*., 1999). There are two kinds of filters, namely the Finite Impulse Response (FIR) filter and the Infinite Impulse Response (IIR) filter (Hua *et al*., 2012; Lai and Lin, 2010; Jiang and Kwan, 2010). Since, the linear phase property is important in wireless communication systems (Proakis, 2000; Ruan *et al*., 2012), the authors pay attention to the linear phase FIR filter in this study. Moreover, because the Low-Pass Filter (LPF) is widely used in wireless communications, only the LPF design is studied in this study.

There are many FIR LPF design algorithms including the window method, the frequency sampling method, the Least Squares (LS) method (Selesnick *et al*., 1996; Zhang and Wu, 2011), PM algorithm (Antoniou, 1982) and the linear programming method (LP) (Rabiner, 1972) (Samueli, 1988). Among these methods, the window method and the frequency sampling method are simple but difficult to produce the equiripple filter. Moreover, previous work indicated that the LPF of wireless systems requests very rigorous performance specification (Lee and Miller, 1998) and these strict requirements make the LS method yields only the coarse equiripple filter. Even though the original PM algorithm is very efficient, it cannot take into account some special frequency domain constraints directly. Thus, by utilizing complicated iterative techniques (Lai, 2002; Lai and Zhao, 2006) proposed constrained Chebyshev methods for equality and inequality constraints which usually required more than ten iterations.

Another optional minimax method in LPF design is the non-iterative LP (NILP) method (Rabiner, 1972) which can make use of both the weight W and the variable frequency domain constraints. However, the NILP method uniformly discretizes the frequency range ($[0, \omega_p]\cup[\omega_s, \pi]$) into G discrete points, denoting as vector ω_D. These operations cause two drawbacks. First, the obtained passband (stopband) ripple would be larger than the desired ripple slightly and the deviation is proportional to 1/G approximately. Second, the number of constraints must be as twice larger as the discrete frequency number (G) which increases the scale of the optimization problem. Aside from the NILP method, Samueli (1988) had exploited non-uniform ω_D and the exchange algorithm in his LP based Nyquist filter design, where he mainly focused on the Inter-Symbol-Interference (ISI). However, the ordinary LPF is quite different from the Nyquist filter, where people care for another three performance specifications instead of the ISI: ripples, transition band and stopband attenuations. Hence, Samueli's method cannot be applied to design the LPF directly.

In this study, after constructing a W-driven optimization model analogous to the previous literature (Rabiner, 1972; Samueli, 1988), an iterative LP (ILP) method is presented to design the linear phase LPF, where

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constraints on the non-uniformly distributed frequency grid are employed to relieve the requirement of discrete frequency number G. In each iteration, the nonlinear root-finding method, i.e., Newton method (Yang *et al*., 2005), is employed to refine the discrete frequency in the local peak of zero-phase responses which improves the precision of frequency vector ω_D and helps to accelerate convergence. In addition, it had been well-known that the stop criterion was important for an iterative algorithm, thus, a comparative study on some stop criteria are also presented in this study in terms of the convergence and the equiripple error. Moreover, in order to provide engineers more flexible alter-natives to confront real-world requirements, another two optimization models are introduced with the above iterative technique, where the proposed algorithm suppress δ_p (δ_s) directly while minimize δ_s (δ_p) thereafter.

Of course, all the methods mentioned above are deterministic in nature. With the rapid development of computation algorithms, the heuristic algorithm had been applied in filter design, such as the particle swarm (PSO) algorithm (Kennedy and Eberhart, 1995; Zhan *et al*., 2009). These heuristic algorithms are non-deterministic and capable of dealing with both convex and non-convex optimization problems. A comparison between the ILP and PSO algorithm is also provided in our study.

CONVENTIONAL LP METHOD

Linear phase FIR filters: For linear phase FIR filters, the filter coefficient vector h must satisfy:

$$h(n) = \pm h(n-N-1),\ 0 \le n \le N-1 \tag{1}$$

where N denotes the filter length and the symmetric coefficient ensures the linear phase (Oppenheim *et al*., 1999). Without loss of generality, the authors focus on the type I lowpass filters, whose zero phase response can be found as:

$$H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} 2h(n)\cos\{\omega[\tfrac{N-1}{2} - n]\} - h(\tfrac{N-1}{2}) \tag{2}$$

where $\omega \in [0, \pi]$. Equation 2 can be written in matrix form:

$$[H(\omega_1)\ H(\omega_2) \ldots H(\omega_G)]^T = [c(\omega_1), c(\omega_2) \ldots c(\omega_G)]^T \times h \tag{3}$$

where, $[.]^T$ represents the transposing operation:

$$h = [h(0), h(1), \cdots, h(\tfrac{N-1}{2})]^T$$

and

$$c(\omega) = 2[\cos[\omega(\tfrac{N-1}{2} - 0)],\ \cos[\omega(\tfrac{N-1}{2} - 1)], \cdots, \cos[\omega], \tfrac{1}{2}]^T.$$

Performance specifications and constraints: Given the filter length N, the filter performance specification includes the stopband attenuation A_s, the ripples (δ_p, δ_s) and the transition width B (that defined by two cutoff frequencies ω_p and ω_s). Their relationships are analogous to three angles in triangle (Losada, 2004), i.e., one can at most select the values of two of the above specifications while the third specification will be determined by the optimization algorithm.

According to Losada (2004), when N and δ_p are fixed, improving δ_s or A_s equivalently will lead to the increase of the transition band width. The following constraints can be used to limit the transition band width:

$$H(\omega_p) \ge 1 - \delta_p;\ H(\omega_S) \ge 1 - \delta_S \tag{4}$$

which is a part of constraints in conventional LP (CLP) methods. It is equivalent to enforce the actual passband cut-off frequency to be no less than the specification ω_p and the actual stopband start frequency to be no more than the specification ω_S.

The CLP method in LPF design: Generally, the weighted approximation error obeys:

$$E(\omega) = W(\omega)\,[H(\omega) - H_d(\omega)] \tag{5}$$

When the error curve of $E(\omega)$ has equal peaks in the passband, the filter is known as the equiripple filter. In Eq. 5, the weighting function:

$$W(\omega) = \begin{cases} 1, & 0 \le \omega \le \omega_p \\ W, & \omega_s \le \omega \le \pi \end{cases}$$

and the desired zero-phase response:

$$H_d(\omega) = \begin{cases} 1, & 0 \le \omega \le \omega_p \\ 0, & \omega_s \le \omega \le \pi \end{cases}$$

Similar to PM algorithm, the CLP optimization model Eq. 6 aims to minimize the weighted ripple both in passband and stopband while ω_p, ω_S and the weight value W (equals to δ_p, δ_S) satisfying the specification:

$$\min_{h,\delta} f = \delta$$
$$\text{st}\begin{cases} c(\omega_k)^T \times h - W \times \delta \le 1, & \omega_k \in [0, \omega_p] \\ -c(\omega_k)^T \times h - W \times \delta \le -1, & \omega_k \in [0, \omega_p] \\ c(\omega_k)^T \times h - 1 \times \delta \le 0, & \omega_k \in [\omega_s, \pi] \\ -c(\omega_k)^T \times h - 1 \times \delta \le 0, & \omega_k \in [\omega_s, \pi] \end{cases} \tag{6}$$

where, the ripple δ is regarded as the optimizing variable and ω_k is an element of the frequency vector ω_D.

ILP METHOD DERIVATION

Evaluating the extreme frequencies: Before detail discussions, a frequency vector Ω_M, corresponding to the extremum of the zero-phase response, should be defined according to the equation:

$$\frac{d}{d\omega}H(\omega)=0 \quad (7)$$

The actual passband and stopband ripple are determined by approximation errors on the frequency Ω_M and the boundary frequency $\{0, \pi\}$. As for the equiripple LPF, the specification ω_p and ω_S must be the alternate points (Oppenheim *et al*., 1999). Hence, people must take into consideration the frequency vector Ω_{extrm} comprised by Ω_M and the boundary frequencies $\{0, \omega_p, \omega_S, \pi\}$. Now Ω_{extrm} can be obtained by solving Eq. 7 by the Newton root-finding method and an acceptable solution can be obtained in about 4~5 iterations with absolute error less than 10^{-12}.

ILP method: Similar to Samueli (1988), Ω_{extrm} is not the subset of ω_D which means that the frequency domain constraints haven't been imposed on Ω_{extrm}. Therefore, the absolute approximation error on frequency Ω_{extrm} tends to be slightly larger than δ_p, δ_S in model Eq. 6, i.e., the real-world ripple value would be larger than the desired value and the deviation is determined by the difference of Ω_{extrm} and the closest element in ω_D. In fact, the above deviation indicates that the filter designed by the CLP method is not the optimum filter in the sense of minimax approximation due to the coarse conversion from the Semi-Infinite Linear Programming (SILP) to Finite Linear Programming (FLP). The conclusion is supported by simulations given in section 4.

Scrutinizing in Eq. 6, people explicitly see that the constraint number is twice times as the element number of ω_D, viz., 2G. The countermeasure to reduce the above deviation is to increase G which leads to the constraint number grows rapidly and results in heavy computation loads. Therefore, the compromise has to be made between the desired performance specification and the computation efficiency which is the bottleneck of the CLP method in the FIR filter design.

One can imagine that if all constraints are imposed on Ω_{extrm}, the ripple in discretized model Eq. 6 can be suppressed precisely. In fact, provided with constraints on Ω_{extrm}, people don't need to import any other frequency constraints. Therefore, it's enough to impose only one constraint on each element of Ω_{extrm} while the CLP method requires two. According to Oppenheim *et al*. (1999), there are no more than (N+5)/2 elements (alternate points) in Ω_{extrm}, resulting in small constraint amounts. This strategy avoids the degeneration of constraints in the conversion from the SILP and to FLP and ensures that the filter coefficient vector h produced by the ILP method is equivalent to solution of the semi-infinite programming model.

In essence, the ILP method is a kind of exchange algorithms which is similar to the Remez exchange algorithm in (Antoniou, 1982) and Multi-exchange algorithm in (Adams, 1991; Selesnick *et al*., 1996). Moreover, the proposed algorithm absorbs the exchange idea and extends CLP method to more general cases. The iterative algorithm is given as follows:

Step 1: Initializing the frequency vector ω_D, i.e., discretizing $[0, \omega_p]\cup[\omega_S, \pi]$ into G points
Step 2: Substituting ω_D into a discretized model, such as Eq. 6 or other models to be presented in the next and calculating the filter coefficient vector h
Step 3: Calculating the new frequency vector Ω_{extrm}
Step 4: Checking the stop criterion. If the stop criterion to be discussed in the next is satisfied, go to (step 7). Otherwise, go to (step 5)
Step 5: Substituting the frequency vector Ω_{extrm} into discretized model. To be specific, the elements corresponding to the local maximum in the zero-phase response use the first or the third inequality constraint in the selected LP model while the elements corresponding to the local minimum in the zero-phase response use the second or the fourth inequality constraint
Step 6: Solving the LP problem and update the filter coefficient vector h. Go to (step 3)
Step 7: Stop

The initial frequency is generated in passband and stopband uniformly, where the passband includes:

$$\left\lceil N\frac{\omega_p}{\pi-(\omega_s-\omega_p)}\right\rceil$$

points and the stopband includes:

$$\left\lceil N\frac{\omega_s}{\pi-(\omega_s-\omega_p)}\right\rceil$$

points ($\lceil x\rceil$ represents the minimum integer no less than x). The above iterative process employs the CLP method in (Step 1, 2) to obtain initial filter coefficients and performs the iterative (exchange) algorithm in step 3 and 7 to realize a simple minimax optimization. Its convergence is guaranteed by the exchange essence according to (Powell, 1981) and extensive simulations demonstrate that this algorithm converges in no more than 10 iterations.

Stop criteria applications: Several stop criteria can be applied in the proposed ILP methods. The first is:

$$\Lambda_\Omega = \left\|\Omega_{extrm}^{k+1} - \Omega_{extrm}^{k}\right\| < \varepsilon_\Omega \tag{8}$$

where, Ω^k_{extrm}, $||x||$ and ε_Ω represent the Ω_{extrm} of the kth iteration, the vector length and a small positive threshold, respectively. The second criteria is similar as that in (Antoniou, 1982), i.e.:

$$\Lambda_W = \frac{\max[E(\Omega_{extrm})] - \min[E(\Omega_{extrm})]}{\max[E(\Omega_{extrm})]} < \varepsilon_W \tag{9}$$

where, max (x) and min (x) represent the maximum and the minimum of vector x. Note that E (x) is defined in Eq. 5. The cost Λ_W characterizes the equiripple condition by taking into account the dynamic range of the weighted approximation error in Ω_{extrm}.

There had been a conclusion on the one-point exchange in by Powell (1981), viz., the exchange algorithm produced a bounded equiripple error (formula 8.16) and the error tends to be the same for adjacent iterations (formula 8.24). Then, the following criterion can be derived:

$$\Lambda_s = |A_s^k - A_s^{k+1}| < \varepsilon_s \tag{10}$$

where, A_s^k and $|x|$ denotes the stop attenuation at the kth iteration and the absolute value of x.

According to Cetin *et al*. (1997), the iteration terminates when there is no significance change in the filter coefficient vector h, such as:

$$\Lambda_h = ||h^{k+1} - h^k|| < \varepsilon_h \tag{11}$$

where, h^k means the filter coefficient vector at the kth iteration. All above stop criteria are suitable for the proposed ILP method and which one produces the fastest convergence speed and the best equiripple performance, will be deeply investigated in the next section.

Additional optimization models: In order to enrich the optimization option, another two optimization models can be as follows:

$$\min_{h,\delta_s} f = \delta_s$$
$$\text{st}\begin{cases} c(\omega)^T \times h + 0\times\delta_s \le 1+\delta_p, & \omega\in[0,\omega_p] \\ -c(\omega)^T \times h + 0\times\delta_s \le -(1+\delta_p), & \omega\in[0,\omega_p] \\ c(\omega)^T \times h - 1\times\delta_s \le 0, & \omega\in[\omega_s,\pi] \\ -c(\omega)^T \times h - 1\times\delta_s \le 0, & \omega\in[\omega_s,\pi] \end{cases} \tag{12}$$

where, both the filter coefficient vector h and the stopband ripple δ_S are regarded as decision variable. The other is to minimize the passband ripple with fixed transition band and stopband ripple δ_S:

$$\min_{h,\delta_p} f = \delta_p$$
$$\text{st}\begin{cases} c(\omega)^T \times h - 1\times\delta_p \le 1, & \omega\in[0,\omega_p] \\ -c(\omega)^T \times h - 1\times\delta_p \le -1, & \omega\in[0,\omega_p] \\ c(\omega)^T \times h + 0\times\delta_p \le \delta_s, & \omega\in[\omega_s,\pi] \\ -c(\omega)^T \times h + 0\times\delta_p \le \delta_s, & \omega\in[\omega_s,\pi] \end{cases} \tag{13}$$

where, the decision variables are the filter coefficient vector h and the passband ripple δ_p.

Since, all three models are convex in nature, no matter which model is used, the resulting tradeoff is the fundamental tradeoff (Davidson, 2010) and cannot be outperformed by any other models (Powell, 1981) which implies that once the specification is given, all three models could yield the same results. This pleasing conclusion gives engineer more flexibility of model choice in practical designs.

EXAMPLES AND ANALYSIS

Here, all frequencies are normalized by the sampling frequency.

Example 1: Stop criterion discussion:

- **Specification A:** N = {49, 75}, $\omega_p = 0.24\pi$, $\omega_S = 0.30\pi$ the logarithmic ripple R_p = 2 dB, where:

$$\delta_p = \frac{1-10^{\frac{-R_p}{20}}}{1+10^{\frac{-R_p}{20}}} \text{ and } \delta_s = (1+\delta_p)\times 10^{\frac{-A_s}{20}}$$

- **Specification B:** N = 49, $\omega_p = 0.24\pi$, $\omega_S = 0.30\pi$, R_p = {0.2, 2} dB
- **Target:** Comparing the cost functions with the ILP method for model (12)

From Fig. 1, all four cost functions (stop conditions) yield similar curves and after six iterations, there exist no significant variations for Λ_Ω, Λ_W, Λ_s and Λ_h which indicates that the proposed ILP algorithm can converged in about six iteration, i.e., the convergence speed is fast and acceptable. Moreover, according to the turning points in Fig. 1, people clearly see that reasonable thresholds {ϵ_Ω, ϵ_W, ϵ_s, ϵ_h} can be {10^{-16}, 10^{-6}, 10^{-6}, 10^{-16}}, respectively. Since, the cost function ϵ_W depicts the equiripple degree

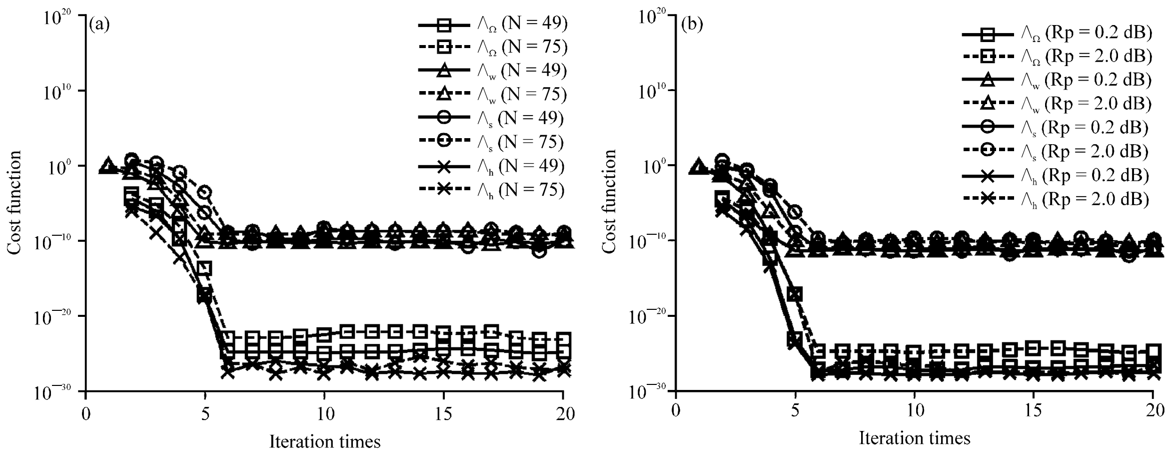

Fig. 1: Stop condition comparisons in example 1

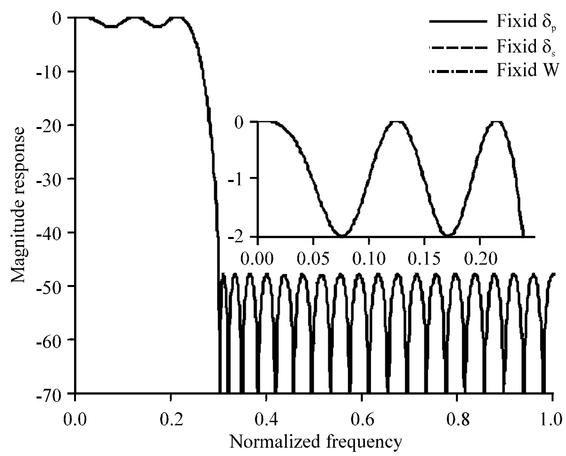

Fig. 2: Magnitude response in example 2: x-axis is the digital frequency and y-axis is the response amplitude

directly, this study chooses $\epsilon_W = 10^{-6}$ as the stop criterion in the next and other three stop criteria can be analyzed similarly.

Example 2: Comparisons of three optimization models:

- **Specification:** N = 49, ω_p = 0.24 π, ω_S = 0.30π, R_p = 2 dB
- **Target:** Comparing the ILP method for model 6, 12 and 13

Test result is shown in Fig. 2 which supports the conclusion that specifications achieved by three different models are the same. It is this equivalence that provides engineers more optimal model alternatives in real-world applications.

Table 1: Test result of example 3

	CLP (G = 2 N)	CLP (G = 8 N)	ILP
ω_P	0.044516	0.044451	0.044444
ω_S	0.066634	0.066662	0.066667
R_p (dB)	0.22198	0.20205	0.20000
A_s (dB)	54.368	54.502	54.537
Λ_W	9.9025e-2	1.0128e-2	5.1890e-10
Time (sec)	2.4390	9.2416	4.6465
I.t.	-	-	4

CLP: Conventional LP method, ILP: Iterative LP method

Example 3: the LPF design in the GMC wireless systems:

- **Specification:** The prototype LPF in the Generalized Multi-Carrier (GMC) system (Hua *et al*. 2004), where, N = 217, $\omega_c = 1/18\pi$, R_p = 0.2 dB, roll-off factor α equals 0.20
- **Target:** Minimizing the stopband attenuation A_s

Here only inequality constraints are imposed on the passband and stopband. Examples with transition constraints, i.e., the constraint on ω_c, are to be given in Example 4.

In the CLP method, there are:

$$\left\lceil \frac{G\omega_p}{\pi-(\omega_s-\omega_p)} \right\rceil \left(G - \left\lceil \frac{G\omega_p}{\pi-(\omega_s-\omega_p)} \right\rceil \right)$$

frequency points uniformly distributed in [0, ω_p] ([ω_S, π]). G may equal 2 N or 8 N for different precisions. Test results are shown in Table 1 and Fig. 3, where CLP and ILP represent the CLP method and the proposed ILP method. Note ‘I.t.’ denotes the iteration times and ‘dB’ is the logarithmic unit.

From Fig. 3 and Table 1, people explicitly see that the CLP method produces frequencies (ω_p, ω_S) slightly deviated from the desired specification and the deviation

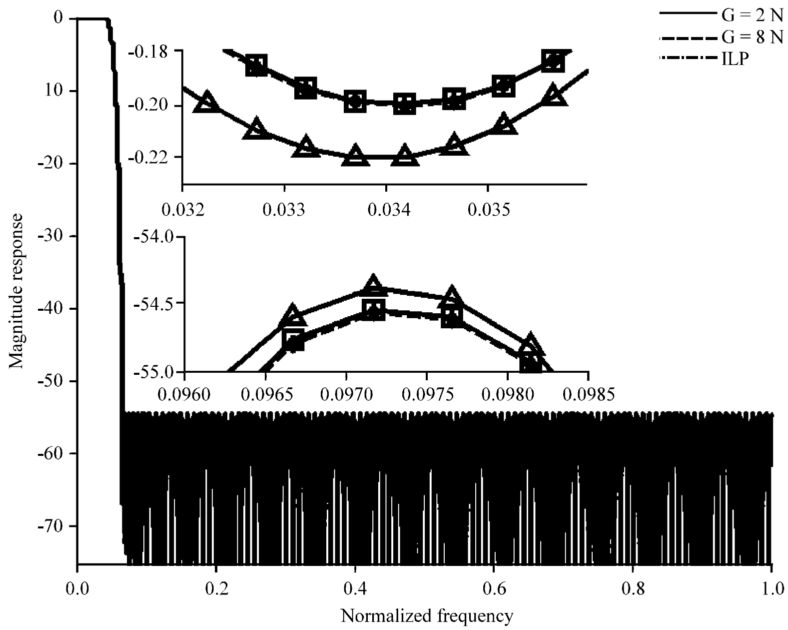

Fig. 3: Magnitude response in example 3: x-axis is the digital frequency and y-axis is the response amplitude

decreases as G increases. However, the proposed ILP method produces accurate frequencies (ω_p, ω_S) exactly equal to the desired specification. Moreover, the proposed method fixed the passband ripple to 0.2 dB precisely while the conventional method yields a slightly larger passband ripple due to that the constraint condition doesn't impose on Ω_{extrm}. In addition, the computation load of the CLP method increases drastically as G increases. For example, in the case that G equals 8 N, it takes about ten seconds, whereas the ILP method only takes about four seconds which once again suggests its superiority to the CLP method.

Example 4: The bandwidth constraint.

In wireless communications, sometimes people care for the signal bandwidth, generally the -3 dB bandwidth denoted by ω_c. if ω_c is constrained, what happens?

- **Specification:** The same as example 3
- **Target:** Investigating the influence of ω_c constraint

Table 2 illustrates the influence of constraint on ω_c. Compared with Table 2, ω_S and R_p are remains unchanged while A_s degrades significantly. If one fixes ω_c = 1/18π precisely, the worst case is presented. Relaxing the constraint ω_c, i.e., allowing ω_c ranges from ω_c (1-ζ) to ω_c (1+ζ), better A_s can be achieved. On the other hand, the constraint on ω_c results in larger R_p which reduces the

Table 2: Test results of example 4

Constraint condition	$\zeta = 0$	$\zeta = 0.005$	$\zeta = 0.05$
ω_p (π)	0.049435	0.049085	0.046288
ω_c (π)	0.055556	0.055278	0.052778
ω_S (π)	0.066667	0.066667	0.066667
R_p (dB)	0.20000	0.20000	0.20000
A_s (dB)	37.479	38.271	47.415
Λ_W	1.0928e-9	4.3146e-11	1.9053e-11
Time (sec)	3.8934	4.8810	5.3027
I.t.	4	4	4

transition band width and brings benefits in some sense. To sum up, if one has to constrain ω_c, he had better throw a range constraint and increase the filter length to ensure $A_s \geq 40$ dB.

Example 5: Comparison with the PSO algorithm:

- **Specification:** The same as example 2
- **Target:** Comparing the PSO method with the ILP method
- **Optimization model:** Model 6

PSO algorithm is one of the most important heuristic algorithms. It was first introduced by Kennedy and Eberhart (1995) and was further refined by many researchers. Among them, the Adaptive Particle Swarm Algorithm (APSO) (Zhan *et al*., 2009) exhibits good performance. Thus, we exploited the APSO algorithm and the canonical PSO algorithm as the optimization tools for FIR filters design.

There are two methods to obtain the filter coefficient by PSO algorithm. The first one is to exploit the PSO algorithm to search h directly (Ababneh and Bataineh, 2008), named the direct-search approach here. The other approach is frequency sampling method, i.e., exploiting the PSO algorithm to search samples in the frequency domain and the filter coefficient h can be obtained by solving a linear matrix equation (Lim, 1990; Gu *et al*., 2012), named the indirect-search approach here.

The simulation results can be found in Table 3 and Fig. 4. As for the direct-search approach, all the elements of h are clamped at 0.5 (Ababneh and Bataineh, 2008). On the other hand, the non-uniformly sampling is exploited for the indirect-search approach, where the passband (stopband) is equivalently divided into:

$$\left\lceil \frac{N-1}{2}\cdot\frac{\omega_p}{\pi-(\omega_s-\omega_p)} \right\rceil \left(\frac{N-1}{2} - \left\lceil \frac{N-1}{2}\cdot\frac{\omega_p}{\pi-(\omega_s-\omega_p)} \right\rceil \right)$$

points and the passband (stopband) samples are clamped within [0.7, 1.3] ([-0.3, 0.3]).

The result shows that the indirect-search approach outperforms the direct-search method and the APSO algorithm outperforms the canonical PSO algorithm. However, even the performance of the best $APSO^F$ algorithm is inferior to that of ILP and the former even takes significant large computation times. Accordingly, we can draw a conclusion that in our study, the ILP algorithm is superior to the PSO algorithm.

Table 3: Test results of example 5

	PSO^D	$APSO^D$	PSO^F	$APSO^F$	ILP
R_p (dB)	9.8974	3.6811	2.0509	2.0077	2.0000
A_s dB)	37.684	43.567	47.877	48.041	48.0710
Λ_W	0.87699	0.82671	0.95239	0.75911	1.3302e-8
Time (sec)	26423	26765	33737	30277	1.5095

D: Direct search method, F: Frequency sampling method

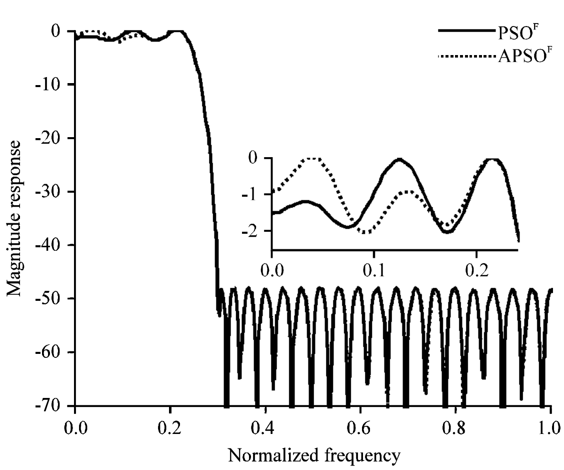

Fig. 4: Magnitude response in example 5: x-axis is the digital frequency and y-axis is the response amplitude

CONCLUSION

An iterative linear programming method for equiripple LPF designing is proposed in this study which exploits the non-uniform frequency sampling and exchange algorithm to reduce the constrain number (the discrete frequency number) and overcomes drawbacks of the conventional method. Numerical computations also show that the proposed method achieves better performance than conventional linear programming method and the modern heuristic algorithms.

ACKNOWLEDGMENTS

This study was supported by Zhejiang provincial NSF under grant No. Y1090645, the key project of Chinese Ministry of Education under grant No. 210087 and in part by the open research fund of National Mobile Communications Research Laboratory, Southeast University (No. 2010D06).

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