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Finite Element Analysis of Deformation of Saturated Soft Clay Foundation in Axisymmetric State

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Abstract: Analysis of foundation deformation is an important part in geotechnical engineering research field. Saturated soft clay has the property of high compressibility and low permeability, therefore it has great academic value and application value to improve the method analyzing deformation of soft clay foundation. In this study, through analyzing the deformation mechanism of soft clay foundation, the interrelation of instantaneous deformation and consolidation deformation of saturated soft clay foundation is studied and a computation model of instantaneous deformation and consolidation deformation is built, based on Biot Consolidation Theory. With FEM Theory, nonlinear program is composed to solve foundation deformation in axi-symmetric state and using this program, deformation of soft clay foundation with tank is analyzed. Results show that the method presented in this study is efficient and suitable in analyzing instantaneous deformation and consolidation deformation.

Key words: Nonlinear numerical simulation, biot consolidation theory, instantaneous deformation, consolidation deformation, pore water pressure

INTRODUCTION

Estimate of sedimentation value and sedimentation rate is an important problem in engineering design, because soft clay foundations have high compressibility, low permeability and long duration of consolidation deformation. We can find a problem from existing theory about deformation analysis of foundation that they research instantaneous or consolidation deformation individually and few researchers about interrelation of both deformations. Therefore, it is of great academic and practical value to improve the analysis method of foundation settlement. A computation model of instantaneous deformation and consolidation deformation is built on the basis of Biot consolidation theory and consideration of mechanism of instantaneous deformation and consolidation deformation and consideration of interrelation of both deformations in this study. By numerical simulation, a nonlinear axi-symmetric FEM program is compiled to solve for foundation deformation.

FUNDAMENTAL EQUATION OF AXI-SYMMETRIC BIOT CONSOLIDATION ANALYSIS

The master-control equation of Biot consolidation theory consists of balance equation, constitutive

equation, geometric equation, effective stress principle, balance equation of void fluid, continuity equation about soil body (Fig. 1). (Gong, 2000) We can get the following calculation equation about the infinitesimal soil body which length of side is dr and dz and included angle is $d\theta$ under axi-symmetric condition:

$$[\partial_r]^T [D] [\partial_r] \{\delta\} + [\partial_r]^T \{M\} p = 0 \quad (1)$$

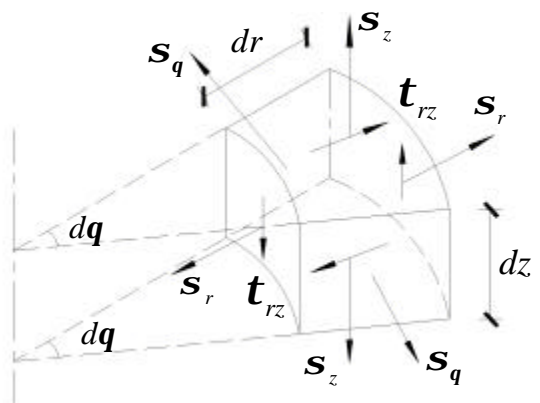


Fig. 1: Stress components of soil unit

$$\{M\}^T [\partial] [k] [\partial'] p + \frac{\partial}{\partial t} \{M\}^T [\partial] \{\delta\} = 0 \quad (2) \quad [K] \{r\} = \{R\} \quad (4)$$

Where:

$$[\delta] = [u \ w]^T; \{v\} = [v_r \ v_z]^T; \{M\} = \{1 \ 1 \ 1 \ 0\}^T$$

$$[D] = \frac{(1-\mu)E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1 & & & & \\ & \frac{\mu}{1-\mu} & & & \\ & & 1 & & \\ & & & \frac{\mu}{1-\mu} & \\ & & & & 1 \\ & & & & & \frac{1-2\mu}{2(1-\mu)} \end{bmatrix}$$

$$[\partial] = \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ \frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix}; [\partial_r] = \begin{bmatrix} \frac{\partial}{\partial r} + \frac{1}{r} & 0 \\ -\frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} + \frac{1}{r} \end{bmatrix}$$

$$[k] = \frac{1}{\gamma_w} \begin{bmatrix} k_r & 0 \\ 0 & k_z \end{bmatrix}; [\partial'] = \begin{bmatrix} \frac{\partial}{\partial r} & \frac{\partial}{\partial z} \end{bmatrix}^T$$

FINITE ELEMENT SOLUTION OF AXI-SYMMETRIC BIOT CONSOLIDATION ANALYSIS

With fundamental equation of axi-symmetric Biot consolidation analysis separated in space and time, we have element incremental equation of axi-symmetric Biot consolidation analysis (Shao *et al.*, 2011):

$$[K]^e \{\Delta U\}^e = \{\Delta R\}^e \quad (3)$$

where, $[K]^e$ is element consolidation matrix, 12×12 step:

$$[K_{ij}]^e = \begin{bmatrix} [K_{ij}]_{2 \times 2} & [K_{ij}]_{2 \times 4} \\ [K_{ij}]_{4 \times 2}^T & -\theta \Delta t K_{ij} \end{bmatrix}, i, j = 1, 2, 3, 4$$

$\{\Delta U\}^e$ is incremental array of unknown quantity of element node, 12×1 step, $\{\Delta u_i\} = [\Delta u, \Delta w_i, \Delta p_i]^T$ $i = 1, 2, 3, 4$, $\{\Delta R\}^e$ is load and flux incremental array of element node, 12×1 step, $\{\Delta R_i\} = [\Delta R_{r,i}, \Delta R_{z,i}, \Delta R_{q,i}]^T$ $i = 1, 2, 3, 4$.

With element consolidation matrix, load incremental array and flux incremental array of element node packed according to locator vector, we have total stiffness matrix, node displacement vector and pore water pressure vector (Wang *et al.*, 1987). We get equation of solving foundation deformation and pore water pressure:

where, $[K]$ is total stiffness matrix, $\{r\}$ is node displacement vector and pore water pressure vector, $r_i = \{r_i, w_i, p_i\}^T$ (the i node), $\{R\}$ is node load and flux matrix, $R_i = \{R_{r,i}, R_{z,i}, R_{q,i}\}^T$ (the i node).

NUMERICAL MODEL OF INSTANTANEOUS DEFORMATION AND CONSOLIDATION DEFORMATION AND CALCULATION FACTOR

Mechanics model calculation: Considering the nonlinearity of stress-strain relationship and the mechanics model calculation based on Duncan-Chang model (Fig. 2), also adopting Mohr-Coulomb failure criterion (Fig. 3) which the processing of damage to cell cube clay (Yu *et al.*, 1998).

Using nonlinearity Duncan-Chang model and Mohr-Coulomb failure criterion workout the tangent modulus value E and secant modulus value \bar{E} , including the 5 factors K, n, c, φ and R_f in Duncan-Chang model:

$$E = KP_a \left(\frac{\sigma_3}{P_a} \right)^n \left[1 - \frac{R_f(1-\sin \varphi)(\sigma_1 - \sigma_3)}{2c \cos \varphi + 2\sigma_3 \sin \varphi} \right]^2 \quad (5)$$

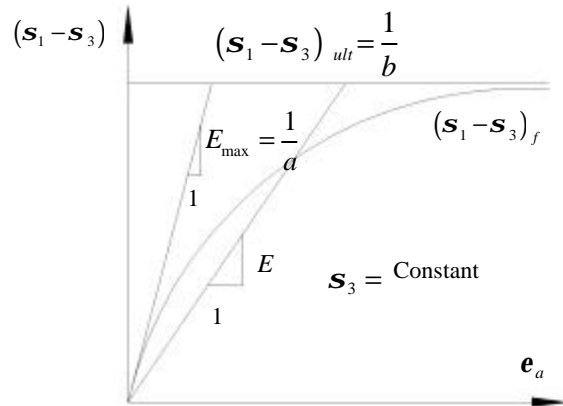


Fig. 2: Hyperbolic stress-strain relationship

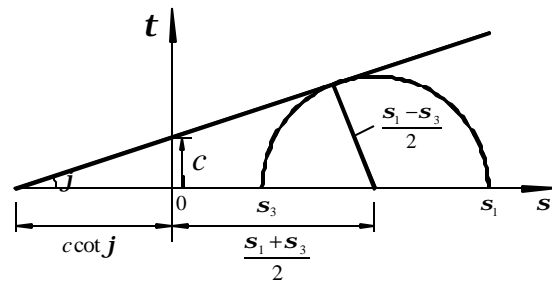


Fig. 3: Mohr-coulomb criterion

$$\bar{E} = K P_a \left(\frac{\sigma_3}{P_a} \right)^n \left[1 - \frac{R_f (1 - \sin \varphi) (\sigma_1 - \sigma_3)}{2c \cos \varphi + 2\sigma_3 \sin \varphi} \right] \quad (6)$$

where p_a is atmospheric pressure.

Model parameters

Model parameters of instantaneous deformation: General stress analysis is used in instantaneous deformation calculation. We have five factors is K , n , c , φ and R_f in Duncan-Chang model by non-drained triaxial consolidation test. In addition, Poisson ratio μ equal 0.48 in this study (Zhang *et al.*, 2000).

Model parameters of consolidation deformation: Effective Stress analysis was used in consolidation deformation calculation. We adopted Duncan-Chang model with volume compression modulus K_v and tangent modulus E expressed. Because μ varies through the derivation processing, from Eq. 7 got constant value K_v to work out Poisson ratio μ of each unit. We have five factors is K , n , c , φ and R_f by drained triaxial shear test and gave compression index C_c and pore-solids ratio e by compression test:

$$K_v = \frac{(1 + e_0)(1 + 2K_0)}{3C_c} p \ln 10 \quad (7)$$

$$\mu = \frac{1}{2} \left(1 - \frac{E}{3K_v} \right) \quad (8)$$

Derivation of initial pore water pressure: A key problem in solving Eq. 4 was to determine increment of initial pore water pressure when clay was loaded instantaneous. In this article, directly used the test results from quote [3] and put forward to calculating increment of pore water pressure. For considered the nonlinearity character of clay, increment of initial pore pressure:

$$\Delta u = \frac{\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3}{3} + \left[\frac{M}{\left(M - N \frac{T}{\sqrt{2}\sigma_c} \right)^2} - \frac{1}{3} \right] \frac{1}{\sqrt{2}} \Delta T \quad (9)$$

where, \bar{M} is the intercept of straight-line in the reference axis after normalization; \bar{N} is the slope of straight-line in the reference axis after normalization; σ_c is isoline of consolidation pressure;

$$M = \frac{1}{\bar{M}}; N = \frac{\bar{N}}{\bar{M}}$$

$$T = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

$$\Delta T = \Delta \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

EXAMPLES

Engineering Survey of Examples: To prove the applicability and efficiency of proposed approaches and programs, a saturation soft clay ground was chosen as deformation analysis. The size of ground is 70m long and 40 m deep, including 5 layers which are sand mat (layer 1), muddy and silty clay (layer 2), muddy clay (layer 3), muddy and silty clay (layer 4) and muddy and silty clay (layer 5). Now list its physics index in Table 1. Loaded radius is 10m. Load is evenly distributed load and it is 10t/m².

Finite elements program computation: Base on Biot Consolidation Theory, a nonlinear axi-symmetric FEM program was complied to calculate instantaneous deformation and consolidation deformation with Matlab. Parameter of pore water pressure \bar{M} is 0.155 and \bar{N} is 1.056. List its parameters in Table 2 and 3.

Where K' , n' , c' and φ' are effective stress coefficients to consolidation calculation and μ , K , n , c and φ are general stress coefficients to instantaneous calculation.

Table 1 Physics index of the clay in examples

Layers	Thickness (m)	Volumetric weight γ (N/m ³)	Horizontal penetration coefficient (m/s)
1	0.6	20000	0.1e-3
2	8.0	18000	0.1e-7
3	8.0	17300	0.1e-8
4	12.0	18100	0.3e-7
5	12.0	18300	0.5e-7

Layers	Vertical penetration coefficient (m/s)	Compression index	Pore-solids ratio e
1	0.1e-3	0.150	0.60
2	0.1e-7	0.219	1.10
3	0.1e-8	0.412	1.30
4	0.3e-7	0.216	1.05
5	0.5e-7	0.166	1.03

Table 2: Calculation parameters in finite elements program

Layers	μ	c (N/m ²)	φ (°)	K	N
1	0.25	0	32	595	0.65
2	0.33	15000	16	73	0.83
3	0.38	18000	14	60	0.85
4	0.33	13000	18	80	0.85
5	0.35	10000	20	85	0.86

Table 3: Calculation parameters in finite elements program

Layers	c' (N/m ²)	φ' (°)	K'	n'	R_r
1	0	32	595	0.65	0.79
2	0	30	26	0.83	0.80
3	0	28	22	0.85	0.85
4	0	26	31	0.85	0.80
5	0	28	35	0.86	0.90

Table 4: Vertical deformation of foundation

f	t		0	4	10	16
1/3	10	g	-0.429	-0.408	-0.280	-0.091
		s	-0.513	-0.468	-0.121	0.005
		z	-0.942	-0.876	-0.401	-0.085
		s/z	0.545	0.535	0.301	-0.063
1/4		g	-0.415	-0.395	-0.274	-0.091
		s	-0.507	-0.463	-0.124	0.006
		z	-0.922	-0.858	-0.399	-0.085
		s/z	0.550	0.540	0.312	-0.071
1/5		g	-0.409	-0.389	-0.272	-0.091
		s	-0.504	-0.461	-0.127	0.006
		z	-0.914	-0.850	-0.398	-0.084
		s/z	0.552	0.542	0.318	-0.072
f	t		20.500	34.000	46.000	70.000
1/3	10	g	-0.045	-0.016	-0.009	-0.006
		s	0.010	0.006	0.005	0.004
		z	-0.035	-0.010	-0.004	-0.002
		s/z	-0.292	-0.615	-1.205	-2.294
1/4		g	-0.045	-0.016	-0.009	-0.006
		s	0.011	0.006	0.005	0.004
		z	-0.035	-0.010	-0.004	-0.002
		s/z	-0.312	-0.619	-1.231	-2.500
1/5		g	-0.046	-0.016	-0.009	-0.006
		s	0.0111	0.0061	0.0049	0.0040
		z	-0.035	-0.010	-0.004	-0.002
		s/z	-0.322	-0.629	-1.290	-2.353

Table 5: Horizontal deformation of foundation

f	t		0	4	10	16
1/3	10	g	0.0000	-0.0184	-0.0916	-0.0944
		s	0.0000	0.0161	0.0503	0.0580
		z	0.0000	-0.0023	-0.0413	-0.0364
1/4		g	0.0000	-0.0194	-0.0949	-0.0959
		s	0.0000	0.0162	0.0513	0.0595
		z	0.0000	-0.0032	-0.0436	-0.0364
1/5		g	0.0000	-0.0197	-0.0963	-0.0964
		s	0.0000	0.0158	0.0514	0.0600
		z	0.0000	-0.0039	-0.0449	-0.0364
f	t		20.5000	34.0000	46.0000	70.0000
1/3	10	g	-0.0704	-0.0239	-0.0111	0.0000
		s	0.0395	0.0143	0.0066	0.0000
		z	-0.0309	-0.0096	-0.0045	0.0000
1/4		g	-0.0712	-0.0242	-0.0112	0.0000
		s	0.0407	0.0146	0.0067	0.0000
		z	-0.0305	-0.0096	-0.0045	0.0000
1/5		g	-0.0714	-0.0243	-0.0113	0.0000
		s	0.0411	0.0148	0.0068	0.0000
		z	-0.0303	-0.0095	-0.0045	0.0000

Loading ratio is 1/3, 1/4 and 1/5, loading interval is 10 days. Assuming horizontal penetration factor k and vertical penetration factor k1 is equal.

Results analysis of FEM program: Ground deformation results when consolidation deformation completed list in Table 4 and 5. Where f is loading ratio, t is loading time, g is consolidation deformation, s is instantaneous deformation, z is total deformation and s/z is instantaneous deformation ratio. The number of first line represents horizontal coordinate (Zhang, 2002).

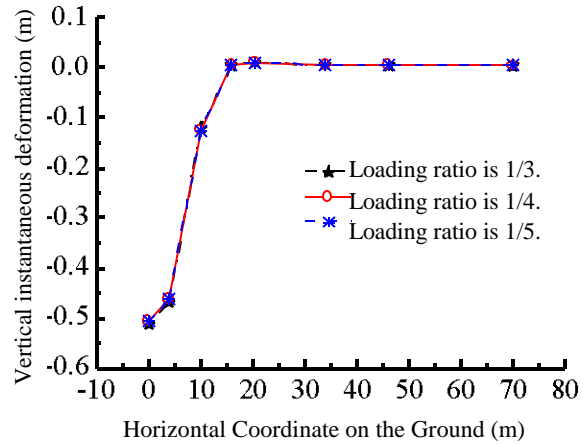


Fig. 4: Instantaneous vertical deformation in different load ratio of each point on the ground

Conclusions were obtained by the Table 4:

- Within side of loading area, direction of instantaneous vertical deformation is vertically downwards and the distance to loading centre further the instantaneous vertical deformation is smaller. Out side of loading area, direction of instantaneous vertical deformation is vertically upwards and firstly increase and then decrease, with the increase of horizontal coordinate
- Direction of consolidation vertical deformation decrease as horizontal and vertical coordinate increasing

Conclusions were obtained by the table five:

- Direction of instantaneous horizontal deformation is positive direction of horizontal coordinate axis. Instantaneous horizontal deformation firstly increase and then decrease as horizontal coordinate increasing and get maximum at the edge of load.
- Direction of consolidation horizontal deformation is negative direction of horizontal coordinate axis. Consolidation horizontal deformation firstly increase and then decrease as horizontal coordinate increasing and get maximum near edge of load

It was expressed by Fig. 4 and 5 that relationship curves between the vertical instantaneous and consolidation deformation and loading distance in different loading ratio. Vertical coordinate is the vertical deformation (m) and horizontal coordinate is coordinate value of each point on the ground (m).

Conclusions were obtained by the Fig. 4 and 5.

Within side of loading area, the results showed that the vertical instantaneous and consolidation deformation

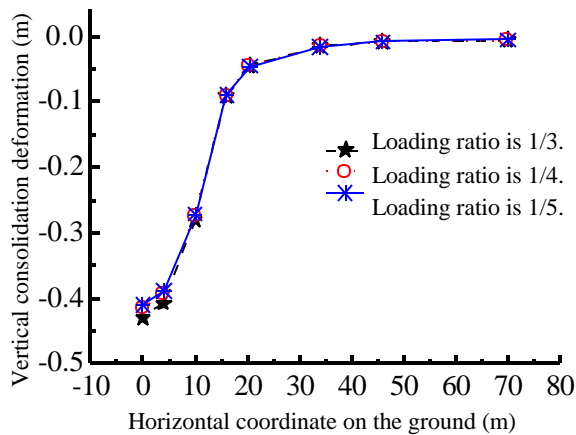


Fig. 5: Consolidation vertical deformation in different load ratio of each point on the ground

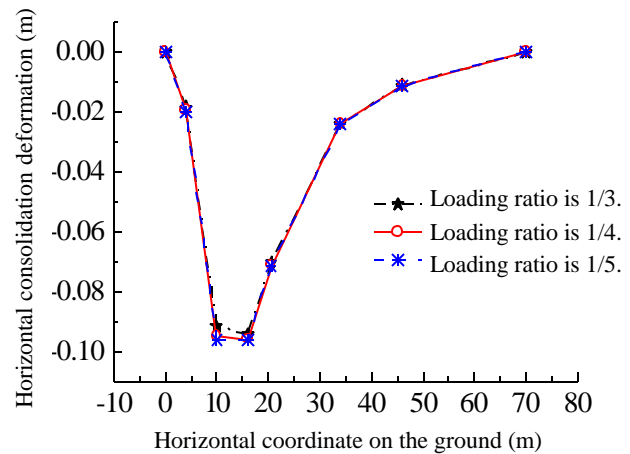


Fig. 7: Consolidation horizontal deformation in different load ratio of each point on the ground

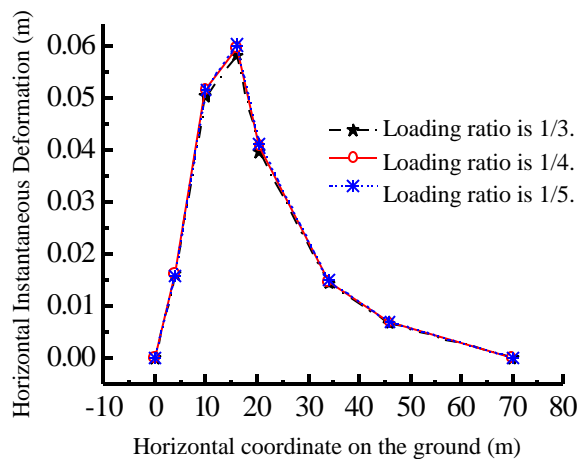


Fig. 6: Instantaneous horizontal deformation in different load ratio of each point on the ground

of each point on the ground decreased with loading ratio decreasing. Out side of loading area, the results showed that loading ratio had not significant influence on the vertical instantaneous and consolidation deformation of each point on the ground.

It was expressed by Fig. 6 and seven that relationship curves between the horizontal instantaneous and consolidation deformation and loading distance in different loading ratio. Vertical coordinate is the horizontal deformation (m) and horizontal coordinate is coordinate value of each point on the ground (m).

Conclusions were obtained by the Fig. 6 and 7.

- The trend of horizontal instantaneous deformation of each point on the ground: The horizontal

instantaneous deformation increased from zero to maximum ($x = 10$ m) with horizontal coordinate increasing, then it decreased from maximum to zero

- The horizontal instantaneous deformation of each point on the ground increased with loading ratio decreasing
- Outside of loading area, the results showed that loading ratio had not significant influence on the horizontal consolidation deformation of each point on the ground

CONCLUSION

In this study, instantaneous deformation, consolidation deformation and pore water pressure of axi-symmetric soft clay foundations were systematically studied with FEM on the basis of Biot Consolidation Theory and consideration of interrelation of instantaneous deformation and consolidation deformation. Nonlinear interrelation of stress and strain was considered by combining Duncan-Zhang model with compaction curve of soil. Results show that the method presented in this study is efficient and suitable to analyze instantaneous deformation and consolidation deformation which can be adopted in analysis of practical engineering.

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