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Lifetime Maximization Algorithm for Chain Wireless Sensor Networks

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Abstract: Many routing algorithms for chain wireless sensor networks were lack of theoretical analysis. It was difficult to judge whether these algorithms maximally prolonged network lifetime. In order to solve the problem, lifetime maximization algorithm for chain wireless sensor networks (LMA_CWSN) was proposed. The optimization method was used to research on the network lifetime maximization problem. Network optimization model was established. Non-negative slack variables and logarithmic barrier function were introduced. Newton method was used to solve the model. Finally, optimal value of network lifetime and optimal routing scheme were obtained. Simulation results show that LMA_CWSN makes full use of nodes' energy to improve network lifetime, converges to the optimal value of network lifetime and optimal routing scheme after iteration calculation and outperforms LEACH (low-energy adaptive clustering hierarchy), PEGASIS (power-efficient gathering in sensor information systems) and Ratio_w (ratio weight routing algorithm). Under certain conditions, LMA_CWSN can guide the data routing for chain wireless sensor networks, try to meet the optimal scheme when node transmission path and data amount are selected and provide reference to assess the performance of other routing algorithms.

Key words: Chain wireless sensor networks, network lifetime, optimization method, Newton algorithm

INTRODUCTION

Tiny sensor technologies and inter-node wireless communication capabilities give the wide application prospect for Wireless Sensor Networks (WSNs). At present, most applications of WSNs are divided into two categories: monitoring and tracking. Monitoring includes indoor and outdoor environmental monitoring, health monitoring, power monitoring, inventory position monitoring, factory automation monitoring, earthquake and structural monitoring. Tracking includes animal, human, vehicle and other targets tracking. In short, WSNs have been applied to many fields such as environmental and weather monitoring, floods warning, farm management, smart home and intelligent transportation. It presents tremendous commercial value and application potential, brings far reaching impact to the field of human production and life and is taken more and more attention by industry and academia (Yick *et al*., 2008).

Currently, the research on WSNs routing algorithm have achieved some results. Lindsey *et al*. (2002) proposes a chain routing algorithm (PEGASIS). All nodes in the monitoring region self-organize into a link by greedy algorithm. In data dissemination phase, each node receives the information from nearest upstream neighbor node. Then it transmits the fused information to sink node through nearest downstream neighbor node. The algorithm assumes that all nodes can communicate with sink node. It is obviously not feasible in the actual network. The nodes which are far away sink node cause excessive data latency. Heinzelman *et al*. (2000) proposes Low-energy Adaptive Clustering Hierarchy (LEACH). LEACH includes three-tier network architecture such as sink node, cluster-head node and sensor node. In the algorithm, sensor nodes transmit the sensed data to cluster-head nodes. Cluster-head nodes fuse the data of sensor nodes and transmit them to sink node by multi-hop among cluster-head nodes. In large-scale WSNs or node energy imbalance WSNs, LEACH algorithm randomly selects cluster-head nodes which may be concentrated in special region and can not cover the entire monitoring region. It easily leads to split network and increase node energy consumption. Zhu *et al*. (2009) proposes ratio weight routing algorithm (Ratio_w) and sum weight routing algorithm (Sum_w). The algorithms consider energy consumption of link communication, node residual energy, energy consumption factor and residual energy factor to construct a new weight function. Dijkstra algorithm is used to construct the shortest path tree whose root is sink node. All nodes transmit the data to sink node along the shortest path tree. Shin and Sun (2011) proposes Chain Routing With Even Energy

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Consumption (CREEC). CREEC improves the network lifetime by two strategies: Firstly, energy distribution of each node is balanced as far as possible. Secondly, the node energy consumption is preliminarily simulated and the feedback mechanism is used to save node energy. Chen *et al*. (2009) and Chen and Lin (2012) divide the sensing region of WSNs into a number of smaller regions. The nodes construct a link in each smaller region. All nodes in the network self-organize into chains. The above references research on the WSNs routing protocol to improve network lifetime and reduce node energy consumption. They focus on transmission path selection of network packet. But the algorithms are lack of theoretical analysis. It is difficult to determine whether these algorithms have maximum network lifetime. Wang *et al*. (2008) breaks multi-sink node routing problem into a number of single-sink node routing problem. It considers constraints with four core parameters such as transmission flow, node energy, signal-to-noise ratio and transmission bandwidth. It establishes the network lifetime optimization model and uses the KKT method to obtain the optimal solution. The algorithm is fit for small WSNs. With the expansion of network, the algorithm needs to consider too many factors and equality constraints. The solution process is complexity.

Some routing algorithms such as PEGASIS, LEACH and Ratio_w for CWSNs are lack of theoretical analysis. It is difficult to judge whether these algorithms maximally prolonged network lifetime. In order to solve the problem, lifetime maximization algorithm for chain wireless sensor networks (LMA_CWSN) is proposed. The LMA_CWSN can obtain the optimal value of network lifetime and optimal routing solution. It can guide the data routing for CWSNs, try to meet the optimal scheme when node transmission path and data amount are selected and provide reference to assess the performance of other routing algorithms.

NETWORK OPTIMIZATION MODEL

WSNs are widely applied to CWSNs, such as unattended dangerous status monitoring of freight train, status monitoring of single street lamp, online monitoring of transmission line, factory automation monitoring. As shown in Fig. 1, the sensor nodes sense and gather the information in the monitoring region with various types of sensors, transmit the sensed information to sink node through CWSNs. The routing of CWSNs has many problems. If traditional stepwise multi-hop routing protocol is used, the routing path will be single and nodes which are near the sink node will frequently forward the information of other nodes. It leads to high node energy

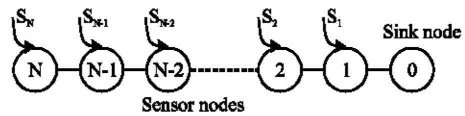

Fig. 1: Chain wireless sensor networks

Table 1: Notations

Symbol	Definition	Symbol	Definition
G (V, L)	Undirected connected graph, represents a WSN	E_i	Initial energy of node i
V	Set of nodes	F_{ij}	Transmission data amount from node i to node j
L	Set of links	S_i	Sensor rate of node i
$\lvert V\rvert$	Number of nodes	R_{max}	Node maximum transmission rate
N(i)	Neighbor node set of node i	E_{elec}	Unit data energy consumption parameter of circuit electronic
U(i)	Upstream neighbor node set of node I	ε_{fs}	Unit data energy consumption parameter of signal amplifier
X(i)	Downstream neighbor node set of node i	γ	Data loss coefficient
d_{max}	Maximum communication distance of node	ξ	Positive coefficient of logarithmic barrier function
d_{ij}	Distance between node i and node j	θ	Positive coefficient of logarithmic barrier function
B	Sink node	d_{in}	Distance between adjacent nodes
T	Network lifetime	I_M	Unit matrix
s^k	Newton step (a positive number) in kth iteration	Δx^k	Newton increment in kth iteration
x^k	Solution vector in kth iteration	ω^k	Dual Newton increment in kth iteration
A	Constraint matrix	f(x)	Objective function

consumption and premature failure and reduces the network lifetime. If each node directly transmits the information to sink node, the nodes which are farther away sink node will consume more energy and fail prematurely. Finding a CWSNs routing scheme to maximize network lifetime is great significance. Therefore LMA_CWSN is proposed in the following. The notations in the article are in Table 1.

System assumptions: Considering the application scenario of CWSNs, it is assumed that CWSNs have following characteristics:

- The positions of sink node and sensor nodes are fixed. All sensor nodes are uniformly distributed on each link. Sink node is distributed on one end of the link
- Because the deployment positions of nodes are pre-planned, sink node and sensor nodes can obtain the entire network topology information
- All sensor nodes have the same performance (such as radio maximum transmission power, maximum communication radius, distance between adjacent nodes, node energy consumption model et al.

- All sensor nodes in the network need to sense and transmit data. They take on data gather and relay tasks and transmit date to sink node by direct or multi-hop mode
- Energy of each sensor node is limited. But energy of sink node is unlimited

Optimization model establishment: As shown in Table 1, if node j is in the communication range of node i, node j is the neighbor node of node i. According to the definition of symbols, the equations $N(i) = \{j|d_{ij}<d_{max}, j\in V\}$, $U(i) = \{k|d_{kB}\geq d_{iB}, N(i)\}$ and $X(i) = \{k|d_{kB}<d_{iB}, N(i)\}$ are obtained. According to the characteristics of CWSNs, node i receives the data from upstream neighbor nodes and transmits data to sink node through downstream neighbor node (Hua and Yum, 2008).

According to the definition of network lifetime, the network lifetime maximization problem is transformed into the following optimization model. The goal of network optimization model 1 is to maximize network lifetime T and obtain optimal value F_{ij}.

$$\max\ T \tag{1}$$

$$\text{s.t.}: \sum_{j\in X(i)} F_{ij} = TS_i + \sum_{j\in U(i)} F_{ji}, \quad \forall i\in V \tag{2}$$

$$\sum_{j\in X(i)} F_{ij} + \sum_{j\in U(i)} F_{ji} \leq T * R_{max}, \quad \forall i\in V \tag{3}$$

$$\sum_{j\in U(i)} F_{ji}E_{elec} + \sum_{j\in X(i)} F_{ij}(E_{elec} + \varepsilon_{fs}d_{ij}^{\gamma}) \leq E_i, \quad \forall i\in V \tag{4}$$

$$F_{ij} \geq 0, \forall i\in V, \quad \forall j\in X(i) \tag{5}$$

where, Eq. 2 follows from flow balance constraint. The transmission data amount of node i consists of the received data amount from upstream neighbor nodes and its sensed data amount. Equation 3 follows from maximum data transmission constraint. Node data transmission bandwidth is limited and the total transmission data amount is also limited. Equation 4 follows from energy constraint. In the network lifetime, for node i the energy consumption of received data is $\Sigma_{j\in U(i)}F_{ji}E_{elec}$, the energy consumption of transmission data is $\Sigma_{j\in X(i)}F_{ij}(E_{elec}+\varepsilon_{fs}d^{\gamma}_{ij})$.

Solution of optimization model: The non-negative slack variables y_i and z_i, $\forall i\in V$ are introduced. The formulas Eq. 4 and 5 are transformed into equation constraints:

$$\sum_{j\in X(i)} F_{ij} + \sum_{j\in U(i)} F_{ji} + y_i - T * R_{max} = 0, \quad \forall i\in V \tag{6}$$

$$\sum_{j\in U(i)} F_{ji}E_{elec} + \sum_{j\in X(i)} F_{ij}(E_{elec} + \varepsilon_{fs}d_{ij}^{\gamma}) + z_i - E_i = 0, \quad \forall i\in V \tag{7}$$

The objective function of optimization model 1 is not concave function. The optimal value obtained by the Newton method may be not the global maximum value. Equation 1 can be transformed into min(-logT). Its optimization model is transformed into:

$$\min(-\log T) \tag{8}$$

s.t.: constraints of Eq. 2, 5, 6 and 7:

$$z_i \geq 0,\ y_i \geq 0,\ \forall i\in V \tag{9}$$

Let:

$$f(x) = -\xi \log T - \theta\sum_{i\in V}\sum_{j\in X(i)} \log(F_{ij}) - \theta\sum_{i\in V}\log(y_i) - \theta\sum_{i\in V}\log(z_i) \tag{10}$$

where, $\theta \geq 1$, ξ is much larger than θ.

The optimization model is modified, i.e.:

$$\min(f(x)) \tag{11}$$
$$\text{s.t.: } A*x = c$$

where, $x = [T\{F_{ij}\}\ y_1\ldots y_{|V|}\ z_1\ldots z_{|V|}]^T$, $c = [0\ \ 0\ \ H]^T$, $H = [E_1, E_2, \ldots, E_{|V|}]$:

$$A = \begin{bmatrix} \begin{bmatrix}-S_1 & -S_2 & \cdots & -S_{|V|}\end{bmatrix}^T & \{C_k\} & 0 & 0 \\ \begin{bmatrix}-R_{max} & -R_{max} & \cdots & -R_{max}\end{bmatrix}^T & \{G_k\} & I_{|V|} & 0 \\ 0 & \{J_k\} & 0 & I_{|V|} \end{bmatrix} \tag{12}$$

The node can communicate with $n = d_{max}/d_{in}$ nodes. Vector x has $(n+2)|V|-(n^2-n)/2+1$ elements. Matrix A has $3|V|*((n+2)|V|-(n^2-n)/2+1)$ elements. According to formula 12, some elements of A are defined as follows:

$$C_k = \begin{cases} 1 & \text{Node i transmits data through link k} \\ -1 & \text{Node i receives data through link k} \\ 0 & \text{Others} \end{cases} \tag{13}$$

$$G_k = \begin{cases} 1 & \text{Node i transmits and receives data through link k} \\ 0 & \text{Others} \end{cases} \tag{14}$$

$$J_k = \begin{cases} E_{elec} & \text{Node i receives data through link k} \\ E_{elec} + \varepsilon_{fs}d_{ij}^{\gamma} & \text{Node i transmits data through link k} \\ 0 & \text{Others} \end{cases} \tag{15}$$

Newton method is used to solve the constrained optimization model 11. An initial solution vector x^0 is determined in any feasible domain. The update iteration formula of Newton method is:

$$x^{k+1} = x^k + s^k\Delta x^k \tag{16}$$

Δx^k is obtained based on the following linear Eq. 17:

$$\begin{pmatrix} \nabla^2 f(x^k) & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} \Delta x^k \\ \omega^k \end{pmatrix} = -\begin{pmatrix} \nabla f(x^k) \\ Ax^k - c \end{pmatrix} \quad (17)$$

Let $H_k = \nabla^2 f(x^k)$. According to Eq. 17, the solution of Δx^k can be divided into the following two formulas:

$$(AH_k^{-1} A^T)\omega^k = (Ax^k\text{-}c)\text{-}AH_k^{-1}\nabla f(x^k) \quad (18)$$

$$\Delta x^k = -H_k^{-1}(\nabla f(x^k)+A^T\omega^k) \quad (19)$$

where, $f(x^k)$ is secondary derivable for x .$^k H_k^{-1}$ is the positive diagonal matrix.

The Newton step s^k is:

$$s^k = \begin{cases} \dfrac{1}{\lambda(x^k)+1} & \lambda(x_i^k) > 0.25 \\ 1 & \lambda(x_i^k) \le 0.25 \end{cases} \quad (20)$$

where, $\lambda(x^k) = \sqrt{(\Delta x^k)^T \nabla^2 f(x^k) \Delta x^k}$. The s^k guarantees that each element of x^{k+1} is non-negative (Ren *et al*., 2012). The following is convergence analysis of LMA_CWSN when s^k is determined by formula 20.

Convergence analysis: To analyze the convergence of LMA_CWSN, the definition of self-concordant function is introduced.

Definition 1: (Wei *et al*., 2010): A convex function $g:R\rightarrow R$, $\forall x \in R$. If the inequality $|g'''(x)| \le 2g''(x)^{3/2}$ holds, the function g is self-concordant function.

Lemma 1: (Wei *et al*., 2010): A convex function $g:R^n\rightarrow R$, $\forall x$, $v \in R^n$ if function $\tilde{g}(t) = g(x+tv)$ is self-concordant function, the function g is also self-concordant function.

Let function:

$$\tilde{f}_k(s) = f(x^k+s\Delta x^k) \quad (21)$$

where, $\tilde{f}_k$ and $\tilde{f}_k(s^k)$, respectively denote $f(x^k)$ and $f(x^{k+1})$:

$$f(x^k) = -\xi \log T^k - \theta \sum_{i\in V} \sum_{j\in X(i)} \log(F_{ij}^k) - \theta \sum_{i\in V} \log(y_i^k) - \theta \sum_{i\in V} \log(z_i^k) \quad (22)$$

Theorem 1: The function $\tilde{f}_k$ in formula 21 is self-concordant function. The function f(x) in formula 10 is also self-concordant function.

Proof: $\tilde{f}_k(s)$ is convex function. The second and third derivatives of $\tilde{f}_k(s)$ are:

$$\tilde{f}_k''(s) = \xi\left(\frac{\Delta T^k}{T^k + s\Delta T^k}\right)^2 + \theta \sum_{i\in V} \sum_{j\in X(i)} \left(\frac{\Delta F_{ij}^k}{F_{ij}^k + s\Delta F_{ij}^k}\right)^2 + \theta \sum_{i\in V} \left(\frac{\Delta y_i^k}{y_i^k + s\Delta y_i^k}\right)^2 + \theta \sum_{i\in V} \left(\frac{\Delta z_i^k}{z_i^k + s\Delta z_i^k}\right)^2 \quad (23)$$

$$|\tilde{f}_k'''(s)| = 2\left|\xi\left(\frac{\Delta T^k}{T^k + s\Delta T^k}\right)^3 + \theta \sum_{i\in V} \sum_{j\in X(i)} \left(\frac{\Delta F_{ij}^k}{F_{ij}^k + s\Delta F_{ij}^k}\right)^3 + \theta \sum_{i\in V} \left(\frac{\Delta y_i^k}{y_i^k + s\Delta y_i^k}\right)^3 + \theta \sum_{i\in V} \left(\frac{\Delta z_i^k}{z_i^k + s\Delta z_i^k}\right)^3\right| \quad (24)$$

Because ξ, θ, T. F_{ij}, y_i, z_i are non-negative and $\xi \ge \theta \ge 1$, we have:

$$\begin{aligned} 2\tilde{f}_k''(s)^{3/2} &= 2\left(\xi\left(\frac{\Delta T^k}{T^k + s\Delta T^k}\right)^2 + \theta \sum_{i\in V} \sum_{j\in X(i)} \left(\frac{\Delta F_{ij}^k}{F_{ij}^k + s\Delta F_{ij}^k}\right)^2 + \theta \sum_{i\in V} \left(\frac{\Delta y_i^k}{y_i^k + s\Delta y_i^k}\right)^2 + \theta \sum_{j\in V} \left(\frac{\Delta z_i^k}{z_i^k + s\Delta z_i^k}\right)^2\right)^{3/2} \\ &\ge 2\left(\xi\left(\frac{\Delta T^k}{T^k + s\Delta T^k}\right)^2\right)^{3/2} + \sum_{i\in V} \sum_{j\in X(i)} \left(\theta\left(\frac{\Delta F_{ij}^k}{F_{ij}^k + s\Delta F_{ij}^k}\right)^2\right)^{3/2} + \sum_{i\in V} \left(\theta\left(\frac{\Delta y_i^k}{y_i^k + s\Delta y_i^k}\right)^2\right)^{3/2} + \sum_{i\in V} \left(\theta\left(\frac{\Delta z_i^k}{z_i^k + s\Delta z_i^k}\right)^2\right)^{3/2} \\ &= 2\xi^{3/2}\left|\left(\frac{\Delta T^k}{T^k + s\Delta T^k}\right)^3\right| + \theta^{3/2} \sum_{i\in V} \sum_{j\in X(i)} \left|\left(\frac{\Delta F_{ij}^k}{F_{ij}^k + s\Delta F_{ij}^k}\right)^3\right| + \theta^{3/2} \sum_{i\in V} \left|\left(\frac{\Delta y_i^k}{y_i^k + s\Delta y_i^k}\right)^3\right| + \theta^{3/2} \sum_{i\in V} \left|\left(\frac{\Delta z_i^k}{z_i^k + s\Delta z_i^k}\right)^3\right| \\ &\ge 2\left|\xi\left(\frac{\Delta T^k}{T^k + s\Delta T^k}\right)^3 + \theta \sum_{i\in V} \sum_{j\in X(i)} \left(\frac{\Delta F_{ij}^k}{F_{ij}^k + s\Delta F_{ij}^k}\right)^3 + \theta \sum_{i\in V} \left(\frac{\Delta y_i^k}{y_i^k + s\Delta y_i^k}\right)^3 + \theta \sum_{i\in V} \left(\frac{\Delta z_i^k}{z_i^k + s\Delta z_i^k}\right)^3\right| \\ &= |\tilde{f}_k'''(s)| \end{aligned} \quad (25)$$

According to Definition 1, $\tilde{f}_k(s)$ is self-concordant function. According to Lemma 2, f(x) is also self-concordant function. Therefore, the Theorem 1 is proved. According to the above definition and theorem, the convergence of optimization model (11) is analyzed in attenuation convergence phase ($\lambda(x^k)>0.25$) and quadratic convergence phase ($\lambda(x^k)\le 0.25$).

Attenuation convergence phase: Attenuation convergence phase researches on the changes of objective function value in each iteration when ($\lambda(x^k)>0.25$) . First Lemma 2 is given.

Lemma 2: (Wei *et al*., 2010): $g:R\rightarrow R$ is self-concordant function, $\forall s \in R$ and $s \ge 0$. If $sg''(0)^{1/2}<1$ holds, the following inequality holds:

$$g(s) \le g(0)+sg'(0)-sg''(0)^{1/2}-\log(1-sg''(0)^{1/2}) \quad (26)$$

Theorem 2: If $\lambda(x^k) = \sqrt{(\Delta x^k)^T \nabla^2 f(x^k) \Delta x^k} > 0.25$ holds, the following inequality of function f(x) in formula (10) holds:

$$f(x^{k+1}) - f(x^k) \le -\frac{(0.25)^2}{2(1+0.25)} = -0.025 \quad (27)$$

Proof: According to Eq. 21, we have:

$$\nabla^2 f(x^k)\Delta x^k + A^T\omega^k = -\nabla f(x^k) \quad (28)$$

Equation 28 multiplies $(\Delta x^k)^T$ on both sides, i.e.:

$$(\Delta x^k)^T\nabla^2 f(x^k)\Delta x^k + (\Delta x^k)^T A^T\omega^k = -(\Delta x^k)^T\nabla f(x^k) \quad (29)$$

$A\Delta x^k = 0$, then:

$$(\Delta x^k)^T\nabla^2 f(x^k)\Delta x^k = -(\Delta x^k)^T\nabla f(x^k) \quad (30)$$

The first and second derivatives of function $\tilde{f}_k(s)$ are obtained according to Eq. 30, i.e.:

$$\tilde{f}_k'(0) = (\nabla f(x^k))^T\Delta x^k = -((\Delta x^k)^T\nabla^2 f(x^k)\Delta x^k)^T = -\lambda(x^k)^2 \quad (31)$$

$$\tilde{f}_k''(0) = (\Delta x^k)^T\nabla^2 f(x^k)\Delta x^k = \lambda(x^k)^2 \quad (32)$$

According to Lemma 2, $0\le s\le 1/(\lambda(x^k)+1)$, we have:

$$\tilde{f}_k(s) - \tilde{f}_k(0) \le -s\lambda(x^k)^2 - s\lambda(x^k) - \log(1 - s\lambda(x^k)) \quad (33)$$

According to Eq. 20 and 21, we have:

$$f(x^k + s^k\Delta x^k) - f(x^k) = \tilde{f}_k(s^k) - \tilde{f}_k(0) \le -s^k\lambda(x^k)^2 - s^k\lambda(x^k) - \log(1 - s^k\lambda(x^k))$$
$$= -\frac{\lambda(x^k)^2}{\lambda(x^k)+1} - \frac{\lambda(x^k)}{\lambda(x^k)+1} - \log(1 - \frac{\lambda(x^k)}{\lambda(x^k)+1}) = \log(\lambda(x^k)+1) - \lambda(x^k) \quad (34)$$

is Taylor expanded, i.e.:

$$\log(\lambda(x^k)+1) \le \lambda(x^k) - \frac{\lambda(x^k)^2}{2(1+\lambda(x^k))} \quad (35)$$

According to Eq. 34 and 35, we have:

$$f(x^{k+1}) - f(x^k) \le -\frac{\lambda(x^k)^2}{2(1+\lambda(x^k))} \quad (36)$$

When $\lambda(x^k)>0.25$, $-\lambda(x^k)^2/2(1+\lambda(x^k))$ is monotonically decreasing function of $\lambda(x^k)$. When $\lambda(x^k)$ infinitely tends to 0.25, it has maximum value, i.e., the inequality Eq. 27 is obtained. Therefore, the Theorem 2 is proved.

In summary, according to the Theorem 2, in attenuation convergence phase, the objective function value drops by at least 0.025 than the previous iteration value. After iteration calculation, the objective function value tends to be minimum value. At that time it enters quadratic convergence phase from attenuation convergence phase.

Quadratic convergence phase: Quadratic convergence phase researches on the changes of objective function value in each iteration when $\lambda(x^k)\le 0.25$. The Lemma 3-5 is given.

Lemma 3: (Wei *et al.*, 2010): $g:R^n\rightarrow R$ is self-concordant function. If $\lambda = \sqrt{(x-y)^T\nabla^2 g(x)(x-y)} \le 1$, $\forall x, y\in R^n$ holds, the following inequality holds:

$$(1-\lambda)^2\tau^T\nabla^2 g(x)\tau \le \tau^T\nabla^2 g(y)\tau \le \frac{1}{(1-\lambda)^2}\tau^T\nabla^2 g(x)\tau \quad (37)$$

where, $\tau\in R^n$.

Lemma 4: (Wei *et al.*, 2010): $g:R^n\rightarrow R$ is self-concordant function, Δx^k is Newton increment. If $\lambda(x^k) = \sqrt{(\Delta x^k)^T\nabla^2 g(x^k)(\Delta x^k)} \le 1$ holds, the following inequality holds:

$$(x^k+\Delta x^k)^T\nabla^2 g(x^k+\Delta x^k)(x^k+\Delta x^k) \le \frac{\lambda^2}{1-\lambda}\sqrt{(x^k+\Delta x^k)^T\nabla^2 g(x^k)(x^k+\Delta x^k)} \quad (38)$$

Lemma 5: (Wei *et al.*, 2010): $g:R^n\rightarrow R$ is self-concordant function, Δx^k is Newton increment, g^* is the minimum value of objective function. If $\lambda(x^k) = \sqrt{(\Delta x^k)^T\nabla^2 g(x^k)(\Delta x^k)} \le 0.68$ holds, the following inequality holds:

$$g^* \ge g(x^k) - \lambda(x^k)^2 \quad (39)$$

According to Lemma 3-5, the Theorem 3 can be deduced.

Theorem 3: f^* is the minimum value of function $f(x)$ in Eq. 10. If $\lambda(x^k) = \sqrt{(\Delta x^k)^T\nabla^2 f(x^k)\Delta x^k} \le 0.25$ holds, the following inequality holds:

$$\lim_{m\to\infty}\sup f(x^{k+m}) - f^* \le \frac{(0.75)^{-2}}{(2.25)^{2^m}} \quad (40)$$

Proof: According to Eq. 24 and Lemma 3-4, we have:

$$\lambda(x^{k+1})^2 = \sqrt{(\Delta x^{k+1})^T\nabla^2 f(x^k+s^k\Delta x^k)\Delta x^{k+1}} \le \frac{\lambda(x^k)^2}{1-\lambda(x^k)}\sqrt{(\Delta x^{k+1})^T\nabla^2 f(x^k)\Delta x^{k+1}}$$
$$\le \frac{\lambda(x^k)^2}{1-\lambda(x^k)} * \frac{1}{1-\lambda(x^k)}\sqrt{(\Delta x^{k+1})^T\nabla^2 f(x^k+s^k\Delta x^k)\Delta x^{k+1}} = \frac{\lambda(x^k)^2}{(1-\lambda(x^k))^2}\lambda(x^{k+1}) \quad (41)$$

Equation 41 is simplified with $\lambda(x^{k+1})\ge 0$, i.e.:

$$\lambda(x^{k+1}) \le \frac{\lambda(x^k)^2}{(1-\lambda(x^k))^2} \quad (42)$$

When $\lambda(x^k)\le 0.25$, Eq. 42 is simplified:

$$\lambda(x^{k+1}) \le \frac{\lambda(x^k)^2}{(1-0.25)^2} = \frac{\lambda(x^k)^2}{(0.75)^2} \quad (43)$$

$$\lambda(x^{k+m+1}) \le \frac{\lambda(x^{k+m})^2}{(0.75)^2} = \frac{\lambda(x^k)^{2^{m+1}}}{(0.75)^{2+\cdots+2^{m+1}}} = \frac{\lambda(x^k)^{2^{m+1}}}{(0.75)^{2^{m+2}-2}} \quad (44)$$

where, m is a positive integer. According to Lemma 5, we have:

$$f(x^{k+m}) - f^* \le \lambda(x^{k+m})^2 \le \frac{\lambda(x^k)^{2^m}}{(0.75)^{2^{m+1}-2}} \le \frac{(0.25)^{2^m}}{(0.75)^{2^{m+1}-2}} = \frac{(0.75)^{-2}}{(2.25)^{2^m}} \quad (45)$$

Then, the Eq. 40 is obtained. In quadratic convergence phase ($\lambda(x^k) \le 0.25$), after iteration calculation, the objective function value converges to the minimum. Therefore, the Theorem 3 is proved.

From the above content, Newton method is used to solve the network optimization model 1 and obtain the optimal value of network lifetime.

Algorithm implementation: LMA_CWSN is a centralized algorithm. After it obtains the node information and executes the following steps to calculate the optimal value of network lifetime:

Step 1: T^0, F^0_{ij}, y^0_i, z^0_i and other parameters are initialized. The feasible initial value of strict positive vector x^0 is determined
Step 2: According to x^k, $\nabla f(x^k)$, H_k^{-1}, ω^k and Δx^k are calculated
Step 3: If k<M, go to step 4, else go to step 5
Step 4: According to Eq. 20, x^{k+1} is updated, k = k+1, go to step 2
Step 5: The optimal value of network lifetime and the corresponding values F_{ij} are obtained. LMA_CWSN is end

Pseudo code of LMA_CWSN is as follows:

Initialization phase;
k = 0;
while (k<M)
According to vector x^k, $\nabla f(x^k)$, H_k^{-1}, ω^k and Δx^k are calculated;
$s^k = \begin{cases} \dfrac{1}{\lambda(x^k)+1} & \lambda(x_i^k) > 0.25 \\ 1 & \lambda(x_i^k) \le 0.25 \end{cases}$;
$x^{k+1} = x^k + s^k \Delta x^k$
k = k+1;
end
The optimal value of network lifetime and the corresponding values F_{ij} are recorded

Time complexity of LMA_CWSN is time complexity of Newton method which iteratively executes M times. Time complexity of Newton method relates to the matrix multiplication in Eq. 22 and 23. Because time complexity of $A_{a,n} \times A_{n,b}$ is $\Theta(anb)$, its time complexity is $\Theta(|V|^3)$. Finally, time complexity of LMA_CWSN is $\Theta(M|V|^3)$. In short, the time complexity of LMA_CWSN is high. Its convergence needs certain time. Only when the energy consumption of data transmission is far greater than the energy consumption of optimal solution calculation, LMA_CWSN can work well.

SIMULATION AND ANALYSIS

Simulation parameters: In the simulation, the energy consumption of wireless data communication is only considered. The energy consumption of calculation, data fusion and query packet transceiver are not considered. The energy consumption of timeout retransmission and debug in the data transmission process are also not considered. In the network simulation region, the ordinates of all nodes are the same and the abscissa interval between adjacent nodes is 20 m. The simulation parameters in Table 2 are selected to iteratively calculate the network lifetime and the transmission data amount of nodes.

Simulation results analysis: In the simulation, the number of nodes is 10 and the related parameters in Table 2 are selected. The node distribution is shown in Fig. 2. The five-pointed star represents sensor node. The square represents sink node. All nodes arrange in the row, transmit the data to sink node with forwarding nodes and form the chain wireless network. The node IDs from left to right are 1-10. Then the LMA_CWSN is used to research on its performance. The network lifetime and transmission data amount of nodes are iteratively calculated 2000 times according to the algorithm implementation. Then, Fig. 3 and Table 2 are obtained.

The network lifetime of each iteration is obtained and normalized. As shown in Fig. 3, when the iteration number increases, the network lifetime increases. When the number of iteration is lower than 1000, the curve of network lifetime is quadratic change. When the number of

Table 2: Simulation parameters

Parameters	Value	Parameters	Value
E_{elec}	50 nJ bit^{-1}	ξ	100
ε_{fs}	100 pJ bit^{-1} m^{-2}	γ	2
d_{max}	80 m	M	2000
d_{in}	20 m	|V|	10-60
E	1000 J	S_i	100 kbit min^{-1}
θ	1	R_{max}	|V|*S_i bit min^{-1}

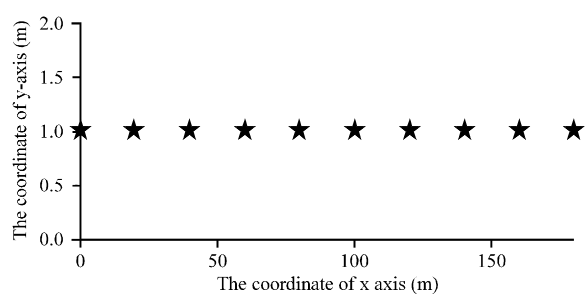

Fig. 2: Node distribution when |V| = 10

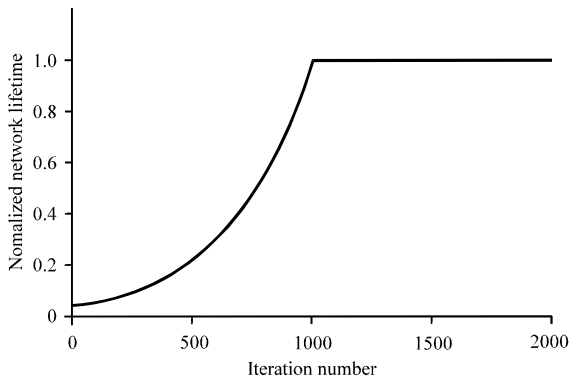

Fig. 3: Convergence of network lifetime

Table 3: Transmission data amount among nodes

Transmission node ID	Received node ID (transmission data amount)
1	2 (1.74), 3 (1.67), 4 (1.68), 5 (7.58)
2	3 (0.04), 4 (0.02), 5 (0.01), 6 (14.3)
3	4 (0.02), 5 (0.01), 6 (0.01), 7 (14.4)
4	5 (0.01), 6 (0.01), 7 (0.03), 8 (14.4)
5	6 (0.02), 7 (6.30), 8 (4.80), 9 (9.17)
6	7 (0.02), 8 (9.07), 9 (4.80), 10 (0.03)
7	8 (0.01), 9 (23.6), 10 (9.78)
8	9 (0.01), 10 (40.9)
9	10 (63.4)

The order of magnitude of transmission data amount is 10^8 bit, 1-9: Sensor nodes, 10: Sink node

iteration is larger than 1000, network lifetime essentially converges to the optimal value. Therefore, according to the analysis of theory and simulation, LMA_CWSN is convergence and has quadratic convergence rate.

Table 3 shows the routing scheme which can obtain optimal network lifetime. As shown in Table 3, each node transmits data to downstream neighbor nodes. When the nodes can not communicate directly with sink node (such as nodes 1-5), they tend to select the farthest downstream neighbor nodes. For example, node 1 is far away from sink node. To transmit the data to sink node, it tends to select the forwarding nodes 5 which is its neighbor node and is the nearest to sink node. It can reduce the energy consumption. When the nodes can directly communicate with sink node and there are several downstream neighbor nodes between them (such as nodes 6-7), to balance network lifetime and energy consumption, they also tend to select the downstream neighbor nodes. When the nodes are close to sink node (such as nodes 8-9), to save energy consumption of commutation, they directly transmit data to sink node in single hop.

In order to reflect the effectiveness of LMA_CWSN, LEACH (the probability which sensor node is selected to be cluster-head node is 0.25), PEGASIS, Ratio_w and LMA_CWSN are compared. The 10, 20, 30, 40, 50, 60 numbers of nodes (includes a sink node and other sensor nodes) are respectively selected. The related parameters in Table 2 are selected to calculate the network lifetime and total node energy consumption.

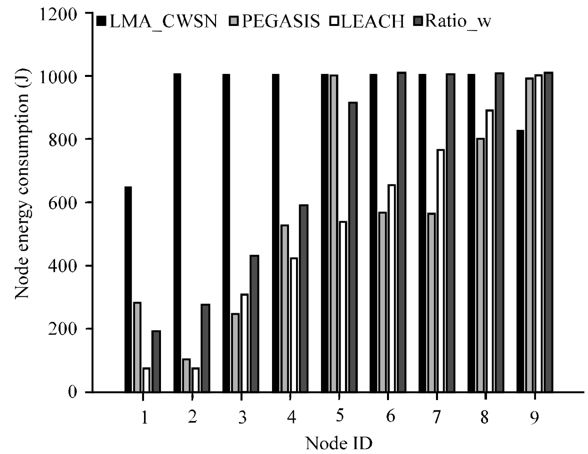

Fig. 4: Total energy consumption of each nodes when $|V| = 10$

As $|V| = 10$ for example, the total energy consumption of each node is shown in Fig. 4 when the first node runs out of energy. The node energy consumption of LMA_CWSN distributes uniformly. Each node consumes about 1000 J and all nodes are almost simultaneous failure. The node energy consumption of Ratio_w and PEGASIS relates to the distance between itself and sink node. The nodes which are closer to sink node consumes more energy. And vice versa the energy consumption is lower. The node energy consumption of LEACH distributes unevenly. Node 5 and 9 consume more energy. Others consume lower energy. It is the reason that LMA_CWSN can find the optimal network lifetime and routing scheme and nodes make full use of its energy to improve network lifetime. In Ratio_w, nodes select the neighbor node with smallest link weight to transmit data. In PEGASIS, nodes select the nearest downstream neighbor node to transmit data. LEACH randomly selects cluster-head nodes to establish cluster. The three algorithms all focus on transmission path selection of network packet and propose various data routing schemes. But these schemes are not the optimal routing scheme. When the network is failure, some nodes still have much energy.

In the simulation of LEACH, PEGASIS, Ratio_w and LMA_CWSN, in order to facilitate the comparison, network lifetime is defined by the number of Data Gathering Cycle (DGC) from the time at which network starts to the time at which the first node runs out of energy. Where, a DGC represents the time all nodes sense 100 kbit data and transmit them to sink node. As shown in Fig. 5, the network lifetime of LMA_CWSN (the number

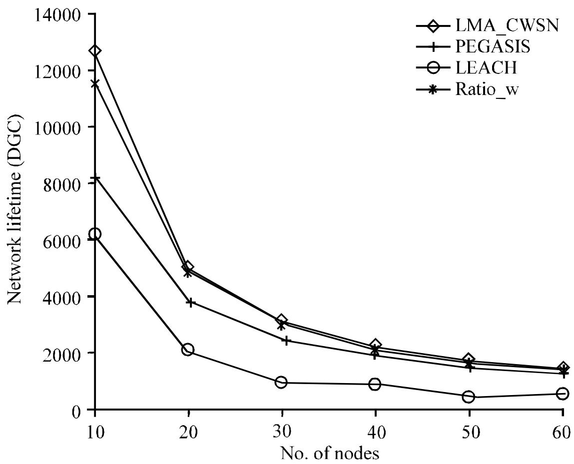

Fig. 5: Network lifetime comparison

of DGC) is larger than that of Ratio_w, PEGASIS and LEACH. It is optimal. The network lifetime of Ratio_w is second. The network lifetime of LEACH is the lowest. It is the reason that LMA_CWSN uses Newton method to maximize network lifetime and obtain optimal value. When the number of nodes increases, the difference in network lifetime between Ratio_w and LMA_CWSN is smaller. It is the reason that when the number of nodes increases, the transmission path selection of Ratio_w is reasonable and is relatively accordant with optimal routing scheme.

CONCLUSION

The network lifetime maximization algorithm for CWSNs is researched. Firstly, the lifetime maximization problem is transformed into network optimization model. Secondly, the Newton method is used to solve the optimization model. Nextly, the convergence of LMA_CWSN is theoretically analyzed, simulated and verified. The influence of node optimal routing scheme and the corresponding node energy consumption on network lifetime is simulated and analyzed. Finally, LEACH, PEGASIS, Ratio_w and LMA_CWSN are compared.

LMA_CWSN provides optimal value of network lifetime and optimal routing scheme, guides the data routing for CWSNs, tries to meet the optimal scheme when node transmission path and data amount are selected and provides reference to assess the performance of other routing algorithms. After further improvement, the algorithm can be applied to other types of WSNs. But the algorithm has large computation. Only when the energy consumption of data transmission is far greater than the energy consumption of optimal solution calculation, LMA_CWSN can work well.

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