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Optimal Inventory Modeling of Multi-ECHELON System for Aircraft Spares Parts

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Abstract: Owing to the expensive and sporadic demand nature of aircraft spare parts, the stock level of spare parts was difficult to confirm. How to maximize the availability of the aircraft fleet with cost constraint for spare parts was an important issue for airlines. A multi-ECHELON inventory model based on METRIC (Multi-ECHELON technology for recoverable items control) was proposed by allocating spare parts among the bases of airlines in this study. The model based on system approach instead of item approach to determine aircraft spare parts stock level in a multi-ECHELON system. First, the key system measure was selected, then, the Negative binomial distribution was employed to more accurately reflect the variance in part failure processes when the value of variance-to-mean ratio was bigger than one, Next, the marginal analysis method was applied to find the optimal position of spare parts among bases and depot, finally, two examples were given and the first one showed that the negative binomial distribution was more reasonable than Poisson distribution to describe the non-stationary demand process. The second one showed that the models in this paper were engineering applicability for airlines and could provide an effective theoretical and technical support.

Key words: Spare parts, multi-ECHELON, system approach, inventory optimization, negative binomial distribution

INTRODUCTION

Equipment-intensive industries such as airlines, nuclear power plants, played an ever more important role in society. The availability of such systems may strongly affect daily operations. As a consequence, it has required large quantities of spare parts to guarantee their availability which in turn resulted in excessive cost. The aviation industry, for example, it must carry about hundreds billion dollar each year to stock the spare parts they need to keep their airplanes flying (Rambau and Schade, 2010). So it was very important for airlines to determine the stock level with reasonable inventory investment.

The importance of service parts management had increased in the past decades. Many models for these kinds of stock allocation problem had been developed. They could be classified into two streams. The works of Yanagi *et al.* (1997), Wong *et al.* (2002) and Van Harten and Sleptchenko (2003) belonged to the first stream. They modeled the problem as a multi-dimensional Markovian problem. All research in the second stream was based on the well-know METRIC model proposed by Sherbrooke (1968) for the US Air force, including the works by Slay (1984), Graves (1985), Sherbrooke (1986, 2004), Axsater (1990), Zamperini and Freimer (2005) and Wong *et al.* (2006). These models were largely focused on spare parts inventories and was considered as system approaches that aiming at a high availability of complete technical systems, as opposed to more classical inventory management approaches, that were primarily directed towards a high availability of individual items. Compared to the models in the second stream, the models in the first stream give more exact results than the METRIC type models. However, they were more difficult to solve because of the huge multi-dimensional state spaces involved same as the multi-objective optimization applying evolutionary algorithms. Due to the complex nature of aircrafts' repair process, the METRIC model which could be evaluated and optimized within reasonable computation times, was chosen to solve this problem.

On the early version of the METRIC model, the number of LRUs in the pipeline which denoted the number of LRUs in repair or being resupplied from a higher ECHELON, was assumed to follow Poisson distribution (Jiang-sheng *et al.*, 2008; Wang and Kang, 2009; Rui *et al.*, 2011), but Sherbrooke (2004) proved that the value of variance-to-mean ratio of backorder at the depot was bigger than one which meant the Poisson distribution was not suitable for the distribution of the number of LRUs in the pipeline of bases which was composed by the backorders. In this study, the negative binomial distribution was employed to more accurately reflect the distribution of the number of LRUs in the pipeline of

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bases and a example was given to show that by taking into account not only the mean value but also the variance, it could obtain a much better estimation for the non-stationary demand.

The aim of this paper was to determine the stock level of those expensive but low-usage aircraft spare parts, LRUs (Line Replaceable Units), in multi-ECHELON maintenance organization with minimum cost. This paper analyzed LRUs demand generation, transfer process and the fault repair process in multi-ECHELON maintenance support organizations. Based on the research conclusion, the spares stock level was given in a numerical study.

MODEL DESCRIPTION AND ASSUMPTIONS

An airline company usually kept a central inventory of parts in its depot. Additionally, it also kept smaller 'outstation' inventory at the other airports bases where its aircraft had regularly scheduled landings and departures.

Generally, after an aircraft had landed at a base, engineering inspection was carried out prior to the next departure. When a LRU on an aircraft become defective, it should be removed and replaced by a serviceable one from the local stock (if available). If there was no available one in the local inventory for the replacement, a backorder was established and the aircraft will remain on the ground and will be delayed until an incoming flight brings a replacement part from the depot. The failed part could be repaired at the base for some minor problems but for the more serious problems it should be sent to the depot or to a special repair facility. At the same time, a functioning LRU was sent from the depot to the base. But if the depot was empty, a depot backorder was established. This does not necessarily imply that an aircraft was grounded but the risk of backorders at the base increases.

The transportation time for a defect LRU from a base to the central depot was assumed to be deterministic and known and the same was assumed for the transportation time from the central depot to a base. For simplicity, it was assumed that there was no difference between the bases in this respect.

The repair time for a defect LRU at the depot was assumed to be a random variable with expected value. An important assumption (approximation) in the model was that these repair time were independent and equally distributed. According to Palm's theorem, if demand for an item was a Poisson process with annual mean m and the repair time for each failed unit was independently and identically distributed according to any distribution with mean T years, then the steady-state probability distribution for the number of LRUs in repair had a Poisson distribution with mean mT.

There were the following variable definitions for the models:

- τ_{ij} = Average order and ship time from the depot to line base j for LRU_i
- $p(x)$ = Probability that the demand for a given LRU during a predefined time
- I = Index of LRU type i, i = 1, 2,..., I, where, I is the total number of LRUs in the system
- λ_{ij} = Average annual demand of the LRU_i at base j
- j = Index of base j, j = 0, 1,..., J, where, 0 is the depot and J is the total number of bases
- v_{ij} = Average repair time (in one year) of the LRU_i at base j
- ρ_{ij} = Probability of repair of the LRU_i at base j
- μ_{ij} = Average pipeline at base j, represents the average demand for the LRU_i under repair or resupply

MODEL TECHNIQUE

In this section we briefly discussed the models of multi-ECHELON inventory for airlines based on the METRIC theory. The problem will be solved in three steps:

Step 1: The definition of the expected backorder (EBO) was given and the relationship between the fleet availability and EBO was deduced

Step 2: For a single LRU, inventory models were developed to allocate stock level between the depot and the bases and marginal analysis was used to find the optimal allocation of stock levels between the bases and the depot with the cost constraints

Step 3: All LRUs were combined on a system using marginal analysis. It was shown how to construct an optimal cost-backorder curve for all LRUs as a system

Mean and variance for the backorders: At a given randomly chosen time, there was a famous balance equation for any spare which must be at one of the state. The balance equation was the basis for all of the analysis to come:

$$s = OH+DI-BO \quad (1)$$

The stock level, s, was the stock level; DI was the number of LRUs coming from repair and resupply, OH was the number of LRUs currently available in the inventory (on hand), BO was the number of backorders. They were all natural random variables which could only take on non-Negative integer values, Moreover, at each time at least one of BO and OH was zero. One of the variables changed, the others would change.

Therefore, BO and OH could be expressed as the following functions of X and s:

$$BO = (DI\text{-}s)^+ = \max\{0, DI\text{-}s\} \quad (2)$$

$$OH = (s\text{-}DI)^+ = \max\{0, s\text{-}DI\} \quad (3)$$

Suppose p(x) was the distribution of the number of LRUs coming from repair and resupply. The expected number of backorders, EBO(s), was thus:

$$EBO(s) = 1 \times P(x = s+1) + 2 \times P(x = s+2) + \ldots = \sum_{x=s+1}^{\infty} (x - s)p(x) \quad (4)$$

And the recursive expression of Eq. 4 was:

$$\begin{aligned} EBO(s) &= \sum_{x=s+1}^{\infty} (x - s)p(x) = \sum_{x=s}^{\infty}[x - (s-1)]p(x) - \sum_{x=s}^{\infty} p(x) \\ &= EBO(s-1) - (1 - \sum_{x=0}^{s-1} p(x) \end{aligned} \quad (5)$$

Referring to the definition of variance, the variance of backorders (VBO) was get:

$$VBO(s) = E[BO^2(s)]\text{-}[EBO(s)]^2 \quad (6)$$

The deriving expression of $E[BO^2(s)]$ is:

$$\begin{aligned} E[BO^2(s)] &= \sum_{x=s+1}^{\infty} (x-s)^2 p(x) = \sum_{x=s+1}^{\infty} [(x-s+1)^2 - 2(x-s) - 1]p(x) \\ &= \sum_{x=s+1}^{\infty} \{[x-(s-1)]^2 - (x-s) - [x-(s-1)]\} p(x) \\ &= \sum_{x=s}^{\infty} [x-(s-1)]^2 p(x) - \sum_{x=s+1}^{\infty} (x-s)p(x) - \sum_{x=s}^{\infty} [x-(s-1)]p(x) \\ &= E[BO^2(s-1)] - EBO(s) - EBO(s-1) \\ &= VBO(s-1) + [EBO(s-1)]^2 - EBO(s) - EBO(s-1) \end{aligned} \quad (7)$$

So the recursive expression of Eq. 6 is:

$$\begin{aligned} VBO(s) = {} & VBO(s\text{-}1)\text{-}EBO(s)\text{-}EBO(s\text{-}1) \\ & \text{-}[EBO(s)]^2 + [EBO(s\text{-}1)]^2 \end{aligned} \quad (8)$$

Objective function and constraints: Availability, A, was the expected percent of the aircraft fleet that was not down for any spare. Treating failures of LRU_i in different installed locations as independent, the probability that no randomly selected element was missing a LRU is:

$$A = 100 \prod_{i=1}^{I} \{1 - EBO(s_i)/(NZ_i)\}^{Z_i} \quad (9)$$

N was the number of aircrafts; Z_i was the number of the LRU_i that installed on an aircraft; i was the number of LRU type, An aircraft will be available only if there was no backorders for any of the occurrences of LRUs, An aircraft will be available only if there was no backorders for any of the occurrences of the ith LRU (which accounts for the exponent Z_i) and for any of the LRUs, Taking logarithms in Eq. 9:

$$\log(A/100) = \sum_{i=1}^{I} Z_i \log\{1 - EBO(s_i)/(NZ_i)\} \approx \sum_{i=1}^{I} EBO(s_i)/N \quad (10)$$

So the relationship between fleet availability and $EBO(s_i)$ was: max (A)_min (sum ($EBO(s_i)$)) and now the optimal mathematical statement for the aircrafts fleet at all bases was:

$$\begin{cases} \min \sum_{i=1}^{I} \sum_{j=1}^{J} EBO(s_{ij}) \\ \sum_{i=1}^{I} \sum_{j}^{J} c_i s_{ij} \le C \end{cases} \quad (11)$$

C is the total system cost targets, c_i was the cost of LRU_i; s_{ij} was the stock level of LRU_i at base j.

The pipeline and backorders at depot: For a single LRU, the pipeline at a base included the LRUs in repair and resupply and the resupply time depended on the EBO at depot.

The defected LRUs arrive to the bases were assumed to according to a Poisson process, then the average demand at depot was the sum of the LRUs that couldn't be repaired at the bases with intensity:

$$\lambda_{i0} = \sum_{j=1}^{J} \lambda_{ij}(1 - \rho_{ij}) \quad (12)$$

As the repair time had been assumed independent, it follows from Palm's theorem that the average number of LRU_i in the pipeline at depot, X_{i0}, was a Poisson random variable with:

$$\mu_{i0} = E[x_{i0}] = \lambda_{i0} v_{i0} \quad (13)$$

The $EBO(s_{i0})$ at depot, with the same recursive equation as Eq. 5 is:

$$EBO(s_{i0}) = EBO(s_{i0}) - [1 - \sum_{k=0}^{s_{i0}-1} Poisson(k, \mu_{i0})] \quad (14)$$

The variance of backorders was get according to Eq. 6:

$$\begin{aligned} VBO(s_{i0}) = {} & VBO(s_{i0}\text{-}1)\text{-}EBO(s_{i0})\text{-}EBO(s_{i0}\text{-}1) \\ & \text{-}[EBO(s_{i0})]^2 + [EBO(s_{i0}\text{-}1)]^2 \end{aligned} \quad (15)$$

The pipeline and backorders at bases: The pipeline of LRU_i at base j consisted of three parts: LRU_i in repair at the base j, LRU_i on order and LRU_i waiting at the depot for backorders. The fraction of all demands at the depot for LRU_i that was resupplied to base j was as below:

$$f_{ij} = \frac{\lambda_{ij}(1-\rho_{ij})}{\lambda_{i0}} \tag{16}$$

So the pipeline could be expressed as:

$$\mu_{ij} = \lambda_{ij}\rho_{ij}v_{ij}+\lambda_{ij}(1-\rho_{ij})\tau_{ij}+f_{ij}EBO(s_{i0}) \tag{17}$$

When s = 0, $EBO(s_{i0})$ was reduced to the mean of probability distribution and $VBO(s_{i0})$ was reduced to the variance of probability distribution, so the ratio $VBO(s_{i0})/EBO(s_{i0})$ was one. The typical behavior for the ratio was to increase as a function of s to a maximum value slightly larger than s and then decrease asymptotically to one, as shown in Fig. 1. Thus the probability distribution of backorder for LRUs at the depot was not Poisson unless LRUs stock level s = 0. Because μ_{ij} was composed by backorder function, so Poisson distribution wasn't suitable unless the stock level at the depot was zero.

The negative binomial was proved that its mean and variance was more suitable for the values of the pipeline than the Poisson by Slay (1984) and later by Graves (1985). So the Negative binomial distribution was used to describe the distribution of the number of LRUs in the pipeline at bases, μ_{ij} but the function had an added work requiring two parameters r_{ij} and p_{ij} which could get from the mean and the variance:

$$r_{ij} = \mu^2_{ij}/[Var_{ij}-\mu_{ij}] \tag{18}$$

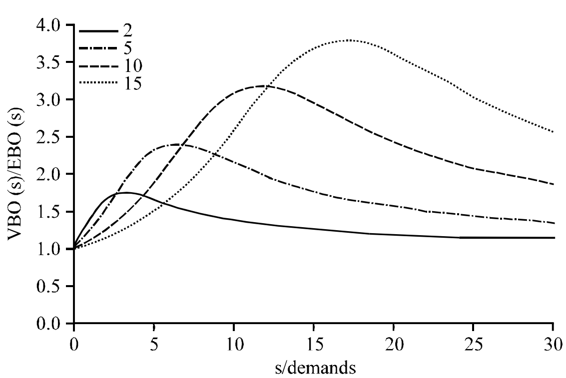

Fig. 1: VBO(s)/EBO(s) for various mean values of the Poisson

$$p_{ij} = \mu_{ij}/Var(s_{ij}) \tag{19}$$

The expression for the variance of the number of LRUs in the pipeline at the base j was got by Sherbrooke (2004):

$$Var_{ij} = \lambda_{ij}[\rho_{ij}v_{i0}+(1-\rho_{ij})\tau_{ij}]+f_{ij}(1-f_{ij})EBO(s_{i0}) +f^2_{ij}VBO(s_{i0}) \tag{20}$$

For computational purposes, it was useful to had recursion formulas. The derivation was below:

$$\begin{aligned} neg(x) &= \binom{x+r-1}{x}p^r(1-p)^x = \frac{(x+r-1)!}{x!(r-1)!}p^r(1-p)^x \\ &= \frac{(x+r-1)\times(x+r-2)!}{x*(x-1)!(r-1)!}p^r(1-p)^{x-1}(1-p) \\ &= \frac{x+r-1}{x}(1-p)neg(x-1) \end{aligned} \tag{21}$$

The Recursive expression of the negative binomial distribution was below:

$$\begin{cases} neg(k) = \dfrac{(k+r-1)}{k}(1-p)\times neg(k-1) \\ neg(0) = p^r \end{cases} \tag{22}$$

where, 0<p<1 and r cannot be integer.

According to Eq. 4 the expected backorders for LRU_i at base j was:

$$EBO(s_{ij}) = \sum_{x=s_{ij}+1}^{\infty}(k-s_{ij})\times neg(x) \tag{23}$$

Calculating and optimization procedure:

Step 1: Start with the depot stock level of zero for one LRU

Step 2: Calculate the number of LRUs in the pipeline for the depot from Eq. 13 and the expected backorders and variance from Eq. 14-15 of the depot

Step 3: Calculate the average number of LRUs in the pipeline for the base j from Eq. 17 and variance from Eq. 20. Then the two parameters r and p could get from Eq. 18 and 19

Step 4: Calculate the expected backorders for each stock level of bases from Eq. 23. Repeat for each base

Step 5: Use marginal analysis to combine the base backorder functions and obtain the minimum backorders for each number of LRU at bases

Step 6: If the level of depot stock was large enough, go to step 7; otherwise, increase the depot stock level by one and go to step 2

Step 7: Find the minimum sum of expected backorders value for each number of the LRU stock

Step 8: Repeats steps 1-7 for each LRU

Step 9: Use marginal analysis to combine the LRU solutions, where the delta of backorder must be divided by the LRU costs

To prove that marginal analysis produces optimal solutions, The EBO function was need to prove convex for any probability distribution:

$$\Delta EBO(s) = EBO(s+1)-EBO(s) = 1\times P(DI = s+2) +2\times P(DI = s+3)+...-1\times P(DI = s+1)-2\times P(DI = s+2) -3\times P(DI = s+3)-... = -P(DI = s+1)-P(DI = s+2) -P(DI = s+3)-...\leq 0 \quad (24)$$

$$\Delta^2 EBO(s) = P(DI = s+3)+2P(DI = s+4)+... -2\times P(DI = s+2)-4\times P(DI = s+3)-6\times P(DI = s+4)-... +P(DI = s+1)+2\times P(DI = s+2)+3\times P(DI = s+3) +4\times P(DI = s+4)+... = P(DI = s+1)\geq 0 \quad (25)$$

The first difference of EBO was less than or equal to zero and the second difference was greater than or equal to zero which was according with the definition of the convex function.

Since the expected backorder function was convex, the marginal analysis values {EBO(s-1) -EBO(s)}/c were non-increasing. The system backorders were convex also.

NUMERICAL EXAMPLE AND RESULTS

Here, we present two examples. The aim of the first one was to show the reasonable of selecting Negative binomical distribution to describe the non-stationary demand process and the second one elaborated the calculating and optimization process of a multi-ECHELON inventory problem.

Example 1: A real sample of spares demand in 36 months was shown in Table 1. The sample mean was [30×0+3×1+1×(2+3+4)]/36 = 0.33, so the column of $(y\text{-mean})^2$ was the deviation degree from which variance could got 0.778. The column of E/V was the value of variance-to-mean which stated clearly the demand process was non-stationary. Table 2 listed the sample frequency, the probability of Poisson distribution and Negative Binomial distribution adjusted to the sample whose mean was 0.33 and variance was 0.778. From this table could be seen there was 83.3% zero demand which was the low-usage nature of those expensive aircraft spare parts. Figure 2 was drawn according to Table 2 and it showed clearly that Negative binomial distribution could more accurately reflect the sample whose variance to mean was bigger than one.

Table 1: A real sample of spares demand

Month (x)	Spare demands (y)	$(y\text{-mean})^2$	E/V	Month (x)	Spares demands (y)	$(y\text{-mean})^2$	E/V
1	0	0.111	0.333	19	0	0.111	0.333
2	1	0.444	1.333	20	0	0.111	0.333
3	0	0.111	0.333	21	0	0.111	0.333
4	0	0.111	0.333	22	0	0.111	0.333
5	0	0.111	0.333	23	3	7.111	21.333
6	0	0.111	0.333	24	0	0.111	0.333
7	0	0.111	0.333	25	0	0.111	0.333
8	0	0.111	0.333	26	0	0.111	0.333
9	0	0.111	0.333	27	0	0.111	0.333
10	4	13.444	40.333	28	0	0.111	0.333
11	0	0.111	0.333	29	1	0.444	1.333
12	0	0.111	0.333	30	0	0.111	0.333
13	0	0.111	0.333	31	0	0.111	0.333
14	0	0.111	0.333	32	0	0.111	0.333
15	0	0.111	0.333	33	0	0.111	0.333
16	1	0.444	1.333	34	2	2.778	8.333
17	0	0.111	0.333	35	0	0.111	0.333
18	0	0.111	0.333	36	0	0.111	0.333

Table 2: Poisson and negative binomial distribution adjust to the sample

Demand	Occurrence	Sample frequency	Poisson	Negative binomial
0	30	0.833	0.717	0.809
1	3	0.083	0.239	0.116
2	1	0.028	0.040	0.041
3	1	0.028	0.004	0.018
4	1	0.028	0.000	0.008

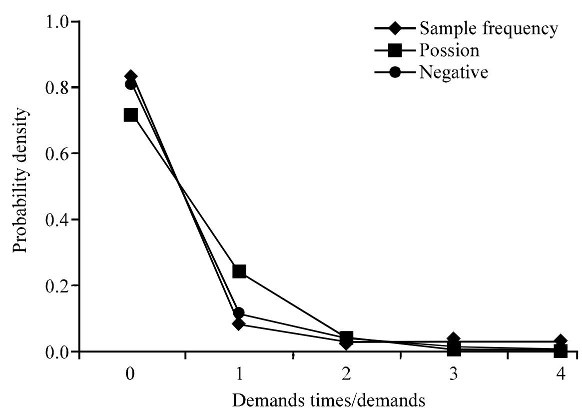

Fig. 2: The comparison of Poisson and negative distribution adjust to the sample

Example 2: In this example, there were two organizational levels as described above, two types of LRUs. See Fig. 3.

The input data for LRU_1: λ_{1j} = 20 demands/year, ν_{1j} = 0.01 years, ρ_{1j} = 0.2, τ_{1j} = 0.01 years, c_1 = 5, ν_{10} = 0.025 years; LRU_2: λ_{2j} = 10 demands/years, ν_{2j} = 0.01 years, ρ_{2j} = 0.1, τ_{2j} = 0.01 years, c_2 = 8, ν_{20} = 0.02 years.

Taking LRU_1 for example, when the depot stock level, s_{10} was zero and the following values λ_{10} = 4×20

×(1-0.2) = 64, μ_{10} = 64×0.025 = 1.6 were determined from Eq. 12 and 13. It was clearly that EBO(s_{10}= 0) was the mean of Poisson distribution which could be get from Eq. 4 and VBO(s_{10} = 0) was the variance of Poisson distribution, so they all equaled 1.6. Poisson distribution was still used to calculate the number of LRUs in the pipeline for the bases. Just taking base 1 for example, the number of LRUs in the pipeline was:

$$\mu_{11} = 20\times0.01\times0.2+20\times0.01\times0.8 +[(20\times0.8)/64]\times1.6 = 0.6$$

from Eq. 14 which was the expected value of backorder, so EBO(s_{11} = 0) = 0.6, Then the stock level increased and EBO(s_{11}) was calculated from Eq. 14. The marginal analysis value, EBO(s_{11}-1)-EBO(s_{11}) (ΔEBO(s_{11})) for any base could be calculated, as shown in Table 3.

The total expected backorders with zero stock was just the sum of the backorders at each base and at the depot as 0.6Δ4+1.6 = 4, seeing the first line of Table 4. Because the cost of one LRU didn't vary by base, so the delta of EBO didn't need to divide the first differences by unit cost to apply marginal analysis. When the first spare was allocate to base one, the delta of EBO was 0.6-0.149 = 0.451 and the total EBO was 4.0-0.451 = 3.549, seeing the second line of Table 4. Then the second spare was allocate to base two and the total EBO was 3.549-0.451 = 3.098, seeing the third line of Table 4. Then the third and the forth spare were allocated to base three and base four. The four bases were assumed to be identical, so the stock was allocated to each base successively.

Table 3: The expected backorders of base 1 for the depot stock level 0

s_{11}	EBO(s_{11})	ΔEBO(s_{11})
0	0.600	-
1	0.149	0.451
2	0.027	0.122
3	0.004	0.023

Table 4: Optimal expected backorders between bases for the depot stock level 0

s	s_{11}	s_{12}	s_{13}	s_{14}	EBO(s)	ΔEBO(s)
0	0	0	0	0	4.000	-
1	1	0	0	0	3.549	0.451
2	1	1	0	0	3.098	0.451
3	1	1	1	0	2.646	0.451
4	1	1	1	1	2.195	0.451
5	2	1	1	1	2.073	0.122
6	2	2	1	1	1.951	0.122
7	2	2	2	1	1.830	0.122
8	2	2	2	2	1.708	0.122
9	3	2	2	2	1.685	0.023
10	3	3	2	2	1.661	0.023
11	3	3	3	2	1.638	0.023
12	3	3	3	3	1.615	0.023
13	4	3	3	3	1.612	0.003
14	4	4	3	3	1.608	0.003
15	4	4	4	3	1.605	0.003
16	4	4	4	4	1.602	0.003

From the above procedure, the optimal solution was got for the depot stock level zero. Then the depot stock level increased to one and the same procedure need to repeat, when s_{10} wasn't zero, the EBO(s_{10}) and VBO(s_{10}) could be got from Eq. 14 and 15. The value of variance-to-mean wasn't one, so Negative binomial was used to calculate the expected values of backorder of the bases, the two parameters p_{11} and r_{11} were calculate from Eq. 18 and 19. The values of these variables for depot stock level 0-6 were shown in Table 5. The expected backorders for the bases, EBO(s_{1j}), calculated by Negative binomial distribution from Eq. 23. When the cost of LRU_1 exceeded the total cost, the total stock of LRU_1 stopped to increase. Then the following lines of Table 6 could be got. For example, when the total stock level was 10, 5 units were allocated to the depot, 2 units were allocated to base one and the other three bases got 1 unit separately.

LRU_2 repeated the same procedure as LRU_1 and the optimal allocation for LRU_2 could be got. Then using marginal analysis as that used in Table 4 to combine the two units but the delta of backorder must be divided by the LRU cost. Then Table 7 could be got.

The curve of system cost-backorder could be drawn according to Table 7, as shown in Fig. 4. It obviously was convex and the solution was the optimal one. This could be converted into an optimal system availability-cost curve in the same manner also which made the model a

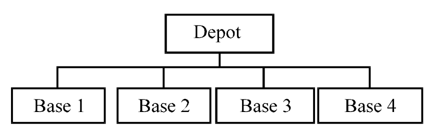

Fig. 3: The level of the organization

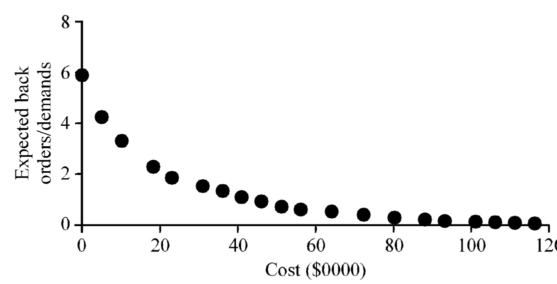

Fig. 4: Optimal system cost-backorder curves

Table 5: The values of variables for the depot stock level 0-6

s_{10}	EBO(s_{10})	VBO(s_{10})	μ_{11}	Var_{11}	p_{11}	r_{11}
0	1.600	1.600	0.600	0.600		-
1	0.802	1.115	0.400	0.420	0.953	8.194
2	0.327	0.523	0.282	0.294	0.958	6.487
3	0.110	0.180	0.228	0.232	0.981	11.828
4	0.031	0.050	0.208	0.209	0.994	37.460
5	0.008	0.012	0.202	0.202	0.999	163.267
6	0.002	0.002	0.200	0.200	1.000	854.245

Table 6: Optimal expected backorders for any stock level

s	s_{11}	s_{12}	s_{13}	s_{14}	s_{10}	EBO	ΔEBO
0	0	0	0	0	0	4.000	-
1	0	0	0	0	1	2.404	1.596
2	0	0	0	0	2	1.454	0.95
3	0	0	0	0	3	1.02	0.433
4	1	0	0	0	3	0.819	0.202
5	1	1	0	0	3	0.617	0.202
6	1	1	1	0	3	0.415	0.202
7	1	1	1	1	3	0.213	0.202
8	1	1	1	1	4	0.114	0.099
9	1	1	1	1	5	0.084	0.030
10	2	1	1	1	5	0.067	0.018
11	2	2	1	1	5	0.049	0.018
12	2	2	2	1	5	0.031	0.018
13	2	2	2	2	5	0.013	0.018
14	2	2	2	2	6	0.007	0.006
15	2	2	2	2	7	0.005	0.001
16	3	2	2	2	7	0.004	0.001

Table 7: Marginal analysis for each LRU

s	s_{LRU1}	s_{LRU2}	s_{11}	s_{12}	s_{13}	s_{14}	s_{10}	EBO(s)
0	0	0	0	0	0	0	0	5.84
1	1	0	0	0	0	0	1	4.24
2	2	0	0	0	0	0	2	3.29
3	2	1	0	0	0	0	1	2.27
4	3	1	0	0	0	0	3	1.83
5	3	2	0	0	0	0	2	1.51
6	4	2	1	0	0	0	3	1.31
7	5	2	1	1	0	0	3	1.10
8	6	2	1	1	1	0	3	0.90
9	7	2	1	1	1	1	3	0.70
`10	8	2	1	1	1	1	4	0.60
11	8	3	1	0	0	0	2	0.50
12	8	4	1	1	0	0	2	0.39
13	8	5	1	1	1	0	2	0.29

very powerful tool for comparing different solutions. For example, when choosing between two components performing the same function, was it more economical from a cost perspective to select the expensive component with low failure rate, or a cheap one with a higher failure rate? Just input their data and see the total backorder result.

DISCUSSION AND CONCLUSION

In this study, it was proved that Negative binomial distribution was more suitable than Poisson for the number of units in the pipeline of bases. The marginal analysis was used to optimize the expensive LRUs inventory with state dependent repair and failure rates. Computational results showed that the proposed approach was efficient in determining the optimal choice of spares for the multi-ECHELON repairable inventory system. The model was not only suitable for confirming initial spare parts list but also well for optimal reallocation and replenishment of existing spares assortments. Further research could be focus on lateral translation, because it could obviously reduce the backorders by pooling the inventory at the same level.

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