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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (4): 696-703, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.696.703

Consensus Tracking of Stochastic Multi-agent Systems Based on Sampled-data

[1]Yaping Liu and [2]Zhihai Wu
[1]Wuxi Research Center of Environmental Science and Engineering,
Wuxi City College, Wuxi 214122, China
[2]School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, China

Abstract: The stochastic bounded consensus tracking problems of leader-follower multi-agent systems were investigated, where the control input of each agent can only use the information measured at the sampling instants from its neighbors or the virtual leader with a time-varying reference state and the measurements are corrupted by random noises. The probability limit theory and algebra graph theory were employed to derive the sufficient conditions guaranteeing the mean square bounded consensus tracking. Simulations were provided to demonstrate the effectiveness of the theoretical results.

Key words: Consensus, coordination tracking, time-varying reference state, multi-agent systems, measurement noises, sampled-data

INTRODUCTION

In recent years, distributed coordinated control of multi-agent systems has attracted a great deal of attention. This is due to its broad applications in various fields including physics, biology, robotics and control engineering. The consensus problem plays an important role in the distributed coordinated control of multi-agent systems. Consensus means that each agent updates its own state based on the states of its local neighbors such that the states of all the agents converge to a common value. For most of the existing consensus protocols, the final common value to be achieved is a function of initial states of all the agents and is inherently a prior unknown constant. This is the so-called χ-consensus (Cortes, 2006), one of important forms of which is the average consensus (Saber and Murray, 2004). Saber and Murray (2004) considered the average-consensus problems of networks of first-order integrator agents with fixed and switching topologies. They proved that if the network is an instantaneous balanced and strongly connected digraph, then the average consensus can be achieved. Saber *et al*. (2007) provided a theoretical framework for the analysis of consensus protocols for multi-agent systems, an overview of basic concepts of information consensus in networks and the methods of convergence and performance analysis for the protocols. Li and Zhang (2009a) designed the optimal weights for guaranteeing the fast average consensus of networked multi-agent systems. More results can be seen in the survey papers (Murray, 2007; Ren *et al*., 2007) and the references therein.

In real applications, it is often required that all the agents converge to a desired common value evolving over time. This is the so-called consensus tracking problem, where all the agents exchange their own state information with their neighbors with the common goal of tracking a time-varying reference state available to only a portion of agents. Thus, the χ-consensus protocols are ineffective to directly deal with the consensus tracking of a desired time-varying reference state. To solve this problem, consensus protocols of leader-follower multi-agent systems have been proposed. Up to now, significant attention has been focused on the consensus tracking of leader-follower multi-agent systems (Jadbabaie *et al*., 2003; Ren and Beard, 2005). Jadbabaie *et al*. (2003) considered the nearest neighborhood principle. They proved that if all the agents are jointly connected with their leader, then their states converge to the state of the leader as time goes on. Ren and Beard (2005) extended the results of Jadbabaie *et al*. (2003) to the directed topology case and gave some more relaxed conditions.

Most researches on the consensus tracking of leader-follower multi-agent systems in the above literature assumed that each agent measures the states of its neighbors and the leader accurately. Obviously, this assumption is only an ideal approximation for real communication channels, since real networks are often in uncertain communication environments with various source noises, channel noises and sink noises. Thus, it is necessary to study the effects of measurement noises on the consensus tracking of leader-follower multi-agent systems (Kingston *et al*., 2005; Ren *et al*., 2005;

Corresponding Author: Yaping Liu, Wuxi Research Center of Environmental Science and Engineering, Wuxi City College, Wuxi 214122, China

Huang and Manton, 2007a, b, 2009, 2010; Xiao *et al*., 2007; Ma *et al*., 2008; Li and Zhang, 2009a, 2010; Hu and Feng, 2010). Recently, consensus of leaderless multi-agent systems with measurement noises has been widely studied (Kingston *et al*., 2005; Ren *et al*., 2005; Huang and Manton, 2007a, b, 2010; Xiao *et al*., 2007; Li and Zhang, 2009a, 2010). Kingston *et al*. (2005) and Ren *et al*. (2005) introduced the time-varying consensus gains and designed the consensus protocols based on a Kalman filter structure. They proved that when there is no noise, the protocols designed can ensure consensus to be achieved asymptotically. Xiao *et al*. (2007) considered the first-order discrete-time average consensus of multi-agent systems with additive input noises. They designed the optimal weighted adjacency matrix to minimize the static mean square error. Since the consensus gain and the adjacency matrix are time-invariant, the average state diverges with probability one as time goes on, even if the noises are bounded. The stochastic approximation type protocols with the decreasing consensus gains were employed to guarantee the mean square or almost sure consensus of the first-order discrete-time multi-agent systems with measurement noises (Huang and Manton, 2007a, b, 2010; Li and Zhang, 2010). Li and Zhang (2009a) investigated the average consensus of the first-order continuous-time multi-agent systems with measurement noises and obtained the convergence conditions guaranteeing the mean square consensus. However, compared with consensus of leaderless multi-agent systems with measurement noises, the consensus tracking of leader-follower multi-agent systems with measurement noises has attracted only a little attention (Ma *et al*., 2008; Huang and Manton, 2009; Hu and Feng, 2010). Huang and Manton (2009) considered the consensus of discrete-time leader-follower multi-agent systems with measurement noises. By using the stochastic Lyapunov analysis, they derived the sufficient conditions guaranteeing the stochastic approximation type protocols with the decreasing consensus gains to reach the mean square consensus. Ma *et al*. (2008) investigated the consensus of continuous-time leader-follower multi-agent systems with measurement noises. By employing the stochastic analysis and algebra graph theory, they obtained the sufficient conditions for the mean square consensus. Hu and Feng (2010) developed a distributed tracking control scheme with distributed estimators based on a novel velocity decomposition technology for a continuous-time leader-follower multi-agent system with measurement noises. They proved that the closed loop tracking control system is stochastically stable in mean square and the estimation errors converge to zero in mean square as well.

For most of the work in the above literature, continuous-time protocols were used for continuous-time agent dynamics and discrete-time protocols were used for discrete-time agent dynamics. However, due to the application of digital sensors and controllers, in many cases, though the system itself is a continuous process, only sampled-data at discrete sampling instants is available for the synthesis of control laws. Compared with full-state information transmission, which might result in high communication traffic, the transmission of the state information at the sampling instants can effectively save the bandwidth of networks and communication cost. Considering the broad applications of digital communication and control in distributed systems, it is of great significance to study the consensus of multi-agent systems based on sampled-data control (Hayakawat *et al*., 2006; Hu and Hong, 2007; Mohanarajah and Hayakawa, 2008; Ren and Cao, 2008; Cao and Ren, 2009; Gao *et al*., 2009; Li and Zhang, 2009b; Cao and Ren, 2010a, b; Gao and Wang, 2010a-c). Up to now, considerable efforts have been focused on the sampled-data consensus of leaderless multi-agent systems without measurement noises (Hayakawat *et al*., 2006; Mohanarajah and Hayakawa, 2008; Ren and Cao, 2008; Cao and Ren, 2009; Gao *et al*., 2009; Cao and Ren, 2010a, b; Gao and Wang, 2010a-c). Till now, only a little attention has been paid to the sampled-data consensus of the leaderless multi-agent systems with measurement noises (Li and Zhang, 2009a,b) band the sampled-data consensus of the leader-follower multi-agent systems without measurement noises (Hu and Hong, 2007). To the best of our knowledge, there is no open report on the consensus tracking of the leader-follower multi-agent systems with measurement noises based on sampled-data.

Based on the above consideration, this study was concerned with the stochastic bounded consensus tracking problems of leader-follower multi-agent systems. Here, the control input of each agent can only use the information measured at the sampling instants from its neighbors or the virtual leader with a time-varying reference state and the measurements are corrupted by random noises. The probability limit theory and algebra graph theory were employed to derive the sufficient conditions guaranteeing the mean square bounded consensus tracking.

PRELIMINARIES AND PROBLEM STATEMENT

Algebra graph theory: Let G = (V, E, A) be a weighted undirected graph with the set of nodes $V = \{v_1, v_2, \ldots, v_N\}$, the set of edges $E \subseteq V \times V$ and the weighted adjacency matrix $A = [a_{ij}]$ with nonnegative adjacency elements a_{ij}. The node indexes of G belong to a finite index set $I = \{1, 2, \ldots, N\}$. An edge of G is denoted by $e_{ij} = (v_i, v_j)$. The adjacency elements associated with the edges are positive, i.e., $e_{ij} \in E \Leftrightarrow a_{ij} > 0$. Moreover, it is assumed that $a_{ii} = 0$ for all $i \in I$. For the undirected graph G, the adjacency matrix A is symmetric, i.e.,$a_{ij} = a_{ji}$. The set of neighbors of node v_i is denoted by $N_i = \{v_j \in V: e_{ij} \in E\}$. The degree of node v_i is defined as:

$$d_i = \sum_{j \in N_i} a_{ij}$$

The Laplacian matrix of G is defined as L = D-A, where D = Diag(d_1, d_2, ..., d_N) is the degree matrix of G with diagonal elements d_i and zero off-diagonal elements. An important fact of L is that all the row sums are zero and thus L has a right eigenvector 1_N associated with the zero eigenvalue, where 1_N denotes the N-dimensional column vector with all ones. A path between two distinct nodes v_i and v_j means a sequence of distinct edges of the form $(v_i, v_{k_1}), (v_{k_1}, v_{k_2}), \cdots, (v_{k_l}, v_j)$. A graph is called connected if there is a path between any two distinct nodes of the graph. An important property of the Laplacian matrix, which is instrumental in the convergence analysis of consensus protocols, is that the graph G is connected if and only if rank(L) = N-1. Thus, for a connected graph G, L has one and only one zero eigenvalue and all the other eigenvalues of L are positive.

Some notations: The following notations are used throughout this paper. 0_N denotes the N-dimensional column vector with all zeros. I_N and $0_{N\times N}$ denote the N-dimensional identity matrix and the N-dimensional matrix with all zeros, respectively. For a given vector or matrix M, M^T and $\|M\|_2$ denote its transpose and 2-norm, respectively. Let R denote the set of real numbers. For a given matrix $M \in R^{N\times N}$, det(M) and $\lambda_i(M)$ denote its determinant and N eigenvalues. For a given random variable X, EX denotes its mathematical expectation.

Problem statement: In a multi-agent system with N agents, an agent and an available information flow between two agents are considered as a node and an edge in an undirected graph, respectively. Consider the system of dynamic agents described by:

$$\dot{x}_i(t) = u_i(t), \quad i \in I \tag{1}$$

where, $x_i(t) \in R$ and $u_i(t) \in R$ represent, respectively, the state of the ith agent and the associated control input.

The ith agent can receive the information $y_{ij}(t) = x_j(t)+\sigma_{ij}\omega_{ij}(t)$, $j \in N_i$ from its neighbors, where $y_{ij}(t)$ denotes the measurement of the jth agent's state $x_j(t)$ by the ith agent, $\{\omega_{ij}(t): j \in N_i,\ i \in N\}$ are the independent standard Gaussian white noises and $\sigma_{ij}>0$ represents the intensity of the measurement noise $\omega_{ij}(t)$. If the ith agent has the access to the virtual leader with a time-varying reference state $x^*(t)$, then it can receive the information from the virtual leader: $y_i^*(t) = x_i^*(t)+\rho_i\eta_i(t)$, where $y_i^*(t)$ denotes the measurement of the virtual leader's state $x_i^*(t)$ by the ith agent, $\{\eta_i(t): i \in I\}$ are the independent standard Gaussian white noises and $\rho_i>0$ represents the intensity of the measurement noise $\eta_i(t)$. In this study, it is assumed that the measurement noises $\{\omega_{ij}(t): j \in N_i,\ i \in I\}$ and $\{\eta_i(t): i \in I\}$ are mutually independent.

In Ma *et al*. (2008), the following continuous-time consensus tracking protocol was introduced:

$$u_i(t) = a(t)[-\alpha_i(x_i(t) - y_i^*(t)) + \sum_{j \in N_i} a_{ij}(y_{ij}(t) - x_i(t))],\ i \in I \tag{2}$$

where, $a(.): [0, +\infty) \to [0, +\infty)$ is a piecewise continuous function, called a time-varying consensus gain and the constant feedback gain α_i is defined as:

$$\alpha_i \begin{cases} > 0, & \text{if the i th agent has the access to the virtual leader} \\ = 0, & \text{otherwise} \end{cases} \tag{3}$$

The protocol Eq. 2 is called a mean square consensus tracking protocol, if the system Eq. 1 using Eq. 2 has the following property: $\text{Lim}_{t\to\infty} E(x_i(t)-x^*(t))^2 = 0$, $\forall i \in I$.

Using the period sampling technology and zero-order hold circuit, the following consensus tracking protocol with sampled data is induced from the continuous-time consensus tracking protocol (2):

$$\begin{aligned} &u_i(t) = a(k)[-\alpha_i(x_i(kh) - y_i^*(kh)) + \sum_{j \in N_i} a_{ij}(y_{ij}(kh) - x_i(kh))] \\ &t \in [kh, kh+h), k = 0,1,2,\cdots; i \in I \end{aligned} \tag{4}$$

The discrete-time model with zero-order holder of the system Eq. 1 is:

$$x_i(kh+h) = x_i(kh) + hu_i(kh), k = 0,1,2,\cdots; i \in I \tag{5}$$

which can be rewritten in the vector form:

$$x(kh+h) = x(kh) + hu(kh), k = 0,1,2,\cdots \tag{6}$$

where, $x(kh) = (x_1(kh), x_2(kh)\ldots, x_N(kh))^T$.

Definition 1: The protocol (4) is called a mean square bounded consensus tracking protocol, if the system (5) using (4) has the following property:

$$\text{Lim}_{k\to\infty} E(x_i(kh)-x^*(kh))^2 = C<\infty,\ \forall i \in I$$

where, C is a bounded positive constant independent of k.

In the following sections, the sufficient conditions guaranteeing the system (5) using the protocol (4) to achieve the mean square bounded consensus tracking will be derived. Before moving on, the following lemma is needed:

Lemma 1 (Hu and Hong, 2007): If the network topology composed of N agents is fixed, undirected and connected and at least one agent has the access to the virtual leader, then the matrix $Diag(\alpha_1, \alpha_2, \ldots, \alpha_N)+L$ must be positive definite, i.e.:

$$Min_{i\in I}\{\lambda_i(Diag(\alpha_1, \alpha_2, \ldots, \alpha_N)+L)\}>0$$

In order to better focus on the effects of the sampled-data and measurement noises on the consensus tracking, throughout this paper it is assumed that the communication network topology composed of N agents is fixed, undirected and connected and at least one agent has the access to the virtual leader.

CONVERGENCE ANALYSIS

In this section, the probability limit theory and algebra graph theory were employed to derive the sufficient conditions that guarantee the multi-agent system (5) applying the consensus tracking protocol (4) to reach the mean square bounded consensus tracking.

Theorem 1: Assume that the time-varying reference state of the virtual leader satisfies:

$$\left|\frac{x^*(kh+h)-x^*(kh)}{h}\right| \le \xi^* < \infty,\ \forall h>0,\ k=0,1,2,\cdots$$

Then the multi-agent system (5), applying the consensus tracking protocol (4), achieves the mean square bounded consensus tracking, provided that the following two conditions are satisfied:

C1: $0 < \inf_{k\ge 0} a(k) \le a(k) \le \sup_{k\ge 0} a(k) < \infty$

C2: $h < \dfrac{1}{\max_{i\in I}\{\lambda_i(Diag(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\sup_{k\ge 0} a(k)}$

Moreover, if the time-varying reference state of the virtual leader further satisfies, then the system (5) under (4) has the following property:

$$Lim_{k\to\infty} E(x_i(kh)-x^*(kh))^2 = o(1),\ h\to 0,\ \forall i\in I \tag{7}$$

Proof: Substituting the protocol (4) into (5) leads to:

$$\begin{aligned} &x(kh+h) = x(kh) - ha(k)[Diag(\alpha_1,\alpha_2,\cdots,\alpha_N) \\ &(x(kh)-x^*(kh)1_N) + Lx(kh)] + ha(k)n(kh),\ k=0,1,2,\cdots, \end{aligned} \tag{8}$$

Where:

$$n(kh) = (n_1(kh), n_2(kh), \ldots, n_N(kh))^T$$

and

$$n_i(kh) = \alpha_i\rho_i\eta_i(kh) + \sum_{j\in N_i} a_{ij}\sigma_{ij}\omega_{ij}(kh),\ i\in I$$

Set $\tilde{x}_i(kh) = x_i(kh)-x^*(kh)$, $i\in I$ and $\tilde{x}(kh) = (\tilde{x}_1(kh), \tilde{x}_2(kh)\ldots, \tilde{x}_N(kh))^T$. Note that $L1_N = 0$, thus:

$$\tilde{x}(kh+h) = P(k)\tilde{x}(kh) - \Delta x^*(kh)1_N + ha(k)n(kh), k=0,1,2,\cdots \tag{9}$$

where, $P(k) = I_N-ha(k)\,(Diag(\alpha_1, \alpha_2, \ldots, \alpha_N)+L)$ and $\Delta x^*(kh) = x^*(kh+h)-x^*(kh)$.

Denote:

$$\begin{cases} \Phi(n,\ k-1) = P(n)P(n-1)\cdots P(k),\ \forall n=0,\ 1,\cdots;\ k=0,\ 1,\cdots,\ n, \\ \Phi(n,\ n) = I_N,\ \forall n=0,\ 1,\cdots \end{cases} \tag{10}$$

From Eq. 9, it gives:

$$\tilde{x}(nh+h) = \Phi(n,\ -1)\tilde{x}(0) + \sum_{k=0}^{n}\Phi(n,k)[-\Delta x^*(kh)1_N + ha(k)n(kh)] \tag{11}$$

It follows from Eq. 11 and the conditions (C1) and (C2) that:

$$\begin{aligned} &E\|\tilde{x}(nh+h)\|_2^2 \le 2\|\Phi(n,\ -1)\|_2^2\|\tilde{x}(0)\|_2^2 \\ &+2E\left\|\sum_{k=0}^{n}\Phi(n,\ k)[-\Delta x^*(kh)1_N + ha(k)n(kh)]\right\|_2^2 \\ &\le 2\|\Phi(n,\ -1)\|_2^2\|\tilde{x}(0)\|_2^2 + 4\left\|\sum_{k=0}^{n}\Phi(n,\ k)\Delta x^*(kh)1_N\right\|_2^2 \\ &+4E\left\|\sum_{k=0}^{n}\Phi(n,\ k)ha(k)n(kh)\right\|_2^2 \\ &\le 2\|\Phi(n,\ -1)\|_2^2\|\tilde{x}(0)\|_2^2 + 4\left\|\sum_{k=0}^{n}\Phi(n,\ k)\Delta x^*(kh)1_N\right\|_2^2 \\ &+4h^2\sup_{k\ge 0} a^2(k)\sum_{k=0}^{n}\|\Phi(n,\ k)\|_2^2 E\|n(kh)\|_2^2 \\ &\le 2\|\Phi(n,\ -1)\|_2^2\|\tilde{x}(0)\|_2^2 + 4Nh^2\xi^2(\sum_{k=0}^{n}\Phi(n,\ k)\|_2)^2 \\ &+4h^2\sup_{k\ge 0} a^2(k)\sum_{i\in I}^{n}(\alpha_i^2\rho_i^2 + \sum_{j\in N} a_{ij}^2\sigma_{ij}^2)\sum_{k=0}^{n}\|\Phi(n,k)\|_2^2 \end{aligned} \tag{12}$$

Employing the conditions (C1) and (C2) yields:

$$\begin{aligned} \|\Phi(n,\ -1)\|_2^2 &\le \Pi_{k=0}^{n}\|P(k)\|_2^2 \\ &\le \Pi_{k=0}^{n}[1-h\min_{i\in I}\{\lambda_i(Diag(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}a(k)]^2 \\ &\le \Pi_{k=0}^{n}\exp\{-2h\min_{i\in I}\{\lambda_i(Diag(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}a(k)\} \\ &= \exp\{-2h\min_{i\in I}\{\lambda_i(Diag(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\sum_{k=0}^{n}a(k)\} \\ &\le \exp\{-2(n+1)h\min_{i\in I}\{\lambda_i(Diag(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\} \end{aligned} \tag{13}$$

$$\sum_{k=0}^{n}\|\Phi(n,k)\|_2 \le \sum_{k=0}^{n}\Pi_{m=k+1}^{n}\|P(m)\|_2$$
$$\le \sum_{k=0}^{n}\exp\{-h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\sum_{m=k+1}^{n}a(m)\}$$
$$\le \sum_{k=0}^{n}\exp\{-h(n-k)\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}$$
$$=\sum_{k=0}^{n}\exp\{-hk\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}$$
$$=\frac{1-\exp\{-h(n+1)\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}}{1-\exp\{-h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}} \quad (14)$$

Which leads to:

$$(\sum_{k=0}^{n}\|\Phi(n,k)\|_2)^2 \le \frac{[1-\exp\{-h(n+1)\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}]^2}{[1-\exp\{-h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}]^2} \quad (15)$$

and

$$\sum_{k=0}^{n}\|\Phi(n,k)\|_2^2 \le \frac{1-\exp\{-2h(n+1)\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}}{1-\exp\{-2h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}} \quad (16)$$

Which can be easily proved by using the analysis similar to one in Eq .14.

Combining Eq. 12, 13, 15 and 16 leads to:

$$\mathrm{Lim\,sup}_{n\to\infty}E\|\tilde{x}(nh+h)\|_2^2$$
$$\le 2\|\tilde{x}(0)\|_2^2\mathrm{Lim}_{n\to\infty}\|\Phi(n,-1)\|_2^2 + 4Nh^2\xi^{*2}(h)\mathrm{Lim}_{n\to\infty}(\sum_{k=0}^{n}\|\Phi(n,k)\|_2)^2$$
$$+4h^2\sup_{k\ge 0}a^2(k)\sum_{i\in I}(\alpha_i^2\rho_i^2+\sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2)\mathrm{Lim}_{n\to\infty}\sum_{k=0}^{n}\|\Phi(n,k)\|_2^2$$
$$\le 2\|\tilde{x}(0)\|_2^2\mathrm{Lim}_{n\to\infty}\exp\{-2h(n+1)\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}$$
$$+4Nh^2\xi^{*2}\mathrm{Lim}_{n\to\infty}\frac{[1-\exp\{-h(n+1)\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}]^2}{[1-\exp\{-h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}]^2}$$
$$+4h^2\sup_{k\ge 0}a^2(k)\sum_{i\in I}(\alpha_i^2\rho_i^2+\sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2)$$
$$\times\mathrm{Lim}_{n\to\infty}\frac{1-\exp\{-2h(n+1)\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}}{1-\exp\{-2h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}}$$
$$=4Nh^2\xi^{*2}\frac{1}{[1-\exp\{-h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}]^2}$$
$$+\frac{4h^2\sup_{k\ge 0}a^2(k)\sum_{i\in I}(\alpha_i^2\rho_i^2+\sum_{j\in N_i}a_{ij}^2\sigma_{ij}^2)}{1-\exp\{-2h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}}\hat{C}<\infty \quad (17)$$

Therefore, under the given conditions (C1) and (C2), the multi-agent system (5), applying the consensus tracking protocol (4), achieves the mean square bounded consensus tracking.

In addition, if the time-varying reference state further satisfies $\xi^* = o(1)$, $h\to 0$, then from Eq. 17 it follows:

$$\mathrm{Lim}_{h\to 0}\mathrm{Lim\,sup}_{n\to\infty}E\|\tilde{x}(nh+h)\|_2^2=0 \quad (18)$$

Here, the following two equations have been used:

$$\mathrm{Lim}_{h\to 0}\frac{h^2}{[1-\exp\{-h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}]^2}$$
$$=\frac{1}{[\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}]^2} \quad (19)$$

and

$$\mathrm{Lim}_{h\to 0}\frac{h^2}{1-\exp\{-2h\min_{i\in I}\{\lambda_i(\mathrm{Diag}(\alpha_1,\alpha_2,\cdots,\alpha_N)+L)\}\inf_{k\ge 0}a(k)\}}=0 \quad (20)$$

Remark 1: If the reference state of the virtual leader is constant, i.e., $x^*(t)\equiv x^*(0)$, $t\ge 0$, then:

$$\left|\frac{x^*(kh+h)-x^*(kh)}{h}\right|=0,\ \forall h>0,\ k=0,1,2,\cdots$$

Thus, under the conditions (C1) and (C2), the results of Theorem 1 naturally hold.

Remark 2: It is obvious that $a(t)\equiv 1$ satisfies the condition (C1), which implies that the consensus tracking protocol (4) has the inner robustness to measurement noises. Different from most of the existing work about consensus of multi-agent systems with measurement noises, where the conditions:

$$\int_0^\infty a(t)dt=\infty,\ \int_0^\infty a^2(t)dt<\infty$$

and

$$\sum_{k=0}^{\infty}a(k)=\infty,\ \sum_{k=0}^{\infty}a^2(k)<\infty \quad (21)$$

are used for the continuous-time system and the discrete-time system, respectively, the condition (C1) used in Theorem 1 can yield:

$$\sum_{k=0}^{\infty}a(k)=\infty,\ \sum_{k=0}^{\infty}a^2(k)=\infty$$

Notice that the condition (21) means $\mathrm{Lim}_{k\to\infty}a(k)=0$ and thus $\mathrm{INF}_{k\ge 0}a(k)=0$. Using the convergence method in Theorem 1 cannot derive the results of Theorem 1 if the condition (C1) is replaced with the condition (Eq. 21). However, the following results can be obtained:

Corollary 1: Assume that the time-varying reference state of the virtual leader satisfies:

$$\sum_{k=0}^{\infty}|\Delta x^*(kh)|\le\zeta^*<\infty$$

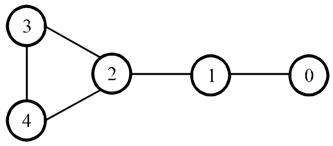

Fig. 1: Network topology composed of four agents and a virtual leader

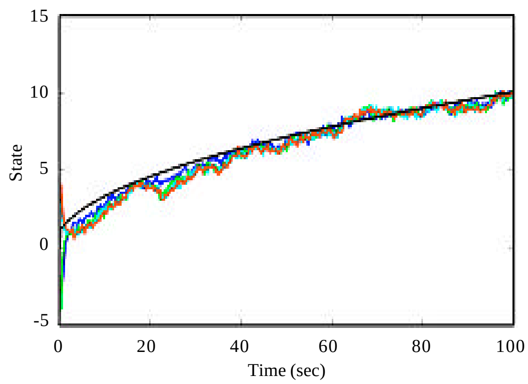

Fig. 2: The sates of the system (5) using Eq. 4 with $h = 0.2$, $a(k) = a_1(k)$ and $x^*(kh) = \sqrt{1+kh}$

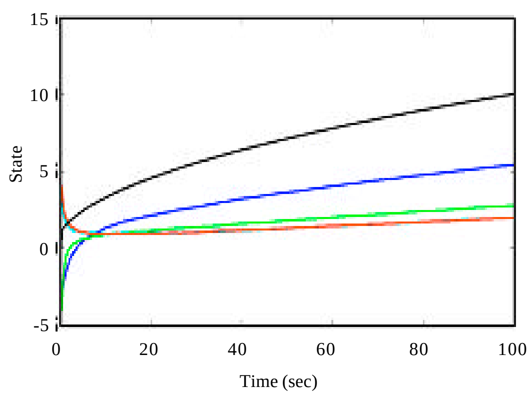

Fig. 3: The sates of the system (5) using Eq. 4 with $h = 0.2$, $a(k) = a_2(k)$ and $x^*(kh) = \sqrt{1+kh}$

Then the multi-agent system (5), applying the consensus tracking protocol (4), achieves the mean square consensus tracking, provided that the conditions (21) and (C2) are satisfied.

SIMULATIONS

Numerical simulations were to illustrate the effectiveness of the above theoretical results. Consider a multi-agent system composed of four agents with the network topology shown in Fig. 1, where the agent 0 represents the virtual leader and all the weights of edges are 1. The measurement noises are independent white Gaussian sequences with the intensity equal to 0.5. The initial states of the four agents are $x_1(0) = -3$, $x_2(0) = -4$, $x_3(0) = 3$, $x_4(0) = 4$. Two candidate sequences of time-varying consensus gains are $a_1(k) = (k+1)/(k+2)$ and $a_2(k) = (1+k)^{-2/3}$. The numerical results are shown in Fig. 2 and 3, respectively.

From Fig. 2 and 3, it can be seen that the system (5) using (4) with $h = 0.2$, $a(k) = a_1(k)$ and $x^*(kh) = \sqrt{1+kh}$ can achieve the mean square bounded consensus tracking, whereas the system (5) using (4) with $h = 0.2$, $a(k) = a_2(k)$ and $x^*(kh) = \sqrt{1+kh}$ cannot achieve the mean square bounded consensus tracking. Thus, the results of Theorem 1 are numerically testified.

CONCLUSIONS

In this study, the stochastic bounded consensus tracking problems of leader-follower multi-agent systems with sampling-data, measurement noises and a time-varying reference state were investigated. The probability limit theory and algebra graph theory were employed for deriving the sufficient conditions guaranteeing the mean square bounded consensus tracking. It turned out that the sufficient conditions for guaranteeing the mean square bounded consensus tracking quite depend on the sampling period and the time-varying consensus gains. A future work is to extend the results in this paper to the second-order multi-agent systems with switching directed network topology and the sampling delay.

ACKNOWLEDGMENTS

This study was supported by National Natural Science Foundation of China under Grant No. 61203147, Fundamental Research Funds for the Central Universities of China under Grant No. JUSRP111A44 and Humanities and Social Sciences Youth Funds of the Ministry of Education under Grant No. 12YJCZH218.

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