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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSI*net*

Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Multi-objective Optimization Problem Based on Genetic Algorithm

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Abstract: Target weighted multi-objective optimization genetic algorithm for solving the problem is to place all aggregated into a target objective function with parameters. In the multi-objective optimization evaluation index system, determine the weights of attributes have a pivotal position. So how to scientifically and reasonably determine the attribute weights, the results related to the multi-objective optimization reliability and validity. The first focuses on the weighted sum of the genetic algorithm, uniform design created by combining the initial population and its standardization of the objective function to create a new fitness function, we propose a dynamic allocation weighting scheme, based on the design of a new weight distribution strategy multi-objective genetic algorithm for solving multi-objective optimization problem. The algorithm can find the sparse regions of non-dominated frontier, to search for sparse areas, making the search to a more uniform distribution of non-dominated solutions, introduces a uniform crossover operator and single point crossover two kinds of hybrid composite operator, to make up for a simulated binary search capability is weak crossover defects and gives proof of convergence of the algorithm by simulation to verify the effectiveness of the algorithm.

Key words: Genetic algorithm, multi-objective optimization, optimal solutions, crossover, mutation

INTRODUCTION

Multi-objective optimization is a rapidly developed disciplines, is an important branch of optimization which in some sense the main research goals while multiple numerical optimization problem attracted many scholars (Liao and Tsao, 2006; Martikainen and Ovaska *et al.*, 2005) In real life, the natural human transformation program planning and design process in general reflect the "maximize efficiency, minimize costs," the basic principle of optimization measures in co-optimal strategies for solving the problem of how to get the win goals, in the non-cooperative game problem of how to make your own interests to maximize, minimize the other's benefit and control engineering steady, accurate and fast time-domain index and degree of stability region, the system bandwidth frequency domain characteristics comprehensive are actually multi-objective optimization problem (Liu and Cao, 2006; Koumoussis and Katsaras, 2006) so multi-objective optimization problem can be seen everywhere in the real world. While the classical approach solves some optimization problem, but for multi-objective optimization problem but no efficient and practical solution, but many originated in the actual design of complex systems, modeling and planning problems, (Jin and Su, 2005; Keedwell and Narayanan, 2005) such as industrial manufacturing, capital operations, urban transport, reservoir design, the new urban layout and landscaping, energy distribution, etc., almost every

important decision problems in real life to be in considering various constraints while optimizing a number of goals and these problems involved in the plurality of target does not exist independently, are often coupled together and in a state of competition, each target has different dimensions and physical meaning, their complexity and competition optimization makes it very difficult. Therefore, the multi-objective optimization problem with a very important practical research Meaning, has become an attractive area of research.

In real life practical applications to solve practical optimization problems, if only to consider a goal which we call single-objective optimization problem otherwise termed A multi-objective optimization problem (Salcedo-Sanz and Yao, 2004; Song *et al.*, 2006) Multi-objective optimization is an important field of optimization research direction, because a lot of scientific research and engineering practice optimization problems can be attributed to a multi-objective optimization problem. These systems are located in the areas of industrial manufacturing, urban transport, capital budgeting, reservoir management, energy distribution, logistics, network communication, etc. (Cakar *et al.*, 2008; Tesfatsion, 2003) can be said that there are multi-objective optimization problem everywhere.

Mathematical models of genetic algorithms: In practical applications it is often encountered in multi-criteria or objectives, design and decision-making problems, "such

as securities investment issues, investors in order to get higher returns, you need to select the best stocks to invest in, in general, a outstanding shares have the following characteristics: Good performance, low price-earnings ratio, growth higher, but usually these goals are in conflict, such as the current domestic steel industry generally better performance of listed companies, earnings are relatively low, but the steel industry is not sunrise industry, the company's growth is not high; while some small and medium sized companies although growth is high, but the performance is poor, the high price-earnings ratio and thus to be able to choose a good stock, you need to make investment decisions among these goals a balanced approach "that more than a numerical target in a given region of the optimization problem is known as multi-objective optimization.

In order to solve multi-objective optimization problem, we need to create a general mathematical model, "we must first determine its decision variables, the general case, the decision variables n dimensional Euclidean space as a point E^n , namely:

$$x = (x_1, x_2, x_3 \dots x_n) \in E^n \tag{1}$$

Second is the objective function, in general it can be assumed with p objective functions and decision variables are all about function, namely:

$$f(x) = [f_1(x), f_2(x), \dots, f_p(x)]^T \tag{2}$$

Finally its constraints, from a mathematical point of view, there are two constraints: Inequality constraints and equality constraints, constraints can be defined as the m inequality constraints and k equality constraints:

$$\begin{cases} g_i(x) \leq 0 & i=1,2,3 \dots m \\ h_j(x) = 0 & j=1,2,3 \dots k \end{cases} \tag{3}$$

If all are the minimization of the objective function value, the multi-objective optimization problem can be described as the following mathematical model:

$$\begin{cases} \min f(x) = [f_1(x), f_2(x), \dots, f_p(x)]^T \\ x_i^a \leq x_i \leq x_i^b \end{cases} \tag{4}$$

where, x is the decision variable, f(x) is the objective function, X represents the decision vector formed by the decision space x, $g_i(x)$ and $h_j(x)$ constraints x feasible decision variables to determine the range, min represents Vector Minimization, namely vector target $f(x) = [f_1(x), f_2$

$(x), \dots, f_p(x)]^T$ in certain constraints as far as possible the various sub-objective function minimization. It can be seen when the $p = 1$, the mathematical model for a single objective optimization problem mathematical model.

Definition multi-objective optimization: Multi-objective optimization problem is that people in the production or frequently encountered problems in life, in most cases, due to multi-objective optimization problem in all its goals are in conflict, a sub-target improvement may cause the performance of other sub-goals reduced, in order to make optimal multiple targets simultaneously is impossible and thus in solving multi-objective optimization problem for each sub-goal can only be coordinated and compromise treatment, so that each sub-objective functions are optimal as possible multi-objective optimization problem with a single objective optimization problem is essentially different, in order to properly solve multi-objective optimization problem the optimal solution, we must first multi-objective optimization of the basic concepts of a systematic exposition.

Definition 1: N Viola Space:

$$\begin{cases} x = (x_1, x_2, x_3 \dots x_n)^T \\ y = (y_1, y_2, y_3 \dots y_n)^T \\ x = y \text{ Iff } x_i = y_i \quad \forall i=1,2,3 \dots n \\ x > y \text{ Iff } x_i > y_i \quad \forall i=1,2,3 \dots n \end{cases} \tag{5}$$

Definition 2: Let $X \subseteq R^m$ is a multi-objective optimization model of the constraint set, $f(x) \in R^p$ is a vector objective function, $x_1 \in X, x_2 \in X$, (a) $f_k(x_1) < f_k(x_2)$ better solution called solution x_1, x_2 , (b) x_1 weak solution of $f_k(x_1) \leq f_k(x_2)$ called superior solution x_2 , (c) $f_k(x_1) \geq f_k(x_2)$ solution called indifference to solution x_1, x_2 .

Definition 3: Let $X \subseteq R^m$ be a multi-objective optimization model constraint set, $f(x) \in R^p$ is a vector objective function, $x^n \in X$ and x^n than the X all the other points are superior, called x^n is the multi-objective minimization model optimal solution.

By definition, multi-objective optimization problem is to make the optimal solution x-vector objective function f(x) for each sub-goal is to achieve the most advantages of the solution, obviously, in most cases, the optimal multi-objective optimization problem solution does not exist.

Definition 4: Pareto optimal solution: Let $X \subseteq R^m$ be a multi-objective optimization model constraint set, $f(x) \in R^p$ is the vector of the objective function. If $\xi \in X, \xi$ and there

is no more than the superiority of x , then ξ is a minimal model of multi-objective Pareto optimal solution, or non-inferior solution.

Definition 5: No inferior set with the front end: Let

$X \subseteq R^m$ be a multi-objective optimization model constraint set, $f(x) \in R^p$ is a vector objective function. $\lambda \in X$ is a minimal model of multi-objective Pareto optimal solution set, then λ is called non-inferior set of X , λ is called Pareto optimal front.

Seen from the above definition: (a) Multi-objective optimization problem with a single objective optimization problem is essentially different, in general, multi-objective optimization problem Pareto "optimal solution is a collection of the μ most cases, similar to the single-objective optimization problem in a multi-objective optimal solution optimization problem does not exist, there is only Pareto optimal "multi-objective optimization problem is just a Pareto optimal solution acceptable" not bad "solution and usually most multi-objective optimization problem with multiple Pareto optimal solution. (b) If a multi-objective optimization problem optimal solution exists, then the optimal solution must be Pareto optimal solution and the Pareto optimal solution is also the optimal solution by only composed of these, do not contain other solutions, so can be so say, Pareto optimal solution is a multi-objective optimization problem reasonable solution set. (c) For practical application, must be based on the level of understanding of the problem and the decision-makers of personal preference, from a multi-objective optimization problem Pareto optimal solution set of one or more selected solution as a multi-objective optimization problem of optimal solution, so seeking more objective optimization problem the first step is to find all its Pareto optimal.

Weight coefficient of genetic algorithm: Weight coefficient variation method is used to solve multi-objective optimization problem of the earliest methods. The basic idea is: For a multi-objective optimization problem, if for each of its sub-objective function $f_i(x)$ given different weights w_i , where the size of w_i represents the corresponding sub-goals $f_i(x)$ in a multi-objective optimization problem in an important degree, the individual sub-goals weighted linear function can be expressed as:

$$\begin{cases} f(x) = w_1 f_1(x) + w_2 f_2(x) + \dots + w_p f_p(x) \\ \quad = \sum_{i=1}^p w_i f_i(x) \\ w_i = \frac{\text{random}_i}{\text{random}_1 + \text{random}_2 + \dots + \text{random}_p} \end{cases} \quad (6)$$

As to the fitness function:

$$\sum_{i=1}^p w_i f_i(x)$$

roulette wheel selection can be determined by hybridization and mutation of the individual involved and so on. Thus, this method can provide a lot of random points to a valid interface search direction; this algorithm is used to Flow-shop scheduling problems and achieved good results.

Here we have normalized the objective function, when the first t Let $(x_1, x_2, x_3, \dots, x_p)$ generation populations, so:

$$h_i(t) = \max \{ |f_i(x_1)|, |f_i(x_2)|, |f_i(x_3)|, \dots, |f_i(x_p)| \}$$

then $f_i(x)$ can be normalized to the new objective function:

$$g_i(x) = \frac{f_i(x)}{h_i(t)} \quad (7)$$

The new fitness function can be redefined as:

$$G(x) = \sum_{i=1}^m w_i g_i(x) \quad (8)$$

Adapted according to the size of the angle $g(x)$ with the roulette wheel selection operator $p_0(t)$ is selected from the initial population of parent and points N for hybridization, parents set point set is P :

$$\begin{cases} x_k^i = r \cdot x^i + (1-r) \cdot x^{i+t} \\ x_k^{i+t} = (1-r) \cdot x^i + r \cdot x^{i+t} \end{cases} \quad (9)$$

where in r is a random number between $[0,1]$, $[x^i, x^{i+t}] \in P$, $[x_k^i, x_k^{i+t}] \in p(t)$, $p(t)$ is set after hybridization offspring.

Let $x = (x_1, x_2, x_3, \dots, x_n)$ be a parent population of the body $p(t)$, $j = 1, 2, 3, \dots, n$, x_j is $x \in p(t)$, page j coordinate; a_j , b_j , respectively, of the search points in the space coordinate j The lower and upper bounds; $x_j^k, x_j^t \in p(t)$ is the first coordinate j , $p(t)$ is the first step Offspring collection; r is between $[0,1]$ random number between; T is the maximum genetic algebra; t for the current genetic algebra; $r(0,1)$ said that produce 0 or 1 random number. There are:

$$\begin{cases} x_j^k = x_j + (b_j - a_j) \cdot r \cdot \left(1 - \frac{t}{T}\right) \\ x_j^t = x_j - (b_j - a_j) \cdot r \cdot \left(1 - \frac{t}{T}\right) \end{cases} \quad (10)$$

Non-uniform mutation is to participate in the weight variation did a random disturbance; this disturbance in the early evolution of a relatively large range, but with the evolution of generation increases, changes in disturbance gradually decreases.

SYSTEM ASSESSMENTS

Fitness function and operator: Let $(x_1^t, x_2^t, x_3^t \dots x_n^t)$ be the population in the first generation of the feasible solutions individual t , N is the population size, x_i^t denotes the first-generation t i individuals, p_i^t is Pareto superior to individual x_i^t population the number of individuals, defined as the first $s_i^t (s_i^t = p_i^t + 1)$ t x_i^t behalf of individuals ordinal values, denoted x_i^t individual fitness function t to:

$$f_i^t = \frac{1}{s_i^t}$$

if the first generation of Pareto superior to the more individuals t , x_i^t order value is larger, the lower the degree of its adaptation, ie individual x_i^t worse quality; Contrary, adapt the higher the better the quality of the individual x_i^t . By the uniform crossover design ideas, assume that hybridizes to two parent individuals were $p_1 = (x_1, x_2, x_3 \dots x_n)^T$, $p_2 = (y_1, y_2, y_3 \dots y_n)^T$ component of the inserted between their corresponding points, towered equally divided into $q-1$:

$$\begin{cases} y_1 = x_1, x_1 + d, x_1 + 2d, \dots + x_1 + (q-1)d \\ y_2 = x_2, x_2 + d, x_2 + 2d, \dots + x_2 + (q-1)d \\ \dots \\ y_n = x_n, x_n + d, x_n + 2d, \dots + x_n + (q-1)d \end{cases} \quad (11)$$

where $d = y_i - x_i / (q-1)$, is the length of the corresponding component of the aliquots, as the individual components of uniform design factor, $x_i + (q-1)d$, i , is the first horizontal factor of q , where q and n must be greater than a prime number. Uniform design method according to the previous chapter, x and y will be generated in the hypercube formed by the q uniformly distributed points, we these points as q , x and y offspring generated after the hybridization. Since the plurality of offspring in its two parent hypercube formed uniformly distributed which is equivalent in the two parent individuals do a local search around.

According to the crossover probability of individual populations to determine whether to participate in the cross and then a random string of individual genes set a crossover point, at the intersection of genes on arithmetic crossover operation, matching the intersection in front of the individual genes directly copied to the offspring, after

the intersection after the exchange of genes assigned to the corresponding offspring. Single-point crossover complex not only maintains a simulated binary crossover advantages, but also by increasing the solving arithmetic crossover search area, enhanced search capabilities of the algorithm.

Genetic algorithm implementation: (a) Encoding: binary encoding. (b) The initial population: For the generation of the initial population, using a random generator of. (c) Fitness function: Fitness function using equation (8) in the design of the fitness function. (d) Selection operator: Use roulette wheel selection and elitist strategy of combining strategy with a parent in the fitness value of the top surface of M individuals through crossover and mutation to replace the population after the fitness value of row M individuals in the final, so that offspring and parent to participate in competition, in order to prevent destruction of crossover and mutation operations populations fine mode. (e) Crossover: Use a single-point crossover operator, in which the crossover probability using Equation (9) in the design. (f) Mutation Operator: Use basic bit mutation operator, mutation probability using Equation (10) in the design:

- According to the size of the fitness value of all individuals in contemporary populations sorted and selected the top surface of M individuals
- Let contemporary populations all individuals are involved in crossover and mutation operations to produce the next generation population
- The next generation of all individuals in the population according to the size of fitness value sorted out at the back of the M individuals
- Selected with step1 replaced step3 M individuals to find the M individuals, generate a new generation

In the optimization process, (Jin and Su, 2005) the literature Improved adaptive genetic algorithm crossover and mutation probability design; paper improved genetic algorithm crossover probability and mutation probability formula using the formula (8), the formula (9) in the design; join fitness function is designed in this paper an improved genetic algorithm crossover and mutation probability design, fitness function formula using the formula (10) in the design.

By Fig. 1 and 2 of the observation can be found using only this improved crossover and mutation probability formulas genetic algorithm IAGA average number of iterations than the literature [10] Improved adaptive genetic algorithm IAGA less; improved this article crossover probability and mutation probability

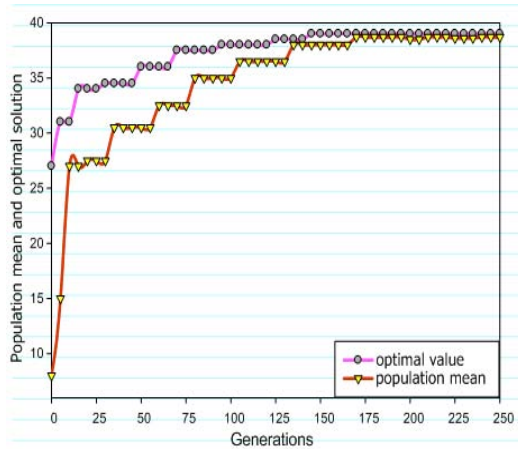


Fig. 1: Population changes in the mean and the optimal value

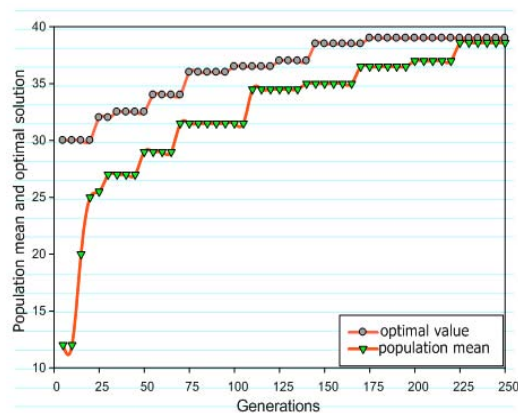


Fig. 2: IAGA population mean and the optimal value changes

formula and join the fitness function designed in this paper an improved genetic algorithm IAGA2 average number of iterations to be less than the algorithm IAGA1. Literature (Jin and Branke, 2005) Improved adaptive genetic algorithm IAGA average number of iterations is about IAGA2 2 times; adaptive genetic algorithm AGA average number of iterations is IAGA2 about four times. This article shows an improved genetic algorithm in computing speed is obviously better than the adaptive genetic algorithm and the literature (Jin and Branke, 2005) improved adaptive genetic algorithm.

CONCLUSION

The main task of multi-objective optimization is to improve the quality of the solution and maintain the

solution of a broad distribution and uniformity, the weighted sum genetic algorithm is a straightforward, practical, strong multi-objective genetic algorithms. This paper focuses on the weighted sum of the genetic algorithm, uniform design created by combining the initial population and their respective objective function Number of standardization, the establishment of a new fitness function and proposes a dynamic allocation weighting scheme, designed a new weight-based allocation strategy Multi-objective Genetic Algorithm for multi-objective optimization problem. Second, the design of a uniform design method based on multi-objective optimization genetic algorithm and gives a proof of convergence of the algorithm by simulation to verify the effectiveness of the algorithm.

ACKNOWLEDGMENTS

This work is supported by Henan University of Science and Technology Youth Fund (2010QN009); National Statistical Research (2012LY132)

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