http://ansinet.com/itj

ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (22): 7019-7023, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.7019.7023

A Grey-box Modeling Approach for the Reduction of Spatially Distributed Processes Using New Basis Functions

Mian Jiang, Xuejun Li and Tiandong Peng
Hunan Provincial Key Laboratory of Health Maintenance for Mechanical Equipment,
Hunan University of Science and Technology, Xiang Tan, 411201, China

Abstract: A grey-box modeling methodology using new basis functions is proposed for the reduction of spatially distributed processes. The methodology amounts to find low-order substitute models for the processes which can balance the dimension, computational efficiency and the accuracy. Model reduction is pursued by time/space separation and projection on the set of new basis functions. Subsequently, an empirical component is identified to substitute the nonlinear terms and un-modeled dynamics. As a consequence, a grey-box model has been developed to approximate the spatially distributed processes. A numerical example is used to demonstrate the effectiveness of the proposed method.

Key words: Spatially distributed processes, grey-box modeling, model reduction, basis functions

INTRODUCTION

In industrial applications, many spatially distributed processes are partially unknown, with model uncertainty usually caused by incomplete system information or simplification made at the first-principle modeling. A number of grey-box models, also referred as the hybrid models in the literature, have been proposed for the industrial spatially distributed processes. The grey-box modeling approaches seeks to combine the advantages of the mechanistic models (first-principle) and black-box models. All available knowledge of the process mechanisms is used to construct a white-box part, while un-modeled dynamic is approximated by identification from the measured input-output data. Psichogios and Ungar (1992) proposed a serial structure where a black-box is used to approximate unknown relations which determine certain model variables. With the nonlinearities can be identified using neural networks, Deng *et al.* (2005) has developed a spectral based intelligent modeling method for the modeling of a class of curing processes. Furthermore, grey-box model of the block-structured type (for example Wiener and Hammerstein models) by Pearson and Pottmann (2000) and developed by Qi *et al.* (2009), Qi and Li (2009). Grey-box models have been widely applied for the modeling of a variety of chemical and biochemical processes. And also have been studied with a focus on applications to control (Alaradi and Rohani, 2002) and identification (Brendel *et al.*, 2004). Grey-box type of models for any type of system which combines linear and nonlinear terms often arises when complex process systems are reduced and identified. Examples of these type of systems are PDE systems with a nonlinear convection term and a linear diffusion term or conversely, a linear convection term and a nonlinear diffusion term. In this note, a grey-box modeling methodology using new basis functions is proposed the reduction of spatially distributed processes. In fact, the dynamical information of the excluded fast modes have been retained when the new basis functions are constructed by transformation. Model reduction is pursued by time/space seperation and projection on the set of new basis functions. This methodology amounts to find low-order substitute models for the processes which can balance the computational efficiency and the accuracy. Subsepuently, an empirical component is identified to substitute the nonlinear terms and un-modeled dynamics. A numerical example is used to demonstrate the effectiveness of the proposed method.

THE NEW BASIS FUNCTIONS

Suppose that the spatially distributed processes, is governed by a PDE with the following state description:

$$\frac{\partial X}{\partial t} = \mathcal{A}X + \mathcal{B}U + \mathcal{F}(X,U) \tag{1}$$

subject to a number of boundary and initial conditions. Here, X (z, t) denotes the vector of the state variables at spatial position Z and at time t. U (z, t) denotes the vector of manipulated spatio-temporal input and only one spatial-dimension is considered. A

Corresponding Author: Mian Jiang, Hunan Provincial Key Laboratory of Health Maintenance for Mechanical Equipment, Hunan University of Science and Technology, Xiang Tan, 411201, China

whichand β which are two linear operators that involve linear spatial derivatives on the state variable and input. F whichrepresents the nominal nonlinear terms. Y (z, t) whichis measured at spatial locations which is assumed to be enough according to the process complexity, the desired accuracy of modeling and control, physical and cost consideration etc. Eq. 1 is considered on a bounded spatial domain $\Omega \in R^n$ which.The phase space of (1) is some infinite-dimensional Hilbert space Λ (Ω) which of sufficiently smooth functions from Ω whichinto real numbers. A scalar product is introduced in Λ (Ω) which which is usually given by:

$$[g,h] = \int_{\Omega} g\,(z) h\,(z)\,dz \tag{2}$$

for two arbitrary functions g, h $\in \Lambda$ which.

Considering the finite-dimensional subspace Φ whichspanned by the first N whichsmooth global spatial orthogonal basis functions:

$$\Phi = \text{span}\{\varphi_1(z), \varphi_2(z), ..., \varphi_N(z)\} \tag{3}$$

The spatio-temporal variable X, U, Y can be expanded into a truncation series with corresponding temporal coefficient α_i (t), u_i (t), y_i (t), respectively. The insertion of the expansion and the application of a Galerkin method yield a system of N which first-order ODE equations and can be rewritten in a general form as follows:

$$\alpha\,(t) = A\alpha\,(t) + Bu\,(t) + f\,(\alpha\,(t), u\,(t)\ y\,(t) = C\alpha\,(t) \tag{4}$$

where, $a(t) = [a_1(t), a_2(t), \cdots, a_N(t)]^T$ A denotes the matrix contains the first N eigenvalues of the operator A; f denote the nominal nonlinearity terms; and $B = [b_1, b_2, \cdots, b_N]^T$ and $b_i = [b_{i1}, b_{i2}, \cdots, b_{im}]$ denote the spatial location information of the input.

Let each new spatial basis function be a linear combination of eigenfunctions for nonlinear DPSs. Define a basis function transform matrix R we have:

$$\{\Psi_1(z), \Psi_2(z), ..., \Psi_k(z)\} = \{\varphi_1(z), \varphi_2(z), ..., \varphi_N(z)\}\ R \tag{5}$$

where, ψ_i and φ_i denote new spatial basis functions and eigenfunctions, respectively. Eq. 5 can be rewritten as follows:

$$\psi_i(z) = \sum_{j=1}^{N} R_{ji}\varphi_j(z) \tag{6}$$

Basis function transform matrix R can be obtained from the linear part of PDEs. Let (A, B, C) be an N whichth order stable state realization of corresponding Linear Time- Invariant (LTI) system of (4). Because A is a diagonal matrix and diagonal entries of are eigenvalues of linear operator in (1), the LTI system is open-loop stable and has unique symmetric positive define controllability gramian P and observability gramian Q which have full rank. Let $P = GG^T$, $Q = HH^T$ be square root decom-positions and defining $GH^T = W\,\Sigma V^T$ as a singular value decomposition, then using the MATLAB style colon notation, the transform matrix $R = [GW\Sigma^{1/2}]$ (:,1:K).

Because of the balanced truncation model reduction, the obtained matrix R whichis of column-orthogonalty. Thus, $R^TR = I_k$.

MODEL REDUCTION USING NEW BASIS FUNCTIONS

Given the column-orthogonality of R and $\{\varphi_1(x), ...\varphi_2(x)\}$, the k new spatial basis functions are shown to be orthogonal toward each other. Using the new spatial orthogonal basis functions, the spatio-temporal variables X, U, Y of Eq. 1 can be expanded into a truncation series with corresponding temporal coefficient $\hat{\alpha}_i$ (t), u_i(t), y_i (t), respectively:

$$\begin{aligned} X(z,t) &= \sum_{i=1}^{k} \bar{a}_i(t)\psi_i(z) \\ U(z,t) &= \sum_{i=1}^{k} u_i(t)\psi_i(z) \\ Y(z,t) &= \sum_{i=1}^{k} \bar{y}_i(t)\psi_i(z) \end{aligned} \tag{7}$$

The insertion of the expansions of Eq. 7 and the application of a Galerkin method, an ODE system with fewer modes is obtained as follows:

$$\begin{aligned} \dot{\bar{a}}(t) &= A_r\bar{a}(t) + B_r u(t) + \bar{f}(\bar{a}(t), u(t)) \\ \bar{y}(t) &= C_r\bar{a}(t) \end{aligned} \tag{8}$$

Where:

$$\bar{a}(t) = [\bar{a}_1(t), \bar{a}_2(t), \cdots, \bar{a}_k(t)]^T$$

$$k < NA_r = R^TAR,\ B_r = R^TB$$

$$\bar{f}(\bar{a}(t), u(t)) = [\bar{f}_1(\bar{a}(t), u(t)), \cdots, \bar{f}_k(\bar{a}(t), u(t))]$$

Where:

$$\bar{f}_i(\bar{a}(t), u(t)) = \int_{\Omega} \mathcal{F}(X, U)\psi_i(z)dz$$

Denotes the nominal nonlinearities.

The nominal nonlinear terms $\bar{f}(\bar{a}(t), u(t))$ in (8) may be have unknown terms or be difficult to have an analytical presentation because of complex spatio-temporal nonlinearity of PDEs and integral for scalar production.For the further application of traditional control techniques, intelligent identification modeling is trained to be a nonlinear model of the nonlinear DPSs in state-space formulation.

GREY-BOX MODELING APPROACH

A low-dimensional grey-box modeling method is proposed to model the nonlinear spatial distributed processes. For hybrid intelligent identification, there are m actuators with implemental temporal signal u (t) and certain spatial distributions. The output Y(z,t) is measured at the n spatial locations $z_1, z_2, \cdots, z_n$ and some sampling time t_1, t_2, ..t_L. For practical implementation, a discrete-time model is often used. The previous reduced model 8 based on the new spatial basis functions is now discretised by Euler forward formula as follows:

$$\begin{aligned} \bar{a}(k+1) &= (I+\Delta t A_r)\bar{a}(k) + \Delta t B_r u(k) \\ &\quad + \Delta t \bar{f}(\bar{a}(k), u(k)) \\ \bar{y}(k) &= \Delta t C_r \bar{a}(k) \end{aligned} \tag{9}$$

with Δt being the sampling time. For simplicity, one can replace $\bar{a}(k), \bar{y}(k)$ with α (k) y (k) and the following can be derived as:

$$\begin{aligned} a(k+1) &= A_0 a(k) + B_0 u(k) + f(a(k), u(k)) \\ y(k) &= C_0 a(k) \end{aligned} \tag{10}$$

With:

$$A_0 = I + \Delta t A_r B_0 = \Delta t B_r C_0 = \Delta t C_r$$

$$f(a(k), u(k)) = \Delta t \bar{f}(\bar{a}(k), u(k))$$

A hybrid intelligent discrete system can be used to model the nonlinear dynamics (10) as shown in Fig. 1, while a feedforward neural network $NN[\hat{a}(k), u(k)]$ is trained to identify the nominal nonlinear terms $\bar{f}(a(k), u(k))$. The advantage of the neural networks is its ability to model complex nonlinear relationships without any assumptions

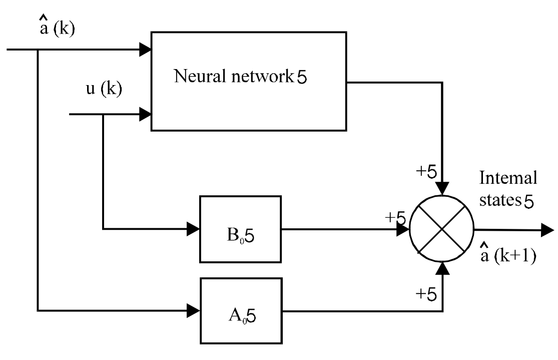

Fig. 1: Neural networks based hybrid modeling

on the nature of these relationships. The most often used neural networks include the Radial Basis Function (RBF) networks, Back Propagation (BP) neural networks, among others. The present study employs a feedforward BP neural network to construct low-dimensional substitute model for the nonlinear dynamics of spatially distributed processes.

The hybrid intelligent model is expressed as follows:

$$\begin{aligned} \hat{a}(k+1) &= A_0 \hat{a}(k) + B_0 u(k) + NN[\hat{a}(k), u(k)] \\ \hat{y}(k) &= C_0 \hat{a}(k) \end{aligned} \tag{11}$$

The above grey-box model is trained to construct low-dimensional approximation for the dynamics of spatially distributed processes. Because of the known $A_0\hat{a}(k)+B_0u(k)$ the neural networks will have high computational efficiency which also can compensate the high order dynamics and decrease the approximation error. The grey-box model can be solved by significantly reduced computational complexity.

The spatio-temporal prediction output $\hat{Y}(z,t)$ is obtained by synthesis of time predicted output $\hat{y}(k)$ and the new spatial basis functions:

$$\hat{Y}(z,k) = \sum_{i=1}^{k} \hat{y}(k)\psi_i(z) \tag{12}$$

Where:

$$\hat{y}(k) = C_0 \hat{a}(k)$$

A NUMERICAL EXAMPLE

In order to evaluate the proposed grey-box modeling reduction strategy for spatially distributed processes, a highly simplified mathematical formulation representing the competition between convection and diffusion, the Burgers equations, is studied. The Burgers equation is a one-dimensional spatial model of a variety of three-dimensional physical phenomena, greatly simplifying the problems while retaining many of the complex behavior characteristics. Recently it is used as a model to study the scaling and intermittency of turbulence. Suppose Y (z, t) and $\hat{Y}(z,t)$ as the measured output and the prediction output at the n spatial locations Z_1, Z_2, ...,Z_n and some sampling times t_{s1} t_{s2} ...,t_{sL} respectively. The Root of Mean Square Error (RMSE) between the real dynamical process and the approximation model is defined as the performance index as follows:

$$RMSE = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{L}\left(Y(z_i,t_j) - \hat{Y}(z_i,t_j)\right)^2 \Big/ nL} \tag{13}$$

Table 1: RMSE of spectral based model and grey-box model

RMSE	k = 1	k = 2	k = 3	k = 4
Spectralbased model	0.912	0.300	0.132	0.083
Grey-box model	0.424	0.231	0.080	0.034

Table 2: RMSE of spectral based model with 5,6,8,10 modes

RMSE	k = 5	k = 6	k = 8	k = 10
Spectralbased model	0.0583	0.0538	0.0509	0.0506

The viscous Burgers equation with spatio-temporal input is considered in one space dimension. This equation contains nonlinear convection and diffusion terms and retains many of the interesting features of the Navier-Stokes equation. The governing equation may be written as:

$$\frac{\partial x}{\partial t} = v\frac{\partial x^2}{\partial z^2} - T\frac{\partial x}{\partial z} + h(z)u(t) \quad (14)$$

In this note, 14 subject to the Dirichlet boundary and initial conditions:

$$X(0, t) = 0,\ x(1,t) = 0,\ x(z,0) = x_0(z) \quad (15)$$

There are available four actuators $u(t) = [u_1(t), \cdots, u_4(t)]^T$ with the spatial distribution functions $h(z) = [h_1(z), \cdots, h_4(z)]^T$, where:

$$h_i(z) = H(z-(i-1)\pi/4),\ i = 1..., 4$$

And H (.) is the standard Heaviside function. In the simulations, the random input signals are used to excite this process.

The testing signals are selected as:

$$u_i(t) = 1+4\sin(\pi t/10)\ (i = 1,..., 4) \quad (16)$$

Suppose 24 sensors uniformly distributed in the space are used for measurement. A noise-free dataset of 500 data is collected from Eq. 14. The sampling intervalΔt is 0.01s and the simulation time is 5s. The viscous coefficient is set to 0.2, the initial condition $X_0(z)$ is set to be sin (πz). For a real system ,sufficient data are always required to obtain a satisfactory model.

A spectral based model is developed for (14), where the family of spatial orthogonal basis functions:

$$\sqrt{2}\sin(k\pi z),\ k = 1, 2, ..., 00$$

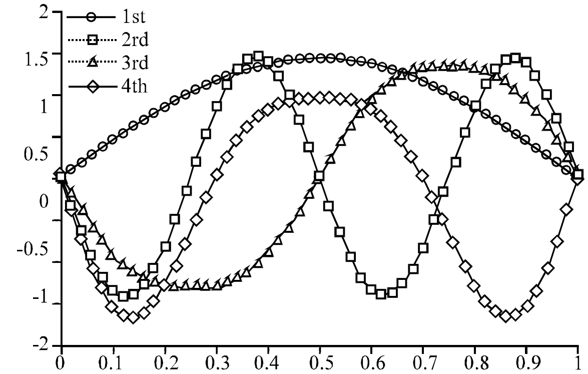

Fig. 2: The first four new spatial basis functions

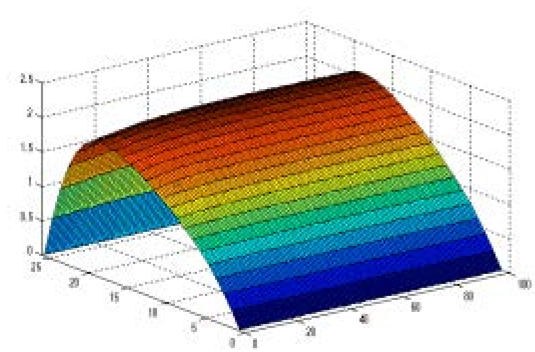

Fig. 3: Measure output for testing

is used for time/space separation and low-dimensional truncation. For synthesis of spatial variables and dynamics of spectral based model and the proposed grey-box model, the RMSE over testing data are compared in Table 1. As shown in Table 1, the modeling error is much smaller than when using the spectral model with the same modes.

To illustrate the effectiveness of the proposed grey-box modeling approach, we have computed the RMSEs of spectral based model with 5, 6, 8, 10 modes which is shown in Table 2. From Table 2, it is obvious that the RMSE of proposed grey-box model with 4 modes is smaller than that of spectral based model based on 10 modes.

From Table 1, the grey-box model up to 4 modes is sufficient to capture the dynamics for control design and its performance is shown. Firstly, 4 new basis functions are shown in Fig. 2.

In order to demonstrate the performance of proposed grey-box model, a new 100 set of data is collected for testing, as shown in Fig. 3. Synthesis of the predicted dynamics of the proposed grey-box model and new spatial basis functions,

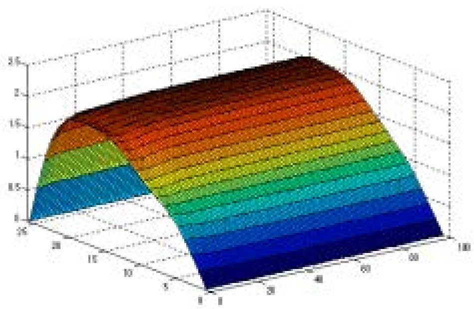

Fig. 4: Prediction distributed output of hybrid intelligent model on testing

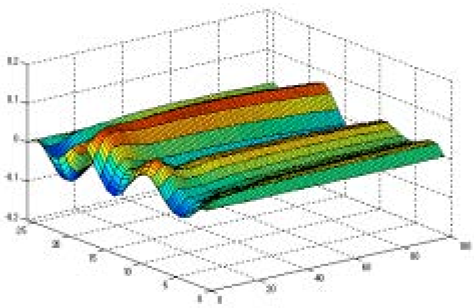

Fig. 5: Prediction distributed error of grey-box model on testing

predicted output and distributed error of the Burgers equation can be estimated as shown in Fig. 4 and Fig. 5, respectively.

CONCLUSIONS

A grey-box modeling methodology using new basis functions is proposed the reduction of spatially distributed processes. The methodology amounts to find low-order substitute models for the processes which can balance the dimension, computational efficiency and the accuracy. A series of new spatial basis functions is developed by basis functions transformation. Model reduction is pursued by time/space separation and projection on the set of new basis functions. Subsequently, an empirical component is identified to substitute the nonlinear terms and un-modeled dynamics. As a consequence, a grey-box model has been developed to approximate the spatially distributed processes. The simulations show that the combination of the model reduction and black-box identification applied to the linear and nonlinear parts of the mechanistic models results in a grey-box model which have less modes and high accuracy.

ACKNOWLEDGEMENTS

This project is supported by National Natural Science Foundation of China (Grant No. 51305133). Financial Support from National Natural Science Foundation of Hunan province (13JJB007), Project of The Education Department of Hunan Province (13A023, 13C317), Aid program for Science and Technology Innovative Research Team in Higher Educational Institutions of Hunan Province are also gratefully acknowledged.

REFFERENCES

Alaradi, A. and S. Rohani, 2002. Identification and control of a riser-type FCC unit using neural networks. Comput. Chem. Eng., 26: 401-421.

Brendel, M., A. Mhamdi, D. Bonvin and W. Marquardt, 2004. An incremental approach for the identification of reaction kinetics. Proceedings of the 7th International Symposium on Advanced Control of Chemical Processes, January 11-14, 2004, Hong Kong, China, pp: 177-182.

Deng, H., H.X. Li and G. Chen, 2005. Spectral-approximation-based intelligent modeling for distributed thermal processes. IEEE Trans. Control Syst. Technol., 13: 686-700.

Pearson, R.K. and M. Pottmann, 2000. Gray-box identification of block-oriented nonlinear models. J. Process Control, 10: 301-315.

Psichogios, D.C. and L. H. Ungar, 1992. A hybrid neural network-first principles approach to process modeling. AIChE J., 30: 1499-1511.

Qi, C. and H.X. Li, 2009. A time/space separation-based Hammerstein modeling approach for nonlinear distributed parameter processes. Comput. Chem. Eng., 33: 1247-1260.

Qi, C., H.T. Zhang and H.X. Li, 2009. A multi-channel spatio-temporal Hammerstein modeling approach for nonlinear distributed parameter processes. J. Process Control, 19: 85-99.