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Stochastic Resonance and Noise Enhancing Signal Transmission

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Abstract: We study the Stochastic Resonance (SR) in the maximizing network with threshold devices and get results on noise enhancing information transmission. Here the input signals are discrete and the results may be useful for code symbols transmission. With proper thresholds, mutual information between input and output signals increases at first as (additive or multiplicative) noise intensity increases and there is a maximal value when noise intensity reaches the optimum. In addition, larger number of threshold devices leads to better efficacy of SR and the maximum of mutual information goes upwards which means a further enhancement of information transmission. At last, we give the optimal combination of additive and multiplicative noise intensity for the maximal mutual information in the global region. In this case, the efficacy of information transmission is improved the most.

Key words: Stochastic resonance, noise intensity, mutual information, transmission

INTRODUCTION

The study of Stochastic Resonance (SR) has lasted for a long time and many results have been achieved (Collins *et al.*, 1995; Jung, 1995; Gammaitoni, 1995; Bulsara and Zador, 1996; Gammaitoni *et al.*, 1994; Gammaitoni *et al.*, 1998; Stocks, 2001a, 2001b; McDonnell *et al.*, 2002). On one hand, different forms of noise are chosen and added into systems to improve systems' performance or enhance information transmission (Bulsara and Zador, 1996; Stocks, 2001b; Wang and Wu, 2005). On the other hand, SR in many kinds of models has been discussed systematically, too (Gammaitoni, 1995; Stocks, 2001a, 2001b; McDonnell *et al.*, 2002). And there are many applications exploiting SR such as sonar arrays, digital-to-analog converters (Stocks, 2001a; McDonnell *et al.*, 2002) and so on.

However, many researches on SR have been assuming there is mere additive or multiplicative noise in systems (Stocks, 2001a, 2001b; Nikitin *et al.*, 2007). Recently, attention is also paid to the situation where additive and multiplicative noises exist at the same time (Guo, 2009; Lv, 2013), because both of the two may affect systems simultaneously. And we discuss SR with both additive and multiplicative noise, too.

MODEL

In our study, we consider a maximizing network of threshold devices whose input signal is discrete. And our results may be useful for coding and the transmission of symbols. The maximizing network of threshold devices is such a multi-threshold model as Fig. 1 shows.

There are N threshold units. And they are subject to the same input signal and independent additive and multiplicative noises.

The input signal x is discrete, whose value is among $\{-m, -m+1,\ldots, 0,\ldots, m-1, m\}$. And λ is a parameter dictating the value of the deterministic signal component. η_i and $\zeta_i (i = 1, 2,\ldots, N)$ are mutually independent standard Gaussian noise, i.e., $\langle\eta_i, \zeta_j\rangle$ and when $i \neq j$, $\langle\eta_i, \eta_j\rangle = 0$, $\langle\zeta_i, \zeta_j\rangle = 0$. And then the input to each threshold device is:

$$z_i = \lambda x + A\eta_i + M\zeta_i x \tag{1}$$

where, A and M are the intensity of additive and multiplicative noise.

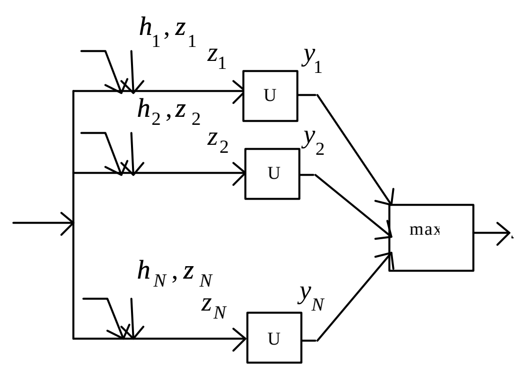

Fig. 1: Maximizing network with N threshold devices

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In each threshold device, U acts as a quantization step determining the interval between two adjoining thresholds and there are 2 m thresholds from $\left(-m+\frac{1}{2}\right)U$ to $\left(m-\frac{1}{2}\right)U$. When inputting z_i, the output of each device y_i can be obtained as follows (Gammaitoni, 1995):

$$y_i = \begin{cases} -m & z_i < (-m+\frac{1}{2})U \\ k(-m<k<m, k\in Z) & (k-\frac{1}{2})U < z_i < (k+\frac{1}{2})U \\ m & z_i > (m-\frac{1}{2})U \end{cases} \tag{2}$$

The output of the system y is the maximal y_i, $y = \max_{1\le i\le N} y_i$.

Since, η_i and ζ_i are mutually independent, the conditional probability density of z_i is Gaussian with a mean of λx and a deviation of $A^2+M^2x^2$ for a given x:

$$p_{z_i|x}(z_i \mid x) = \frac{1}{\sqrt{2\pi(A^2+M^2x^2)}}\exp(-\frac{(z_i-\lambda x)^2}{2(A^2+M^2x^2)}) \tag{3}$$

And according to Eq. 2, probability of each y_i with a determinate x can be calculated as this: for $-m+1 \le k \le m-1$:

$$\begin{aligned} P(y_i = k \mid x) &= P((k-0.5)U < z_i < (k+0.5)U \mid x) \\ &= \int_{(k-0.5)U}^{(k+0.5)U} p_{z_i|x}(z_i \mid x)dz_i \\ &= 0.5\mathrm{erf}(\frac{(k+0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}}) - 0.5\mathrm{erf}(\frac{(k-0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}}) \end{aligned} \tag{4}$$

For k = m:

$$\begin{aligned} P(y_i = m \mid x) &= P(z_i > (m-0.5)U \mid x) \\ &= 0.5 - 0.5\mathrm{erf}(\frac{(m-0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}}) \end{aligned} \tag{5}$$

and for k = -m:

$$\begin{aligned} P(y_i = -m \mid x) &= P(z_i < (-m+0.5)U \mid x) \\ &= 0.5 + 0.5\mathrm{erf}(\frac{(-m+0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}}) \end{aligned} \tag{6}$$

where, erf is an error function defined as:

$$\mathrm{erf}(x) = \int_0^x \frac{2}{\sqrt{\pi}}\exp(-t^2)dt$$

Noting that y is also one member of {-m, -m+1,…, 0,…, m-1, m}:

$$\begin{aligned} P(y = k \mid x) &= P(\max_{1\le i\le N} y_i = k \mid x) \\ &= P(\max_{1\le i\le N} y_i \le k \mid x) - P(\max_{1\le i\le N} y_i \le k-1 \mid x) \\ &= \prod_{1\le i\le N} P(y_i \le k \mid x) - \prod_{1\le i\le N} P(y_i \le k-1 \mid x) \end{aligned}$$

Deducing from Eq. 4-6, we have corresponding results about y.

For $-m+1 \le k \le m-1$, P (y = k | x)

$$\begin{aligned} &= [0.5+0.5\mathrm{erf}(\frac{(k+0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}})]^N \\ &-[0.5+0.5\mathrm{erf}(\frac{(k-0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}})]^N \end{aligned} \tag{7}$$

for k = m, P(y = m|x):

$$= 1-[0.5+0.5\mathrm{erf}(\frac{(m-0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}})]^N \tag{8}$$

and for k = − m, P (y = − m | x):

$$= [0.5+0.5\mathrm{erf}(\frac{(-m+0.5)U-\lambda x}{\sqrt{2(A^2+M^2x^2)}})]^N \tag{9}$$

By Eq. 7-9, the probability of y = k is:

$$P(y = k) = \sum_{h=-m}^{m} P(y = k \mid x = h)P(x = h) \tag{10}$$

RESULTS

Mutual information: The mutual information (Shannon, 1948) between input and output signals is expressed as:

$$I(X; Y) = H(Y)-H(Y|X) \tag{11}$$

where, H(Y) is the output entropy and H(Y|X) is the output conditional entropy. In our model:

$$H(Y) = -\sum_{k=-m}^{m} P(y = k)\log_2 P(y = k)$$

and:

$$H(Y \mid X) = -\sum_{h=-m}^{m} P(x = h)\sum_{k=-m}^{m} P(y = k \mid x = h)\log_2 P(y = k \mid x = h)$$

Here, we consider the input signal is uniform. This condition is common especially when the input signal is binary. So, for each x = h:

$$P(x=h)=\frac{1}{2m+1}$$

And then Eq. 11 has a simplified form I(X; Y):

$$=\frac{1}{2m+1}\sum_{k=-m}^{m}\sum_{h=-m}^{m}P(y=k\mid x=h)\log_2\frac{P(y=k\mid x=h)}{P(y=k)} \qquad (12)$$

Stochastic resonance in the system: Let $\lambda = 1$ and $m = 7$, Fig. 2-3 show the curves of mutual information I as noise intensity increasing. And these points around or on the curves are obtained by simulation.

Figure 2 describes that I decreases monotonously as noise intensity increases in case of small U (U = 7 in (a) and U = 7,15 in (b)) which implies the absence of SR. While for a large U (and every threshold also becomes large since threshold is a multiple of U), SR occurs and is reflected by convex curves in the figures. Increasing the value of U, mutual information rises at first to the maximum, and then decreases. Here, the noise intensity with which mutual information reaches its peak is optimal.

Both two of Fig. 2 indicate that a larger U demands stronger optimal noise. However, there is an essential difference between them. In Fig. 2a, though the optimal noise intensity increases when enlarging U, the maximal value of mutual information nearly stays the same. On the contrary, (b) shows obviously that peak value of mutual information curve drops a lot and when U rises up to 63, the maximum approximates to zero. So, the efficacy of SR weakens and signals are almost buried in strong additive noise. In some degree, these phenomena demonstrate the system is affected more greatly by additive than multiplicative noise.

Figure 3 depicts images of I as a function of noise intensity with a fixed U and various N.

It's clear that efficacy of SR is enhanced with more threshold devices reflecting by the improvement of maximal mutual information. In addition, the optimal noise intensity reduces due to the rise of N which means a better effect may emerge by using less noise. Similarly, we can also draw the conclusion that additive noise plays a more notable impact on this system, for a little additive noise may promote it to an optimal level.

As Fig. 2-3 display, strong noise exceeding the optimum will diminish information. And efficacy of the system turns to the worse. So it is significant to determine the combination of optimal noises. Generally, we can get extremum of I by derivation of Eq. 12. That is to say, the root of equations:

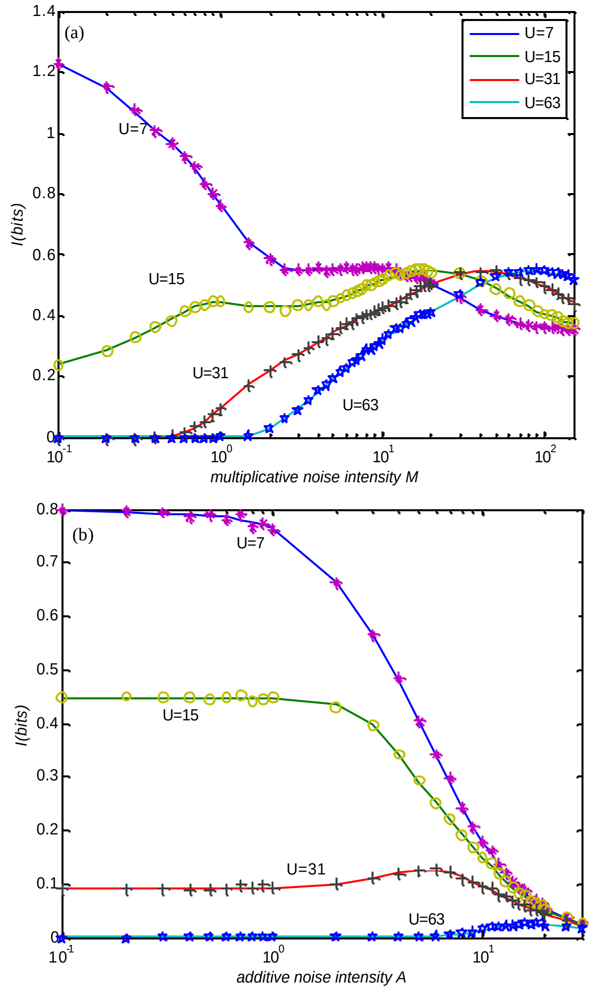

Fig. 2(a-b): Mutual information I as a function of (a) multiplicative noise intensity M (fixed A = 1); (b) additive noise intensity A (fixed M = 1) with various U when N = 3 maximum and then decreases. Here, the noise intensity with which mutual information reaches its peak is optimal

$$\begin{cases}\dfrac{\partial I}{\partial M}=0\\[2mm]\dfrac{\partial I}{\partial A}=0\end{cases}\quad (*)\ (M_0, A_0)$$

is likely to satisfy the maximal I. However that may not work all the time. Figure 4 shows the image of I as a function of two variables. In this case, U = 31, N = 15, m = 7, λ = 1 and multiplicative and additive noise intensities increase at the same time.

From Fig. 4 we can't get M_0 and A_0 fit for (*) at the same time. But it is obvious that the maximal I is situated

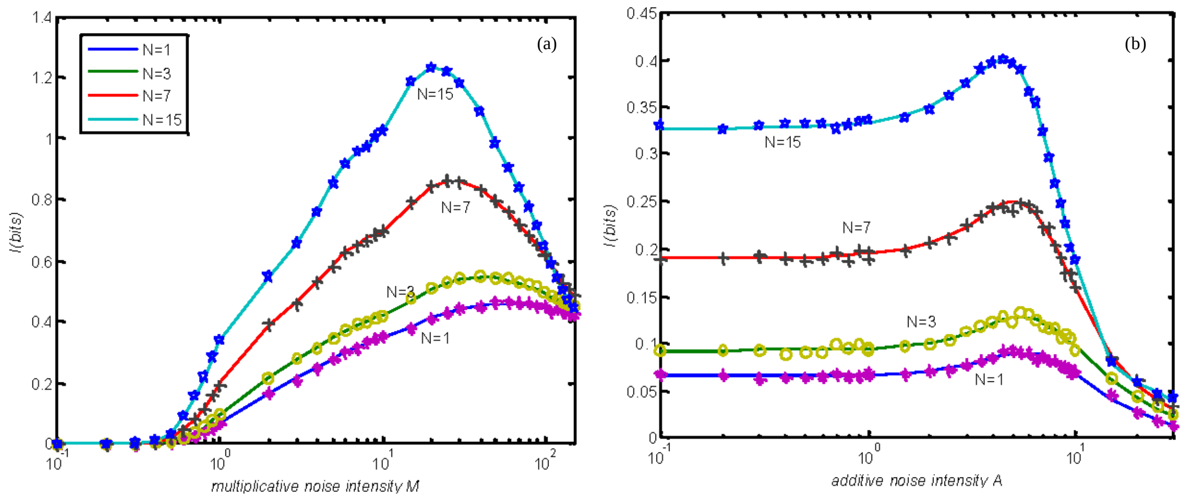

Fig. 3(a-b): Mutual information I as a function of (a) Multiplicative noise intensity M (fixed A = 1) and (b) Additive noise intensity A (fixed M = 1) with various N when U = 31

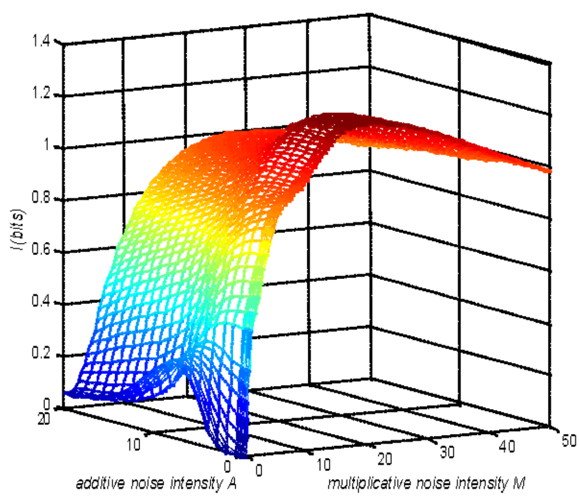

Fig. 4: Mutual information I as a function of multiplicative and additive noise intensities M, A

on A = 0. Incidentally, Fig. 4 indicates that for almost any fixed additive noise intensity, SR will appear by adding multiplicative noise and that for some multiplicative noise intensity, SR won't occur by adding additive noise. These indicate the effectiveness of multiplicative noise.

Since, the combination of optimal noises is located in the curve A = 0. In Eq. 12, let A = 0 and the optimal M can be given by computer: $M_0 = 20.17$. With ($M_0 = 20.17$, $A_0 = 0$), I reaches the maximum, $I_{max} = 1.2354$ bits. And the system in Fig.1 is at its best, where noise (multiplicative noise in fact) enhances information transmission to the utmost. Another way to get the optimal intensity is to search step by step in the global region for a maximal value of I. And this method gives the same result. So, it is available to quantify the noise to get the best effect. However, because of the complexity of systems and unknown of signals, we aren't aware of the exact quantity usually in fact, though it may well exists. And it is necessary to do further investigations.

CONCLUSION

In this study, we study the phenomenon of SR in a multi-threshold system. Input signals are discrete and so are output ones. When U, the interval between thresholds, is small, SR doesn't exist. And mutual information as a function of noise intensity decreases monotonously. Enlarging U, the curve of I becomes convex and there is a peak value denoting the optimal state of the system. Besides, increasing the number of threshold devices can improve the efficacy of SR. And thes maximal mutual information goes upwards which means information transmission is enhanced. What's more, it is easier for mutual information to increase to the maximum by adding additive noise than multiplicative noise. Finally, we give optimal combination of noise intensity to enhance information transmission most greatly.

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