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ITJ

ISSN 1812-5638

INFORMATION TECHNOLOGY JOURNAL

ANSInet
Asian Network for Scientific Information
308 Lasani Town, Sargodha Road, Faisalabad - Pakistan

Information Technology Journal 12 (23): 7476-7480, 2013
ISSN 1812-5638 / DOI: 10.3923/itj.2013.7476.7480

An Improved Leaf Model based on Chaos

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Abstract: There is a strong correlation between the leaf shapes and the venation patterns. A model is needed that can produce these shapes with a set of parameters. This study presents an improved model of leaf shape based on chaos theory, which governs the growth rate of leaf tissue in two dimensions, hences the outline of the leaf and exhibits chaotic behaviour. It is more suitable in biology background and the improved model can be extended to more complex leaf types.

Key words: Venation patterns, leaf model, chaos, runions model

INTRODUCTION

When you walk into the forest, bent down and picking up leaves, you will be surprised to find that no two leaves are the same. When you are astonished at the innumerable shapes of tree leaves in nature, it is high time for you to explore the leaf world. Why the shape is countless? It is seems to be a simply question, but actually it is still a challenge to the explanatory power of biophysical theory. It is perhaps now time for a more convinced model to explain the variety of shapes.

Theoretical studies are valuable in providing models of pattern formation that stimulate new research and that can be tested against experimental data (Meinhardt, 1982; Murray, 1989). Theory offers the promise of bridging the very large gap between the expression of genes and the final shape of an organ. There are a number of divergent hypotheses explaining leaf form. One of the earliest is the work of Thompson in his famous book, On Growth and Form. Other models include the *Lindenmayer* L-system (Thompson, 1992), fractal analysis (Prusinkiewicz and Lindenmayer, 1990), a Turing reaction-diffusion process (Franks and Britton, 2000), an iterative space-filling branching process (Wolfram, 2002) and a linear force-relaxation model (Coen *et al*., 2004). The formation of the vein network of leaves, which is closely related to leaf growth, has also been studied in recent theoretical work (Runions *et al*., 2005; Dimitrov and Zucker, 2006; Young, 2010).

The research by Dengler and Kang (2001) has shown that there is a strong correlation between the leaf shapes and the venation patterns. In Fig. 1 the *Runions* model is a suitable model for solving this problem. However, in Fig. 2, the sidestream vein (level 2 vein stands for sidestream vein in the following) in the *Runions* model does not conform to the real leaf owing to the mainstream vein (level 1 vein stands for mainstream vein in the following) in the *Runions* model growing slowly (Young, 2010).

In this study we would like to propose an improved model based on *Runions* model and chaos theory. Chaos theory is a field of study in mathematics, with applications in several disciplines including physics, engineering, economics, biology and philosophy. Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions, an effect which is popularly referred to as the butterfly effect (Young, 2010).

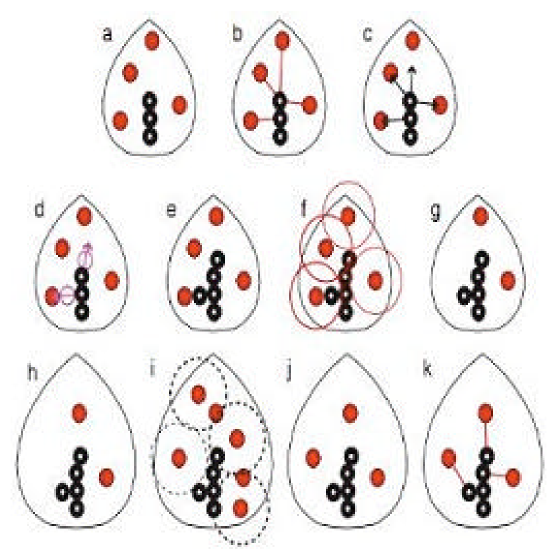

Fig. 1: Runions model (Runions *et al*., 2005)

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Fig. 2(a-b): Simulation Results of Vein Distribution, (a) is the original leaf and b is the result of the *Runions* model, We can see it clearly that the mainstream grows slowly and the sidestream grows so fast that it becomes distorted

MATERIALS AND METHODS

Assumption: The model described here was motivated by a small number of well-established facts. First we only take monocotyledon and dicotyledon into consideration. Second, leaf shape is not rigidly programmed by the genetics and there both certainty and uncertainty at the same time just like wave-particle duality-like 'particle' and 'wave' to fully describe the behavior of quantum-scale objects. Third, assume that the auxin and other inner factors play a major role in the growth of vein cells, whose total affect are called the nutrients. Practical application of model and development principles can be summarized as follows:

- **Principle 1:** There are equal the nutrients and primordium effect at every iteration, which means allowing leaf adequate rest intervals between next and close to utilize a high percentage of the growth but leaving adequate for vigorous new vein growth
- **Principle 2:** The distribution of the nutrients and primordium have the no effect on the growing speed and the growing speed has the no effect on the distribution of the nutrients and primordium. Having the notation in Table 1, this means $S_{(s)}$ and $V_{(v)}$ have the no effect to Dv1, Dv2 and D. And the Dv1, Dv2 and D have the no effect to $S_{(s)}$ and $V_{(s)}$ neither
- **Principle 3:** The mainstream should grow faster than the sidestream in general (VanCleave, 1993). In the model, corresponding, the value of Dv1 should be no smaller than Dv2 and the value of D should be smaller than Dv1 and Dv2 in Table 1

Table 1: Summary of the main mathematical and physical quantities considered in the development of the proposed models

Symbol	Description
Dv1	Level 1 vein growth spacing
Dv2	Level 2 vein growth spacing
D2	The threshold distance
D	The vein growth spacing in runions model
T	The number of iterations
$S_{(s)}$	The set of the leaf primordium
$V_{(v)}$	The set of vein nodes

Environment factor: On the one hand, we know that the environment will impact on the leaf shape (Dengler and Kang, 2001). We assume that the environment factor change the growth spacing so that the shape changes. According to the *Runions* model, all streams in a leaf share the same growth speed to grow which is apparently opposed to reality. That is the reason why mainstream grows slowly. It is relatively slow. VanCleave (1993) shows us the rate of leaf growth along the stem:

if we separate the growth spacing D into Dv1 for level 1 and Dv2 for level 2, we will get a better result.

Gene factor: On the other hand, the gene has a strong influence on it (Micol and Hake, 2003) and we would like to use logistic map, one of the classic methods in chaos theory, to reflect it. It is the key of this mode and it is known to all that the chaos system must be sensitive to initial conditions and topologically mixing and its periodic orbits must be dense. And the auxin also have such characteristics too. We dare to use the logistic map to describe the coordinates of the auxin.

Logistic map can be described as (1-5) and t_{loc} and l_{loc} are the coordinates of the auxin:

$$x = \mu x(1-x) \quad (1)$$

$$\mu = 3.9+now \quad (2)$$

$$0<now<0.1 \quad (3)$$

$$t_{loc} \propto xt_n \quad (4)$$

$$l_{loc} \propto xl_n \quad (5)$$

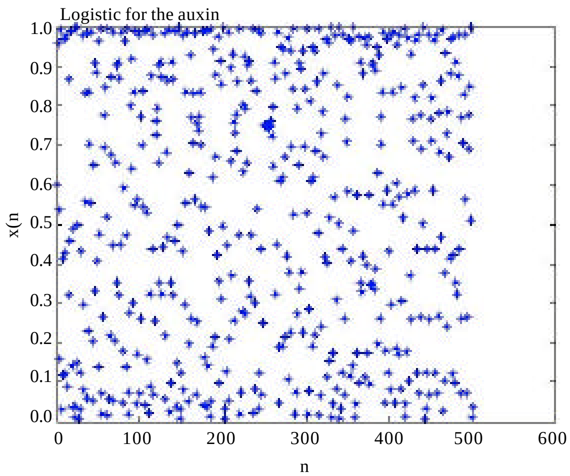

Fig. 3: μ = 3.9+now, x = 0.6, Logistic map for The Nutrients Distribution: We can see all the Nutrients is within 0-1 along x(n), which means certainty on the whole but uncertainty for a single. We can change value x to change distribution but we can't determine the location for random one. That's the reason why not controlled exactly by gene

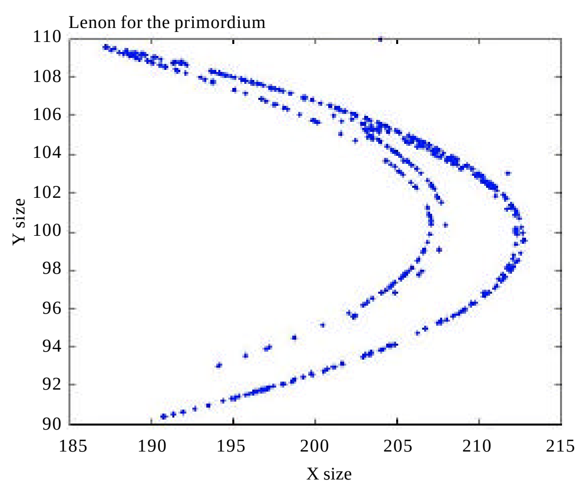

Fig. 4: Henon Map for The Primordium Distribution: The principle is the n-early same with Fig. 3. We set the value a and b to control the whole distribution but have on idea with random one

Each vertical slice shows the attractor for a specific value of μ. The diagram displays period doubling when μ increases, eventually producing chaos. now shows the data is different at the different time reflecting the random process. Therefore, in Fig. 3 we can apply the logistic map to the leaf model showing a state of disorder.

By observing the leaves in nature, we know that the vein grows from a smaller vein, whose name is leaf primordium. We use *Henon* map to control the primordium in Fig. 4. The *Henon* map is a discrete time dynamical system. It is one of the most studied examples of dynamical systems that exhibit chaotic behavior. The map depends on two parameters a and b, which for the classical Henon map have values of a = 1.4 and b = 0.3. For the classical values the Henon map is chaotic. The Henon map takes a point (x_n, y_n) in the plane and maps it to a new point according to (6) and (7). Then change the value according to the range of the primordium as (8) and (9) and k is integer. x_{loc} and y_{loc} are the coordinates of the primordium:

$$x_{n+1} = 1-ax^2_n+y_n \quad (6)$$

$$yn+1 = bx_n \quad (7)$$

$$x_{loc} \propto kx_n \quad (8)$$

$$Y_{loc} \propto ky_n \quad (9)$$

RESULTS

According to the features of the improved *Runions* model, the ways and means of achieving comprehensive and integrated are concluded as follow in Table 1 and Fig. 5. Firstly Initialize the collection of nutritional points and predefined nutrients point is placed in the leaf according to the chaos theory. Secondly the vein grows from the leaf primordium and the leaf primordium is also chosen according to the chaos theory. Thirdly each nutrient point is associated with the vain node that is closet to it and record their corresponding relations.

Then calculate the growth vector for each 'vein junction point' and grow a new vein nodes by the fixed spacing Dv1 and Dv2. Given the threshold distance D, we need to judge whether the nutritional point is covered by the vein; if within, delete this point.

The step can be summarized as follows:

- **Step 1:** Initialize nutrients points
- **Step 2:** Grow from the leaf primor-dium
- **Step 3:** Each point is associated with the vain node which is closest
- **Step 4:** Calculate the growth vector for each "vein junction point" and grow a new node by the fixed spacing Dv1, Dv2
- **Step 5:** If the nutritional point is covered by the vein, delete this point
- **Step 6:** Back step 2 and prepare the next cycle time

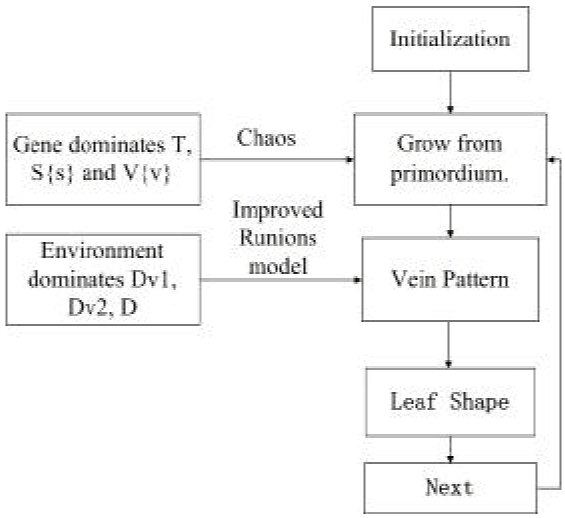

Fig. 5: Flow chart of the improved model: In this chart we can see the leaf is dominated by environment and gene from original in this model. Gene controls the number of iterations, the set of the leaf primordium and the set of vein nodes. We use the chaos theory to react it. Environment controls level 1 vein growth spacing, level 2 vein growth spacing and the threshold distance.We use the improved *Runions* model to form the vein pattern. This model try to explain the countless leaf shape from the modeling

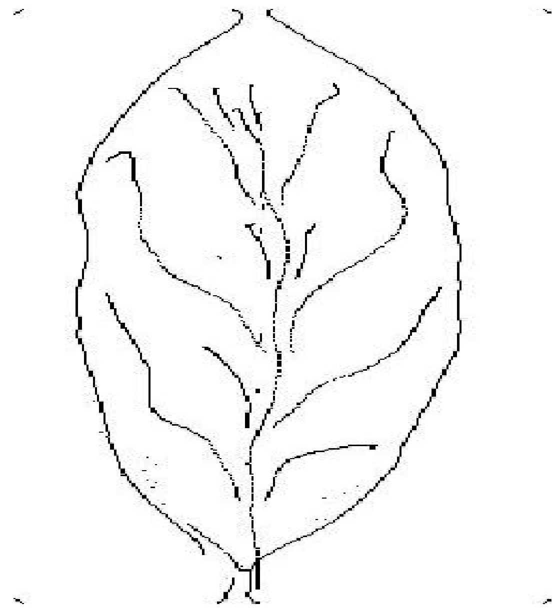

Fig. 6: Dv1 = 50, Dv2 = 50, T = 100 and in this picture the ratio between Dv1 and Dv2 is 1.0 which means Dv1 is equal to Dv2 and this is the same as the Runions model in Fig. 2b

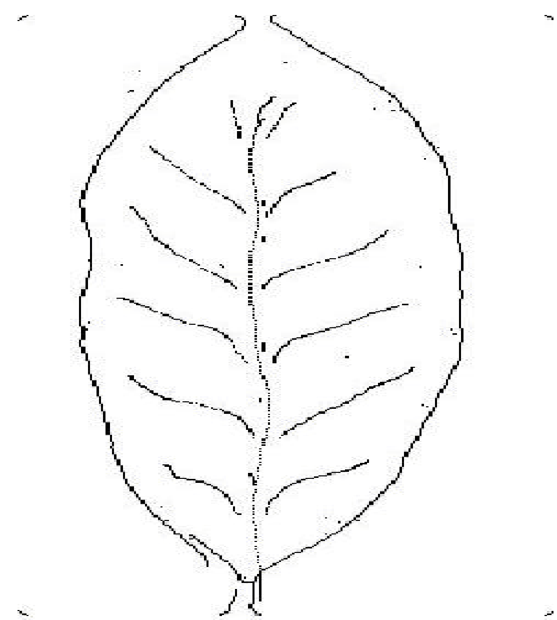

Fig. 7: Dv1 = 70, Dv2 = 50, T = 100 and in this picture ratio between Dv1 and Dv2 is 1.4, It becomes obviously that the level 2 is more similar to the original figure in Fig. 2a

From Fig. 6 to 8, these pictures show how the vein pattern changes along the Dv1 and the Dv2. Compared to the the *Runions* model, we find the result is closer to the real leaf Fig. 2a.

These figures above are all following the principles.

DISCUSSION

Tree leaves only: We just consider tree leaves in this study and we don't know whether it is suitable for other leaves or not. In the further research we will try to explain not only the range of leaf shapes found in the higher plants, but also the large variations in shape seen on closely related plants.

Chaos and distribution: This study only proposes an assumption and get a simulation model based on a tree leaf about the relationship between auxin and primordium distribution and chaos theory. It is not proved that the equations has strong relationship with the vein. Next, we will try to collect the large data about auxin and primordium distribution and make it sure.

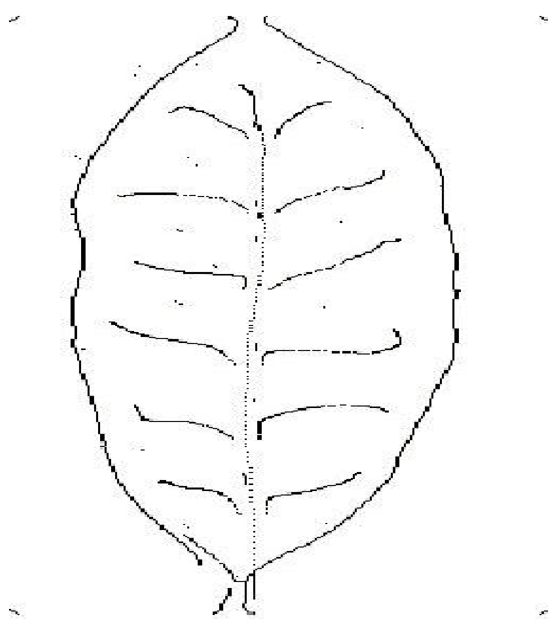

Fig. 8: Dv1 = 100, Dv2 = 50, T = 100 and in this picture ratio between Dv1 and Dv2 is 2.0. Level 2 in this figure is distorted but it is still strictly controlled under the D

Separate the growth spacing: One of the highlights is separating the growth spacing differently for Level 1 and Level 2. However, it is not evident whether there is a certain ratio between Level 1 and Level 2 for different kinds of shapes. It is need to be proved.

ACKNOWLEDGMENT

This study was supported by the Natural Science Foundation of China (11271104).

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